Reinforced Concrete

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Day 1

- Introduction
- Flexure
- Shear
- Columns
- Decks
Topics

Day 2
- Strut and Tie
- Retaining walls
- Footings
- Development (if time permits)

Day 3 (1/2 day)
- Review
- Quiz
Topics

Course covering

AASHTO LRFD Bridge Design Specifications, 3rd Edition, 2004

including 2005 and 2006 interim revisions

4th edition, 2007 presented where applicable

ODOT exemptions also presented
Topics

Sections within AASHTO LRFD

5. Concrete (R/C & P/C)
3. Loads and Load Factors
4. Structural Analysis and Evaluation
9. Decks and Deck Systems
11. Abutments, Piers and Walls
13. Railings
Compressive Strength (5.4.2.1)

- $f'_c > 10$ ksi used only when established relationships exist
- $f'_c < 2.4$ ksi not used for structural applications
- $f'_c < 4$ ksi not used for prestressed concrete and decks
Properties - Concrete

Modulus of Elasticity (5.4.2.4)

- For unit weights, \( w_c = 0.090 \) to \( 0.155 \) kcf and \( f'_c < 15 \) ksi

\[
E_c = 33,000 \times K_1 \times w_c^{1.5} \times f'_c
\]

(5.4.2.4-1)

where

- \( K_1 = \) correction factor for source of aggregate, taken as 1.0 unless determined by test.
- \( f'_c = \) compressive strength (ksi)

- For normal weight concrete \( (w_c = 0.145 \) kcf)

\[
E_c = 1,820 \times f'_c
\]

(C5.4.2.4-1)
Properties - Concrete

Modulus of Rupture, $f_r$, (5.4.2.6)

Used in cracking moment

- Determined by tests

  or

- Normal weight concrete (w/ $f'_c < 15$ ksi):
  
  • Crack control by distribution of reinforcement (5.7.3.4) & deflection / camber (5.7.3.6.2)
    
    $$f_r = 0.24 \sqrt{f'_c}$$
  
  • Minimum reinforcement (5.7.3.3.2)
    
    $$f_r = 0.37 \sqrt{f'_c}$$
  
  • Shear Capacity, $V_{ci}$
    
    $$f_r = 0.20 \sqrt{f'_c}$$
**Properties - Concrete**

**Modulus of Rupture (5.4.2.6)**

- For lightweight concrete:
  
  - Sand-lightweight concrete
    
    \[ f_r = 0.20 \sqrt{f'_c} \]
  
  - All-lightweight concrete
    
    \[ f_r = 0.17 \sqrt{f'_c} \]

*Note:* \( f'_c \) is in ksi for all of LRFD including \( \sqrt{f'_c} \)
Properties - Reinforcing Steel

General (5.4.3.1)

- \( f_y \leq 75 \text{ ksi} \) for design
- \( f_y \geq 60 \text{ ksi} \) unless lower value material approved by owner
Limit States

Service Limit State (5.5.2)

- Cracking (5.7.3.4)
- Deformations (5.7.3.6)
- Concrete stresses (P/C)
Limit States

Service Limit State (5.5.2) - Cracking

Distribution of Reinforcement to Control Cracking (5.7.3.4)

- Does not apply to deck slabs designed per 9.7.2 Empirical Design (Note: ODOT does not allow Empirical Design)

- Applies to reinforcement of concrete components in which tension in cross-section > 80 % modulus of rupture (per 5.4.2.6) at applicable service limit state load combination per Table 3.4.1-1

\[ f_r = 0.24 \sqrt{f'c} \]
Limit States

Service Limit State (5.5.2) - Cracking

Distribution of Reinforcement to Control Cracking (5.7.3.4)

Spacing, $s$, of mild steel reinforcement in layer closest to tension face shall satisfy:

$$s \leq \frac{700 \gamma_e}{\beta f_{ss}} - 2d_c$$  (5.7.3.4-1)

where:

- $\gamma_e = \text{exposure factor (0.75 for Class 2, 1.00 for Class 1)}$
- $f_{ss} = \text{tensile stress in steel reinforcement at service limit state (ksi)}$
- $d_c = \text{concrete cover from center of flexural reinforcement located closest to extreme tension fiber (in.)}$
**Limit States**

**Service Limit State (5.5.2) - Cracking**

Distribution of Reinforcement to Control Cracking (5.7.3.4)

\[
\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}
\]

- where:

\[h = \text{overall thickness} / \text{depth of component (in.)}\]
Limit States

**Service Limit State (5.5.2) - Cracking**

Class 2 exposure condition ($\gamma_e = 0.75$) - increased concern of appearance and/or corrosion *(ODOT - concrete bridge decks. Also 1” monolithic wearing surface not considered in $d_c$ and $h$)*

Class 1 exposure condition ($\gamma_e = 1.0$) - cracks tolerated due to reduced concerns of appearance and/or corrosion *(ODOT – all other applications unless noted)*

For $f_{ss}$, axial tension considered; axial compression may be considered

Effects of bonded prestressing steel may be considered. For the bonded prestressing steel, $f_s = stress beyond decompression calculated on basis of cracked section or strain compatibility
Limit States

**Service Limit State (5.5.2) - Cracking**

Min and max reinforcement spacing shall comply w/ 5.10.3.1 & 5.10.3.2, respectively.

Minimum Spacing of Reinforcing Bars (5.10.3.1)

- **Cast-in-Place Concrete (5.10.3.1.1)** - Clear distance between parallel bars in a layer shall not be less than:
  - 1.5 * nominal bar diameter
  - 1.5 * maximum coarse aggregate size
  - 1.5 in.

- **Multilayers (5.10.3.1.3)**
  - Bars in upper layers placed directly above those in bottom layer
  - Clear distance between layers $\geq 1.0$ in. or nominal bar diameter
  - Exception: Decks w/ parallel reinforcing in two or more layers w/ clear distance between layers $\leq 6.0$ in.
Limit States

Service Limit State (5.5.2) - Cracking

Maximum Spacing of Reinforcing Bars (5.10.3.2)

- Unless otherwise specified, reinforcement spacing in walls and slabs $\leq 1.5 \times$ member thickness or 18.0 in.
- Max spacing of spirals, ties, and temperature shrinkage reinforcement per 5.10.6, 5.10.7, and 5.10.8
**Limit States**

**Service Limit State (5.5.2) - Cracking**

T & S Steel: (5.10.8)

\[
A_s \geq \frac{1.30 \, b \, h}{2 \, (b + h) \, f_y} \quad 5.10.8 - 1
\]

\[
0.11 \leq A_s \leq 0.60 \quad 5.10.8 - 2
\]

where

- \(A_s\) = area of reinforcement in each direction and each face (in²/ft)
- \(b\) = least width of component (in)
- \(h\) = least thickness of component (in)
Limit States

**Service Limit State (5.5.2) - Cracking (5.7.3.4)**

For flanges of R/C T-girders and box girders in tension at service limit state, flexural reinforcement distributed over lesser of:

- Effective flange width, per 4.6.2.6
  - Interior beams (4.6.2.6) - least of:
    - ¼ effective span (span for simply supported or distance between permanent load inflection points for continuous spans)
    - 12* avg. slab depth + greater of (web thickness or ½ of girder top flange width)
    - Avg. spacing of adjacent beams
Limit States

Service Limit State (5.5.2) - Cracking (5.7.3.4)

- Exterior beams (4.6.2.6) - \( \frac{1}{2} \) of adjacent interior beam effective flange width + least of:
  - \( \frac{1}{8} \) effective span
  - \( 6 \times \) avg. slab depth + greater of (1/2 web thickness or 1/4 of girder top flange width)
  - Width of overhang
    - width = 1/10 of the average of adjacent spans between bearings
- If effective flange width > 1/10 span, additional longitudinal reinforcement shall be provided in the outer portions of the flange with area \( \geq 0.4\% \) of excess slab area
Limit States

Service Limit State (5.5.2) - Cracking (5.7.3.4)

If \( d_e > 3.0 \text{ ft.} \) for nonprestressed or partially P/C members:

- longitudinal skin reinforcement shall be uniformly distributed along both side faces for distance \( \frac{d_e}{2} \) nearest flexural tension reinforcement

- area of skin reinforcement \( A_{sk} \) (\( \text{in.}^2/\text{ft. of height} \)) on each side face shall satisfy:

\[
A_{sk} = 0.012 \left( d_e - 30 \right) \leq \frac{A_s + A_{ps}}{4} \quad (5.7.3.4-2)
\]

- where:
  - \( d_e \) = effective depth from extreme compression fiber to centroid of tension steel (\( \text{in.} \))
Limit States

Service Limit State (5.5.2) - Cracking (5.7.3.4)

- Max spacing of skin reinforcement $\leq \frac{d_e}{6}$ or 12.0 in.
- Skin reinforcement may be included in strength computations if strain compatibility analysis used to determine stresses in individual bars / wires
Limit States

Example - Skin Reinforcement

\[ d_e = 44.5'' \]

48”

7 #9’s

36”
Limit States

\[ A_s = 7(\# \, 9's) = 7 \, \text{in}^2 \]
\[ A_{SK} = 0.012 \, (d_e - 30) = 0.012 \, (44.5 - 30) = 0.174 \, \text{in}^2/\text{ft} \]

\[ \frac{A_{st}}{4} = \frac{7}{4} = 1.75 \, \text{in}^2 \]

Spacing:
\[ d_e/6 \text{ or } 12'' \Rightarrow (44.5)/6 = 7.42'' \text{ or } 12'' \]
\[ (7.42'' \text{ controls}) \Rightarrow \text{Say } 6'' \]
\[ \therefore \, \# \, 3 @ 6'' \, (0.22 \, \text{in}^2/\text{ft}) \]
Limit States

\[ 44.5" = d_e \]

\[ A_{sk} = #3's \ @ \ 6" \]

\[ 24" > \frac{d_e}{2} = 22.25" \]
Limit States

**Service Limit State (5.5.2)**

**Deformations (5.7.3.6)**

*General (5.7.3.6.1)*

- Provisions of 2.5.2.6 shall be considered
- Deck joints / bearings shall accommodate dimensional changes caused by loads, creep, shrinkage, thermal changes, settlement, and prestressing

**Deformations (2.5.2.6)**

*General (2.5.2.6.1)*

Bridges designed to avoid undesirable structural or psychological effects due to deformations

While deflection /depth limitations are optional large deviation from past successful practice should be cause for review

If dynamic analysis used, it shall comply with Article 4.7
Limit States

Service Limit State (5.5.2)

Criteria for Deflection (2.5.2.6.2)

The criteria in this section shall be considered optional, except for the following:

- Metal grid decks / other lightweight metal / concrete bridge decks shall be subject to serviceability provisions of Article 9.5.2

In applying these criteria, the vehicular load shall include the dynamic load allowance.

If an Owner chooses to invoke deflection control, the following principles may be applied:

- When investigating max. absolute deflection for straight girder system, all design lanes loaded and all supporting components assumed to deflect equally
Limit States

Service Limit State (5.5.2)

Criteria for Deflection (2.5.2.6.2)

(cont)

• For composite design, stiffness of design cross-section used for the determination of deflection should include the entire width of the roadway and the structurally continuous portions of the railings, sidewalks, and median barriers

  ODOT – Do not include stiffness contribution of railings, sidewalks, and median barriers

• For straight girder systems, the composite bending stiffness of an individual girder may be taken as the stiffness determined as specified above, divided by the number of girders
Limit States

Service Limit State (5.5.2)

Criteria for Deflection (2.5.2.6.2)

(cont)

- When investigating max. relative displacements, number and position of loaded lanes selected to provide worst differential effect
- Live load portion of Load Combination Service I used, including dynamic load allowance, IM
- Live load shall be taken from Article 3.6.1.3.2
- For skewed bridges, a right cross-section may be used, and for curved skewed bridges, a radial cross-section may be used
Service Limit State (5.5.2)

Criteria for Deflection (2.5.2.6.2)

Required by ODOT

• In the absence of other criteria, the following deflection limits may be considered for concrete construction:

  • Vehicular load, general........................................Span/800
  • Vehicular and/or pedestrian loads.........................Span/1000
  • Vehicular loads on cantilever arms.......................Span/300
  • Vehicular and pedestrian loads on cantilever arms....Span/375
### Service Limit State (5.5.2)

Optional Criteria for Span-to-Depth Ratios (2.5.2.6.3)

*Required by ODOT*

#### Table 2.5.2.6.3-1 Traditional Minimum Depths for Constant Depth Superstructures

<table>
<thead>
<tr>
<th>Material</th>
<th>Superstructure</th>
<th>Simple Spans</th>
<th>Continuous Spans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced Concrete</td>
<td>Slabs with main reinforcement parallel to traffic</td>
<td>( \frac{1.2(S + 10)}{30} )</td>
<td>( \frac{S + 10}{30} ) ( \geq 0.54\text{ft} )</td>
</tr>
<tr>
<td></td>
<td>T-Beams</td>
<td>0.070 L</td>
<td>0.065 L</td>
</tr>
<tr>
<td></td>
<td>Box Beams</td>
<td>0.060 L</td>
<td>0.055 L</td>
</tr>
<tr>
<td></td>
<td>Pedestrian Structure Beams</td>
<td>0.035 L</td>
<td>0.033 L</td>
</tr>
</tbody>
</table>
Limit States

Service Limit State (5.5.2)

Optional Criteria for Span-to-Depth Ratios (2.5.2.6.3)

where

S = slab span length (ft.)
L = span length (ft.)

limits in Table 1 taken to apply to overall depth unless noted
Limit States

Service Limit State (5.5.2)

Deflection and Camber (5.7.3.6.2)

- Deflection and camber calculations shall consider dead, live, and erection loads, prestressing, concrete creep and shrinkage, and steel relaxation
- For determining deflection and camber, 4.5.2.1 (Elastic vs. Inelastic Behavior), 4.5.2.2 (Elastic Behavior), and 5.9.5.5 (nonsegmental P/C) shall apply
Limit States

Service Limit State (5.5.2) - Deformations (5.7.3.6)

In absence of comprehensive analysis, instantaneous deflections computed using the modulus of elasticity for concrete as specified in Article 5.4.2.4 and taking moment of inertia as either the gross moment of inertia, \( I_g \), or an effective moment of inertia, \( I_e \), given by Eq. 1:

\[
I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g \quad (5.7.3.6.2-1)
\]
Limit States

Service Limit State (5.5.2) - **Deformations (5.7.3.6)**

in which:

\[ M_{cr} = f_{r} \frac{g}{y_{t}} \]  \hspace{1cm} (5.7.3.6.2-2)

where:

- \( M_{cr} = \) cracking moment (kip-in.)
- \( f_{r} = \) concrete modulus of rupture per 5.4.2.6 - \( f_{r} = 0.24 \sqrt{f'_{c}} \) (ksi)
- \( y_{t} = \) distance from the neutral axis to the extreme tension fiber (in.)
- \( M_{a} = \) maximum moment in a component at the stage for which deformation is computed (kip-in.)
**Limit States**

**Service Limit State (5.5.2) - Deformations (5.7.3.6)**

Effective moment of inertia taken as:

- For prismatic members, value from Eq. 1 at midspan for simple or continuous spans, and at support for cantilevers
- For continuous nonprismatic members, average values from Eq. 1 for critical positive and negative moment sections

Unless more exact determination made, long-term deflection may be taken as instantaneous deflection multiplied by following factor:

- If instantaneous deflection based on $I_g$: 4.0
- If instantaneous deflection based on $I_e$: $3.0 - 1.2(A'_s / A_s) \geq 1.6$
Service Limit State (5.5.2) - Deformations (5.7.3.6)

Axial Deformation (5.7.3.6.3)

- Instantaneous shortening / expansion from loads determined using modulus of elasticity at time of loading
- Instantaneous shortening / expansion from temperature determined per Articles:
  - 3.12.2 (Uniform Temp) \((\text{ODOT} – \text{Procedure A for cold climate})\)
  - 3.12.3 (Temp gradient)
  - 5.4.2.2 \((\mu = 6\times10^{-6}/\text{oF})\)
- Long-term shortening due to shrinkage and creep determined per 5.4.2.3
Limit States

Fatigue Limit State (5.5.3)

General (5.5.3.1)

- Not applicable to concrete slabs in multi-girder applications
- Considered in compressive stress regions due to permanent loads, if compressive stress < 2 * max tensile live load stress from the fatigue load combination (Table 3.4.1-1 and 3.6.1.4)
- Section properties shall be based on cracked sections where tensile stress (due to unfactored permanent loads and prestress and 1.5 * the fatigue load) > 0.095 $\sqrt{f'_{c}}$,
- Definition of high stress region for application of Eq. 1 for flexural reinforcement, taken as 1/3 of span on each side of section of maximum moment
Limit States

**Fatigue Limit State (5.5.3)**

**Reinforcing Bars (5.5.3.2)**

- Stress range in straight reinforcement from fatigue load shall satisfy:

\[
 f_f \leq 21 - 0.33 f_{\text{min}} + 8 \left( \frac{r}{h} \right) \quad \text{2006 Interim}
\]

\[
 f_f \leq 24 - 0.33 f_{\text{min}} \quad \text{2007- 4th Edition (5.5.3.2-1)}
\]

where

- \( f_f \) = stress range (ksi)
- \( f_{\text{min}} \) = min live load stress combined w/ more severe stress from either permanent loads or permanent loads, shrinkage and creep-induced external loads (tension + and compressive -) (ksi)
Fatigue Limit State (5.5.3)

Welded or Mechanical Splices of Reinforcement (5.5.3.4)

- Stress range in welded or mechanical splices shall not exceed values below

<table>
<thead>
<tr>
<th>Type of Splice</th>
<th>$f_r$ for &gt; 1,000,000 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grout-filled sleeve, w/ or w/o epoxy coated bar</td>
<td>18 ksi</td>
</tr>
<tr>
<td>Cold-swaged coupling sleeves w/o threaded ends and w/ or w/o epoxy coated bar; Integrally forged coupler w/ upset NC threads; Steel sleeve w/ a wedge; One-piece taper-threaded coupler; and Single V-groove direct butt weld</td>
<td>12 ksi</td>
</tr>
<tr>
<td>All other types of splices</td>
<td>4 ksi</td>
</tr>
</tbody>
</table>
Fatigue Limit State (5.5.3)

Welded or Mechanical Splices of Reinforcement (5.5.3.4)

- where $N_{\text{cyc}} < 1\text{E}6$, $f_f$ may be increased to $24(6 - \log N_{\text{cyc}})$ ksi but not to exceed the value found in 5.5.3.2

- Higher values up to value found in 5.5.3.2 if verified by fatigue test data
Limit States

**Strength Limit State (5.5.4)**

**Resistance Factors (5.5.4.2)**

- $\Phi$ shall be taken as:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension-controlled section in RC</td>
<td>0.90</td>
</tr>
<tr>
<td>Tension-controlled section in PC</td>
<td>1.00</td>
</tr>
<tr>
<td>Shear and Torsion</td>
<td></td>
</tr>
<tr>
<td>Normal weight concrete</td>
<td>0.90</td>
</tr>
<tr>
<td>Lightweight concrete</td>
<td>0.70</td>
</tr>
<tr>
<td>Compression-controlled w/ spirals/ties</td>
<td>0.75</td>
</tr>
<tr>
<td>Bearing</td>
<td>0.70</td>
</tr>
<tr>
<td>Compression in Strut and Tie models</td>
<td>0.70</td>
</tr>
</tbody>
</table>
## Limit States

### Strength Limit State (5.5.4)

**Resistance Factors (5.5.4.2) (cont.)**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression in Anchorage zones</td>
<td></td>
</tr>
<tr>
<td>Normal weight concrete</td>
<td>0.80</td>
</tr>
<tr>
<td>Lightweight concrete</td>
<td>0.65</td>
</tr>
<tr>
<td>Tension in steel in anchorage zones</td>
<td>1.00</td>
</tr>
<tr>
<td>Resistance during pile driving</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Limit States

Strength Limit State (5.5.4)

Resistance Factors (5.5.4.2)

• Tension-controlled:
  – extreme tension steel strain $\geq 0.005$ w/ extreme compression fiber strain $= 0.003$ ($\Phi = 0.9$ R/C)

• Compression-controlled
  – extreme tension steel strain $\leq$ its compression controlled strain limit as extreme compression fiber strain $= 0.003$. For Grade 60 reinforcement and all prestressing steel, compression controlled strain limit can be taken as 0.002 ($\Phi = 0.75$)

• Transition region
  – tension strains between the tension and compression controlled limits (5.7.2.1)
Limit States

**Strength Limit State (5.5.4)**

- \( c \) ≤ 0.002
- \( 0.002 < \varepsilon < 0.005 \)
- \( \geq 0.005 \)

- Compression - controlled
- Transition
- Tension - controlled
Limit States

**Strength Limit State (5.5.4)**

\[
\frac{c}{d_t} \leq \frac{0.003}{0.003 + 0.005} = 0.375 \quad \text{Tension – controlled} \quad (\Phi = 0.9)
\]

\[
\frac{c}{d_t} \geq \frac{0.003}{0.003 + 0.002} = 0.6 \quad \text{Compression – controlled} \quad (\Phi = 0.75)
\]

\[0.375 < \frac{c}{d_t} < 0.6 \quad \text{Transition}\]
Limit States

**Strength Limit State (5.5.4)**

![Diagram of a structural component with dimensions de and dt labeled.]
Limit States

**Strength Limit State (5.5.4)**

*Resistance Factors, Φ (5.5.4.2)*

- R/C sections in transition region (Grade 60 only!):

\[
0.75 \leq \phi = 0.65 + 0.15 \left( \frac{d}{t} - 1 \right) \leq 0.9
\]  
(5.5.4.2.1-2)

- P/C sections in transition region:

\[
0.75 \leq \phi = 0.583 + 0.25 \left( \frac{d}{t} - 1 \right) \leq 1.0
\]  
(5.5.4.2.1-1)
Limit States

R/C:
Strain = 0.004
\( \phi = 0.85 \)

Grade 60
Limit States

Strength Limit State (5.5.4)

Stability (5.5.4.3)

- Whole structure and its components shall be designed to resist
  - Sliding
  - Overturning
  - Uplift
  - Buckling
Limit States

**Extreme Event Limit State (5.5.5)**

- Entire structure and components designed to resist collapse due to extreme events (earthquake and vessel/vehicle impact)
Flexure
Flexure

Assumptions for Service & Fatigue Limit States (5.7.1)

- Concrete strains vary linearly, except where conventional strength of materials does not apply.
- Modular ratio, $n$, is
  - $E_s/E_c$ for reinforcing bars
  - $E_p/E_c$ for prestressing tendons
- Modular ratio rounded to nearest integer.
- Effective modular ratio of $2n$ is applicable to permanent loads and Prestress.
Assumptions for Strength & Extreme Event Limit States (5.7.2)

General (5.7.2.1)

- For fully bonded reinforcement or prestressing, strain directly proportional to distance from neutral axis, except in deep members or disturbed regions
- Maximum usable concrete strain:
  - $\leq 0.003$ (unconfined) (as in Std. Spec)
  - $\geq 0.003$ (confined, if verified)
- Factored resistance shall consider concrete cover lost
Assumptions for Strength and Extreme Event Limit States (5.7.2)

General (5.7.2.1)

- Stress in reinforcement based on stress-strain curve of steel except in strut-and-tie model
- Concrete tensile strength neglected
- Concrete compressive stress-strain assumed to be rectangular, parabolic, or any other shape that results in agreement in the prediction of strength
- Compression reinforcement permitted in conjunction with additional tension reinforcement to increase the flexural strength
Assumptions for Strength and Extreme Event Limit States (5.7.2)

Rectangular Stress Distribution (5.7.2.2)

- $\varepsilon_S$
- $\varepsilon_s$
- $c$
- $d_e$
- $a$
- $0.003$
- $0.85f_c$
- $f_s$
- $NA$
Assumptions for Strength and Extreme Event Limit States (5.7.2)

Rectangular Stress Distribution (5.7.2.2)

where (as in Std. Spec)

\[ c = \text{distance from extreme comp. fiber to neutral axis, NA} \]

\[ d_e = \text{distance from extreme comp. fiber to centroid of tension steel} \]

\[ a = \text{depth of concrete compression block} = \beta_1 c \]

\[ \beta_1 = \begin{align*}
0.85 & \text{ for } f'c \leq 4 \text{ ksi} \\
1.05 - 0.05 f'c & \text{ for } 4 \leq f'c \leq 8 \text{ ksi} \\
0.65 & \text{ for } f'c \geq 8 \text{ ksi}
\end{align*} \]
Flexure

Flexural Members (5.7.3)

Components w/ Bonded Tendons (5.7.3.1.1)

- Depth from compression face to NA, c:
  - For T- sections

\[
c = \frac{A_{ps} f_{pu} + A_{ss} f_{ss} - A'_{ss} f'_{ss} - 0.85 f'_{c} \left( b - b_{w} \right)_{h} f_{c}^{f}}{0.85 \beta_{1} f_{p}^{d} b_{w} + k A_{ps} \frac{d_{p}}{w}}
\]  

(5.7.3.1.1-3)
Flexural Members (5.7.3)

Components w/ Bonded Tendons (5.7.3.1.1)

- For rectangular section behavior:

\[
c = \frac{A_f \frac{f_{ps}}{s} + A_f \frac{f_{pu}}{s} - A'_f \frac{f_{ps}}{s}}{0.85 \beta \frac{f'_{ps}}{b} \frac{1}{c} \frac{w}{w} + k A \frac{f_{pu}}{d}}
\]

\[(5.7.3.1.1-4)\]
**Flexural Members (5.7.3)**

- where
  - $A_{ps} =$ area of prestressing steel
  - $f_{pu} =$ tensile strength of prestressing steel
  - $A_s =$ area of mild tension reinforcement
  - $A'_s =$ area of compression reinforcement
  - $f_s =$ stress in mild tension reinforcement at nominal resistance
  - $f'_s =$ stress in mild compression reinforcement at nominal resistance
  - $b =$ width of compression flange
  - $b_w =$ width of web
  - $h_f =$ height of compression flange
  - $d_p =$ distance from the extreme compression fiber to the centroid of the prestressing steel
Flexural Members (5.7.3)

Flexural Resistance (5.7.3.2)

• Factored resistance, $M_r$, is:

$$M_r = \Phi M_n \quad (5.7.3.2.1-1)$$

– where

• $M_n = \text{nominal resistance}$

• $\Phi = \text{resistance factor} \Rightarrow 0.9 \text{ Tension Controlled}$

 Transition

$0.75 \text{ Compression controlled}$
Flexure

**Flexural Members (5.7.3)**

**Flexural Resistance (5.7.3.2)**

- Flanged sections where \( a = \beta_1 c > \) compression flange depth:

\[
M_n = A_{ps} f_{ps} \left( \frac{d}{p} - \frac{a}{2} \right) + A_{ss} f_{ss} \left( \frac{d}{s} - \frac{a}{2} \right) - A_{fs} f_s \left( \frac{d'}{s} - \frac{a}{2} \right)
\]

\[
+ 0.85 f'_c \left( b - b_w \right) h_f \left( \frac{a}{2} - \frac{h_f}{2} \right)
\]

(5.7.3.2.2-1)

- where
  - \( f_{ps} \) = average stress in prestressing steel at nominal bending resistance
  - \( d_s \) = distance from extreme compression fiber to centroid of nonprestressed tensile reinforcement
  - \( d'_s \) = distance from extreme compression fiber to the centroid of compression reinforcement
Flexural Members (5.7.3)

General (5.7.2.1)

- $f_s$ can be replaced with $f_y$ in 5.7.3.1 and 5.7.3.2 if $c/d_s \leq 0.6$
- $f'_s$ can be replaced with $f'_y$ in 5.7.3.1 and 5.7.3.2 if $c \geq 3d'_s$
Flexural Members (5.7.3)

Flexural Resistance (5.7.3.2)

- Rectangular sections where $a = \beta_1 c <$ compression flange depth, set $b_w$ to $b$

Strain Compatibility (5.7.3.2.5)

- Strain compatibility can be used if more precise calculations required
Flexural Members (5.7.3)

Limits for Reinforcement (5.7.3.3)

- Max. reinforcement was limited based on:

  \[ \rho \leq 0.75 \rho_b \Rightarrow \frac{c}{d_e} \leq 0.45 \quad \text{Std. Spec} \]

  \[ \frac{c}{d_e} \leq 0.42 \quad \text{Prior to the 2006 Interim} \]

Requirement eliminated because reduced ductility of over-reinforced sections accounted for in lower \( \phi \) factors
Flexural Members (5.7.3)

Limits for Reinforcement (5.7.3.3)

- Amount of prestressed / nonprestressed tensile reinforcement sufficient to assure (as in Std. Spec):

\[ \phi M_n \geq 1.2 \times M_{cr} \] based on elastic stress distribution and modulus of rupture, \( f_r \), determined by:

\[ f_r = 0.37 \sqrt{f'} / c \]

If cannot be met, then

\[ \phi M_n \geq 1.33 \times M_u \]
Flexural Members (5.7.3)

Limits for Reinforcement (5.7.3.3)

\[
M_{cr} = S_c \left( f_r + f_{cpe} \right) - M_{dnc} \left( \frac{S_c}{S_{nc}} - 1 \right) \geq S_c f_r \tag{5.7.3.3.2-1}
\]

where

- \( f_{cpe} \) = concrete compressive stress due to effective prestress forces at extreme fiber of section
- \( M_{dnc} \) = total unfactored dead load acting on monolithic or noncomposite section
- \( S_c \) = composite section modulus
- \( S_{nc} \) = monolithic or noncomposite section modulus

**Note:** \( f_{cpe}, S_c, \) and \( S_{nc} \) found where tensile stress caused by externally applied loads
Determine the resistance flexure factor, $\phi$, for the beam provided below assuming:

- $f'_c = 4$ ksi
- $f_y = 60$ ksi
- $d_t = 20.5''$
- $d_e = 19.5''$
- $A_s = 8 \# 9$ Bars
Resistance Factor Example

24”

18”

d_e  d_t
Resistance Factor Example

\[ a = \frac{A f_y}{0.85 f'_c b} = \frac{(8 \text{ in}^2)(60 \text{ ksi})}{0.85 (4 \text{ ksi})(18 \text{ in})} = 7.84" \]

\[ c = \frac{a}{\beta_1} = \frac{7.84"}{0.85} = 9.23" \]

\[ \frac{0.003}{c} = \frac{0.003 + \varepsilon_s}{d_t} \]
Resistance Factor Example

\[ \varepsilon_s = \frac{d_t (0.003)}{t} - 0.003 \]

\[ \varepsilon_s = \frac{20.5''}{9.23''} (0.003) - 0.003 = 0.0037 < 0.005 \quad \therefore \phi \neq 0.9 \]

\[ > 0.002 \quad \therefore \text{Transition} \]

or

\[ \frac{c}{d} = \frac{9.23}{20.5} = 0.45 > 0.375 \quad \phi \neq 0.9 \]

\[ < 0.6 \quad \phi \neq 0.75 \]

Therefore, transition
Resistance Factor Example

\[ \phi = 0.65 + 0.15 \left( \frac{d}{t}c - 1 \right) \leq 0.9 \text{ but } \geq 0.75 \]

\[ = 0.65 + 0.15 \left( \frac{20.5''}{9.23''} - 1 \right) = 0.83 \]
Shear
Shear

Design Procedures (5.8.1)

Flexural Regions (5.8.1.1)

Shear design for plane sections that remain plane done using either:
  – Sectional model (5.8.3)
    or
  – Strut-and-tie model (5.6.3)

Deep components designed by strut-and-tie model (5.6.3) and detailed per 5.13.2.3

Components considered deep when
  – Distance from zero V to face of support < 2d
    or
  – Load causing > 1/2 of V at support is < 2d from support face
Shear

Design Procedures (5.8.1)

Regions Near Discontinuities (5.8.1.2)

- Plane sections assumption not valid
- Members designed for shear using strut-and-tie model (5.6.3) and 5.13.2 (Diaphragms, Deep Beams, Brackets, Corbels and Beam Ledges) shall apply

Slabs and Footings (5.8.1.4)

Slab-type regions designed for shear per 5.13.3.6 (shear in slabs and footing) or 5.6.3 (Strut and Tie)
Shear

General Requirements (5.8.2)

General (5.8.2.1)

- Factored shear resistance, $V_r$, taken as:

$$V_r = \phi V_n$$  \hspace{1cm} (5.8.2.1-2)

- where:
  - $V_n = \text{nominal shear resistance per 5.8.3.3 (kip)}$
  - $\phi = \text{resistance factor (0.9)}$
Shear

Sectional Design Model (5.8.3)

Nominal Shear Resistance (5.8.3.3)

• The nominal shear resistance, $V_n$, determined as lesser of:

\[ V_n = V_c + V_s + V_p \quad (5.8.3.3-1) \]

and

\[ V_n = 0.25 f'_c b v d_v + V_p \quad (5.8.3.3-2) \]
Shear

Sectional Design Model (5.8.3)

Nominal Shear Resistance (5.8.3.3)

- in which:

\[
V_c = 0.0316 \beta \sqrt{f'_{c,v} b d} \quad (5.8.3.3 - 3)
\]

if 5.8.3.4.1 or 5.8.3.4.2 is used or

lesser of \(V_{ci}\) and \(V_{cw}\) if 5.8.3.4.3 is used

\[
V_s = \frac{A_f d (\cot \theta + \cot \alpha) \sin \alpha}{s} \quad (5.8.3.3 - 4)
\]
Sectional Design Model (5.8.3)

Nominal Shear Resistance (5.8.3.3)

where:

\[ b_v = \text{effective web width within the depth } d_v \text{ per 5.8.2.9 (in.)} \]

- minimum web width \( \parallel \) to neutral axis between resultants of tensile and compressive forces due to flexure
- diameter for circular sections
- for ducts, 1/2 diameter of ungrouted or 1/4 diameter of grouted ducts subtracted from web width

\[ d_v = \text{effective shear depth per 5.8.2.9 (in.)} \]

- distance \( \perp \) to neutral axis between resultants of tensile and compressive forces due to flexure (internal moment arm)
- need not be taken \(<\) greater of 0.9 \( d_e \) or 0.72h

\[ s = \text{spacing of stirrups (in.)} \]
Sectional Design Model (5.8.3)

Nominal Shear Resistance (5.8.3.3)

where:

\[ A_v = \text{area of shear reinforcement within } s \text{ (in.}^2) \]

\[ V_p = \text{component in direction of applied shear of effective prestressing force; positive if resisting the applied shear (kip)} \]

\[ \alpha = \text{angle of transverse reinforcement to longitudinal axis (°)} \]

\[ \beta = \text{factor indicating ability of diagonally cracked concrete to transmit tension as specified in 5.8.3.4} \]

\[ \theta = \text{inclination angle of diagonal compressive stresses per 5.8.3.4 (°)} \]
Sectional Design Model (5.8.3)

Determination of $\beta$ and $\theta$ (5.8.3.4)

Simplified Procedure for Nonprestressed Sections (5.8.3.4.1)

For:

- Concrete footings in which distance from point of zero shear to face of column/pier/wall $< 3d_v$ w/ or w/o transverse reinforcement
- Other non P/C sections not subjected to axial tension and containing $\geq$ minimum transverse reinforcement per 5.8.2.5, or an overall depth of $< 16.0$ in.
  
  $\beta = 2.0$
  
  $\theta = 45^\circ$
Shear

Sectional Design Model (5.8.3)

Nominal Shear Resistance (5.8.3.3)

– becomes:

\[
V_c = 0.0316 \left( \frac{f_c b d}{\sqrt{v_c v_v}} \right) (5.8.3.3-3)
\]

and with \( \alpha = 90^\circ \) & \( \theta = 45^\circ \)

\[
V_s = \frac{A f d}{v y v_v} \frac{1}{s} (5.8.3.3-4)
\]
Sectional Design Model (5.8.3)

General (5.8.3.1)

In lieu of methods discussed, resistance of members in shear may be determined by satisfying:

- equilibrium
- strain compatibility
- using experimentally verified stress-strain relationships for reinforcement and diagonally cracked concrete

where consideration of simultaneous shear in a second direction is warranted, investigation based either on the principles outlined above or on 3-D strut-and-tie model.
Sectional Design Model (5.8.3)

Sections Near Supports (5.8.3.2)

- Where reaction force in the direction of applied shear introduces compression into member end region, location of critical section for shear taken $d_v$ from the internal face of the support (Figure 1).
- Otherwise, design section taken at internal face of support.
Shear

Critical Section for shear:

\[ dv = (A_{ps})(f_{ps}) + (A_{fy}) \]

For top bars (Varies)

For bottom bars (Varies)

Effective shear depth, \( dv \):

\[ dv = x \]

0.5\( dv \) cot(\( \theta \))

0.72h

0.90d_e

For top bars (Varies)
Sectional Design Model (5.8.3)

Sections Near Supports (5.8.3.2)

- Where beam-type element extends on both sides of reaction area, design section on each side of reaction determined separately based upon the loads on each side of the reaction and whether their respective contribution to total reaction introduces tension or compression into end region.

- For nonprestressed beams supported on bearings that introduce compression into member, only minimal transverse reinforcement needs to be provided between inside edge of the bearing plate / pad and beam end.
Shear

Sectional Design Model (5.8.3)
Sections Near Supports (5.8.3.2)

Minimal Av

d_v
General Requirements (5.8.2)

Regions Requiring Transverse Reinforcement (5.8.2.4)

• Except for slabs, footings, and culverts, transverse reinforcement shall be provided where:

\[ V_u > 0.5 \phi (V_c + V_p) \]  \hspace{1cm} (5.8.2.4-1)

where:

• \( V_u \) = factored shear force (kip)
• \( V_c \) = nominal concrete shear resistance (kip)
• \( V_p \) = prestressing component in direction of shear (kip)
• \( \phi \) = resistance factor (0.9)
General Requirements (5.8.2)

Minimum Transverse Reinforcement (5.8.2.5)

- Area of steel shall satisfy:

\[ A_v \geq 0.0316 \sqrt{\frac{b \cdot s}{f_{vc}}} \frac{f_y}{f} \]  

(5.8.2.5-1)

where:

- \( A_v \) = transverse reinforcement area within distance \( s \) (in.\(^2\))
- \( b_v \) = width of web (adjusted for ducts per 5.8.2.9) (in.)
- \( s \) = transverse reinforcement spacing (in.)
- \( f_y \) = transverse reinforcement yield strength (ksi)
Shear

General Requirements (5.8.2)

Maximum Spacing of Transverse Reinforcement (5.8.2.7)

• Maximum transverse reinforcement spacing, $s_{max}$, determined as:

  If $v_u < 0.125 \ f'_c$, then:

  \[ S_{max} = 0.8 \frac{d_v}{v} \leq 24.0 \text{ in.} \quad (5.8.2.7-1) \]

  If $v_u \geq 0.125 \ f'_c$, then:

  \[ S_{max} = 0.4 \frac{d_v}{v} \leq 12.0 \text{ in.} \quad (5.8.2.7-2) \]

  where

  $v_u = \text{shear stress per 5.8.2.9 (ksi)}$

  $d_v = \text{effective shear depth (in.)}$
General Requirements (5.8.2)

Shear Stress on Concrete (5.8.2.9)

• Shear stress on the concrete determined as:

\[
\frac{V}{u} = \frac{V - \phi V_p}{\phi b_v d_v} \tag{5.8.2.9-1}
\]

where:

\( \phi \) = resistance factor (0.9)

\( b_v \) = effective web width (in.)

\( d_v \) = effective shear depth (in.)
Shear

General Requirements (5.8.2)

Design and Detailing Requirements (5.8.2.8)

• Transverse reinforcement anchored at both ends per 5.11.2.6
• Extension of beam shear reinforcement into the deck slab for composite flexural members considered when checking 5.11.2.6
• Design yield strength of nonpresstressed transverse reinforcement:
  \[ = f_y \text{ when } f_y \leq 60.0 \text{ ksi} \]
  \[ = \text{stress @ strain } = 0.0035, \text{ but } \leq 75.0 \text{ ksi when } f_y > 60.0 \text{ ksi} \]
Shear

Anchorage of Shear Reinforcement (5.11.2.6)

Single leg, simple or multiple U stirrups (5.11.2.6.2)

- No. 5 or smaller and No. 6 - 8 w/ $f_y \leq 40$ ksi
  - Standard hook around longitudinal steel

- No. 6 – 8 w/ $f_y > 40$ ksi
  - Standard hook around longitudinal bar plus embedment between midheight of member and outside end of hook, $l_e$, satisfying

$$l_e \geq \frac{0.44 \, d \, f_y}{\sqrt{f'_c}}$$
Anchorage of Shear Reinforcement (5.11.2.6)

Closed stirrups (5.11.2.6.4)

• Pairs of U stirrups placed to form a closed stirrup are properly anchored if the lap lengths $> 1.7 \ell_d$ ($\ell_d =$ tension development length)

• For members $> 18”$ deep, closed stirrup splices w/ tension force from factored loads, $A_b f_y$, $< 9$ k per leg considered adequate if legs extend full available depth of member
Columns
Columns: Compression Members

**General (5.7.4.1)**

- Compression members shall consider:
  - Eccentricity
  - Axial loads
  - Variable moments of inertia
  - Degree of end fixity
  - Deflections
  - Duration of loads
  - Prestressing
Columns: Compression Members

**General (5.7.4.1)**

- Nonprestressed columns with the slenderness ratio, $KL_u/r < 100$, may be designed by the approximate procedure per 5.7.4.3
  - where:
    - $K = \text{effective length factor per 4.6.2.5}$
    - $L_u = \text{unbraced length (in.)}$
    - $r = \text{radius of gyration (in.)}$
Columns: Compression Members

**Limits for Reinforcement (5.7.4.2)**

- Maximum prestressed and nonprestressed longitudinal reinforcement area for noncomposite compression components shall be such:

\[
\frac{A_s}{A} + \frac{A_{ps \ f_{pu}}}{A_{g \ y}} \leq 0.08 \quad \text{(5.7.4.2-1)}
\]

and

\[
\frac{A_{ps \ f_{pe}}}{A_{g \ c \ f'}} \leq 0.30 \quad \text{(5.7.4.2-2)}
\]
Limits for Reinforcement (5.7.4.2)

- Minimum prestressed and nonprestressed longitudinal reinforcement area for noncomposite compression components shall be such that:

\[
\frac{A f_s}{A f_y} + \frac{A f_{ps}}{A f_{pu}} \geq 0.135 \quad (5.7.4.2-3)
\]
Columns: Compression Members

Limits for Reinforcement (5.7.4.2)

where:

- $A_s = \text{nonprestressed tension steel area (in.}^2\text{)}$
- $A_g = \text{section gross area (in.}^2\text{)}$
- $A_{ps} = \text{prestressing steel area (in.}^2\text{)}$
- $f_{pu} = \text{tensile strength of prestressing steel (ksi)}$
- $f_y = \text{yield strength of reinforcing bars (ksi)}$
- $f'_{c} = \text{compressive strength of concrete (ksi)}$
- $f_{pe} = \text{effective prestress (ksi)}$
**Columns: Compression Members**

*Limits for Reinforcement (5.7.4.2) (as in Std. Spec)*

- Minimum number of longitudinal reinforcing bars in a column:
  - 6 for circular arrangement
  - 4 for rectangular arrangement
- Minimum bar size: No. 5
Approximate Evaluation of Slenderness Effects
(5.7.4.3) (as in Std. Spec)

• Members not braced against sidesway, slenderness effects neglected when $KL_u/r, < 22$

• Members braced against sidesway, slenderness effects neglected when:
  – $KL_u/r < 34−12(M_1/M_2)$, in which $M_1$ and $M_2$ are the smaller and larger end moments, respectively

  and

  – $(M_1/M_2)$ is positive for single curvature flexure
Approximate Evaluation of Slenderness Effects (5.7.4.3)

- Approximate procedure may be used for the design of nonprestressed compression members with $KL_u/r < 100$:
  - Unsupported length, $L_u$, = clear distance between components providing lateral support (taken to extremity of any haunches in plane considered)
  - Radius of gyration, $r$, computed for gross concrete section (0.25*diameter for circular cols)
  - Braced members, effective length factor, $K$, taken as 1.0, unless a lower value is shown by analysis
Approximate Evaluation of Slenderness Effects (5.7.4.3)

- Unbraced members, $K$ determined considering effects of cracking and reinforcement on relative stiffness and taken $\geq 1.0$

- Design based on factored axial load, $P_u$, determined by elastic analysis and magnified factored moment, $M_c$, per 4.5.3.2.2b

**Magnified Moment (4.5.3.2.2b)**

$$M_c = \delta_2 M_b + \delta_2 M_s$$  \hspace{1cm} (4.5.3.2.2b-1)
Columns: Compression Members

*Magnified Moment (4.5.3.2.2b)*

where

\[
\delta_b = \frac{C}{{mP}} \geq 1.0 \quad (4.5.3.2.2b-3)
\]

\[
1 - \frac{u}{\phi K e}
\]

\[
\delta_s = \frac{1}{\sum P} \quad (4.5.3.2.2b-4)
\]

\[
1 - \frac{u}{\phi \sum P e}
\]

\(\delta_s = 1\) for members braced against sidesway. \(\Sigma\) is for group of compression members on 1 level of a bent or where compression members are intergrally connected to same superstructure.
Columns: Compression Members

**Magnified Moment (4.5.3.2.2b)**

where

- \( P_u \) = factored axial load (kip)
- \( C_m = \) (4.5.3.2.2b-6)
- \( C_m = 1 \) for all other cases

\[
C_m = 0.6 + 0.4 \frac{M_{1b}}{M_{2b}}
\]

for members braced against sidesway and w/o loads between supports

- \( M_{1b} / M_{2b} = \) smaller / larger end moment

Note: \( M_{1b} / M_{2b} + \) for single curvature and - for double curvature
Columns: Compression Members

**Magnified Moment (4.5.3.2.2b)**

- \( P_e = \) Euler Buckling load (kip) = \( \frac{\pi^2 EI}{(KL_u)^2} \) \( (4.5.3.2.2b-5) \)

- \( \phi_K = \) stiffness reduction factor (0.75 for concrete)
- \( M_{2b} = \) moment due to factored gravity loads resulting in negligible sidesway, positive (kip-ft)
- \( M_{2s} = \) moment due to factored lateral or gravity loads resulting in sidesway > \( L_u/1500 \), positive (kip-ft)
- \( L_u = \) unsupported length (in)
Columns: Compression Members

Magnified Moment (4.5.3.2.2b)

- $K = \text{effective length factor (4.6.2.5)}$
  - In absence of a more refined analysis, $K$ can be taken as:
    - $= 0.75$ for both ends being bolted or welded
    - $= 0.875$ for both ends pinned
    - $= 1.0$ for single angles regardless of end conditions
**Columns: Compression Members**

**Magnified Moment (4.5.3.2.2b)**

- The Structural Stability Council provides theoretical and design values for $K$ in Table C1 of the spec.

<table>
<thead>
<tr>
<th>End Conditions</th>
<th>Fixed-Fixed</th>
<th>Fixed-Pinned</th>
<th>Fixed-Lat. Translation</th>
<th>Pinned-Pinned</th>
<th>Fixed-Free</th>
<th>Pinned-Lat. Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical K</td>
<td>0.50</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Design K</td>
<td>0.65</td>
<td>0.8</td>
<td>1.2</td>
<td>1.0</td>
<td>2.1</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Columns: Compression Members

**Magnified Moment (4.5.3.2.2b)**

- Assuming only elastic action occurs, $K$ can also be found from:

\[
\frac{G_a G_b \left( \frac{\pi}{K} \right)^2 - 36}{6 \left( G_a + G_b \right)} = \frac{\pi}{K} \tan \left( \frac{\pi}{K} \right)
\]

where $a$ and $b$ represent the ends of the column.
Magnified Moment (4.5.3.2.2b)

G can be found by:

\[ G = \frac{\sum \left( \frac{E \ I \ c \ c}{L \ c} \right)}{\sum \left( \frac{E \ I \ g \ g}{L \ g} \right)} \]

subscripts c and g represent the column and girders, respectively, in the plane of flexure being considered. Two previous equations result in commonly published nomographs.
**Columns: Compression Members**

*Magnified Moment (4.5.3.2.2b)*

*In absence of a refined analysis, ODOT allows following values:*

<table>
<thead>
<tr>
<th></th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread footings on rock</td>
<td>1.5</td>
</tr>
<tr>
<td>Spread footings on soil</td>
<td>5.0</td>
</tr>
<tr>
<td>Footings on multiple rows of piles or drilled shafts:</td>
<td></td>
</tr>
<tr>
<td>End Bearing</td>
<td>1.0</td>
</tr>
<tr>
<td>Friction</td>
<td>1.5</td>
</tr>
<tr>
<td>Footings on a single row of drilled shafts/friction piles</td>
<td>1.0</td>
</tr>
<tr>
<td>Footings on a single row of end bearing piles</td>
<td>refined analysis reqd.</td>
</tr>
</tbody>
</table>
Magnified Moment (4.5.3.2.2b)

**ODOT -**

For columns supported on a single row of drilled shafts / friction piles include the depth to point of fixity when calculating effective column length. Refer to Article 10.7.3.13.4 to determine depth to point of fixity.

For drilled shafts socketed into rock, point of fixity should be no deeper than top of rock.

List in Table assumes typical spread footings on rock are anchored when footing is keyed $\geq 3$ in. into rock.
Columns: Compression Members

Approximate Evaluation of Slenderness Effects (5.7.4.3)

- In lieu of more precise calculation, $EI$ for use in determining $P_e$, as specified in Eq. 4.5.3.2.2b-5, taken as greater of:

\[
EI = \frac{E}{c} \frac{g}{5} + E \frac{l}{s} \frac{s}{1+\beta d} \tag{5.7.4.3-1}
\]

\[
EI = \frac{E}{c} \frac{g}{2.5} \frac{2.5}{1+\beta d} \tag{5.7.4.3-2}
\]
Approximate Evaluation of Slenderness Effects (5.7.4.3)

- where:
  - $E_c$ = concrete modulus of elasticity (ksi)
  - $I_g$ = gross moment of inertia of concrete section (in.$^4$)
  - $E_s$ = steel modulus of elasticity (ksi)
  - $I_s$ = longitudinal steel moment of inertia about centroidal axis (in.$^4$)
  - $\beta_d$ = ratio of maximum factored permanent load moments to maximum factored total load moment; positive (accounts for concrete creep)
Columns: Compression Members

**Factored Axial Resistance (5.7.4.4)**

- Factored axial resistance of concrete compressive components, symmetrical about both principal axes, shall be taken as:

\[ P_r = \phi P_n \] \hspace{1cm} (5.7.4.4-1)

in which:

\[ P_n = e \left[ 0.85 f'c (A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps} (f_{pe} - E_p \varepsilon_{cu}) \right] \] \hspace{1cm} (5.7.4.4-2&3)

with \( e \)

- = 0.85 for members w/ spiral reinforcement
- = 0.80 for members w/ tie reinforcement
Columns: Compression Members

**Factored Axial Resistance (5.7.4.4)**

where:

- \( P_r \) = factored axial resistance, w/ or w/o flexure (kip)
- \( P_n \) = nominal axial resistance, w/ or w/o flexure (kip)
- \( A_{ps} \) = prestressing steel area (in.\(^2\))
- \( E_p \) = prestressing tendons modulus of elasticity (ksi)
- \( f_{pe} \) = effective stress in prestressing steel (ksi)
- \( \varepsilon_{cu} \) = concrete compression failure strain (in./in.) (0.003)
**Columns: Compression Members**

**Biaxial Flexure (5.7.4.5)**

- In lieu of equilibrium and strain compatibility analysis, noncircular members subjected to biaxial flexure and compression may be proportioned using the following approximate expressions:
  
  - If the factored axial load is \( \geq 0.10 \phi f'c A_g \):

    \[
    \frac{1}{P_{rx/y}} = \frac{1}{P_{rx}} + \frac{1}{P_{ry}} - \frac{1}{\phi P_o} \tag{5.7.4.5-1}
    \]
    
    in which: (5.7.4.5-2)

    \[
    P_o = 0.85 f'_c \left( A_g - A_{st} - A_{ps} \right) + f_{y'} A_{st} - A_{ps} \left( f_{pe} - E \varepsilon_p \right) \tag{5.7.4.5-2}
    \]
Columns: Compression Members

Biaxial Flexure (5.7.4.5)

- If the factored axial load is < 0.10 \( \phi f'c A_g \):

\[
\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}} \leq 1.0 \tag{5.7.4.5-3}
\]
Columns: Compression Members

**Biaxial Flexure (5.7.4.5)**

- where:
  - $P_{rx} = \text{factored axial resistance in biaxial flexure (kip)}$
  - $P_{rx} = \text{factored axial resistance based on only } e_y \text{ is present (kip)}$
  - $P_{ry} = \text{factored axial resistance based on only } e_x \text{ is present (kip)}$
  - $P_u = \text{factored applied axial force (kip)}$
  - $M_{ux} = \text{factored applied moment about } X\text{-axis (kip-in.)}$
  - $M_{uy} = \text{factored applied moment about } Y\text{-axis (kip-in.)}$
  - $e_x = \text{eccentricity in } X \text{ direction, } \left(\frac{M_{uy}}{P_u}\right) (\text{in.})$
  - $e_y = \text{eccentricity in } Y \text{ direction, } \left(\frac{M_{ux}}{P_u}\right) (\text{in.})$
  - $P_o = \text{nominal axial resistance of section at 0.0 eccentricity}$
Biaxial Flexure (5.7.4.5)

Factored axial resistance $P_{rx}$ and $P_{ry}$ shall be $\leq \phi P_n$ where $P_n$ given by either Eqs. 5.7.4.4-2 or 5.7.4.4-3, as appropriate.

Spirals and Ties (5.7.4.6)

Area of steel for spirals & ties in bridges in Seismic Zones 2, 3, or 4 shall comply with the requirements of Article 5.10.11.

Where the area of spiral and tie reinforcement is not controlled by:

- Seismic requirements
- Shear or torsion per Article 5.8
- Minimum requirements per Article 5.10.6
Columns: Compression Members

Spirals and Ties 5.7.4.6

Ratio of spiral reinforcement to total volume of concrete core, measured out-to-out of spirals, shall satisfy:

\[ \rho_s \geq 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f'_c}{f_{yh}} \]  

(5.7.4.6-1)

where:

- \( A_g \) = gross area of concrete section (in.\(^2\))
- \( A_c \) = area of core measured to the outside diameter of spiral (in.\(^2\))
- \( f''_c \) = 28 day strength, unless another age is specified (ksi)
- \( f_{yh} \) = specified yield strength of reinforcement (ksi)

Other details of spiral and tie reinforcement shall conform to Articles 5.10.6 and 5.10.11
Columns: Compression Members

\[ r = \frac{\pi}{2bA} \]

\[ \rho = \frac{\text{Steel Volume}}{\text{Concrete Volume}} \]

\[ = \frac{A}{\pi r^2} \cdot \frac{2\pi r}{\text{pitch}} \]

Note: The steel volume calc does not account for the pitch but this is minor with smaller pitches.
Spirals and Ties 5.7.4.6

**ODOT –**

- Provision only applies to columns where ratio of axial column capacity to axial column load < 1.5
- For all other column designs, spiral reinforcement detailed as specified in BDM Section 303.3.2.1
Columns: Compression Members

Column Example:

Take:

\[ M_{\text{Permanent } x} = 113 \text{ k-ft} \]
\[ M_{\text{Permanent } y} = 83 \text{ k-ft} \]
\[ M_{\text{Total } x} = 750 \text{ k-ft} \]
\[ M_{\text{Total } y} = 260 \text{ k-ft} \]
\[ f'_c = 4 \text{ksi} \]

Diameter of column = 36 in (3 ft) \[ A_g = \pi \times \left( \frac{36\text{in}}{2} \right)^2 = 1,018\text{in}^2 \left(7.07\text{ft}^2\right) \]

Reinforcement steel = 12 # 9’s \[ A_s = 12 \text{ in}^2 \]
\[ f_y = 60 \text{ ksi} \]
\[ P_u = 927 \text{ kips} + \text{self weight of column} \]
\[ L = 17.65 \text{ ft.} \]
Columns: Compression Members

Column Example:

Self weight of column = 7.07 ft² × 17.65 ft × 0.15 kcf × 1.25 = 23.4 k

\[ P_u = 927 \text{ k} + \left( 7.07 \text{ ft}^2 \times 17.65 \times 0.15 \times 1.25 \right) = 950 \text{ k} \]

\[ \frac{A_s}{A_g} = \frac{12 \text{ in}^2}{1,018 \text{ in}^2} = 0.012 < 0.08 \Rightarrow \text{O.K..... (5.7.4.2-1)} \]

\[ \frac{A_{s f y}}{A_{g f c}} = \frac{12 \text{ in}^2 (60 \text{ ksi})}{1,018 \text{ in}^2 (4 \text{ ksi})} = 0.177 \geq 0.135 \Rightarrow \text{O.K..... (5.7.4.2-3)} \]
Columns: Compression Members

Column Example:

Slenderness

L = Length of column
k = Effective length factor
r = Radius of gyration of cross-section of the column

L = 17.65 ft = 212 in
Column Example:

**Slenderness**

Effective length factor (k): **Pier Cap**

Plane of bent (⊥ to bridge)

Unbraced w/ k = 1.2 (highly rigid pier cap & footing)……. (C4.6.2.5)
Column Example:

Slenderness

Plane ⊥ to bent (∥ to bridge)

Unbraced with $k = 2.1 \Rightarrow$ free cantilever ............ (C4.6.2.5)
Columns: Compression Members

**Column Example:**

**Slenderness**

\[ r = 0.25(d) \] .......................... (C5.7.4.3)

where \( d \) = column diameter

\[ r = 0.25(36 \text{ in}) = 9 \text{ in} \]

Plane of Bent

\[ \frac{kL}{r} = \frac{1.2 \times (212 \text{ in})}{9 \text{ in}} = 28.3 < 100 \text{ O.K, but > 22} \]

\[ \therefore \text{Consider slenderness} \]
Columns: Compression Members

Column Example:

Slenderness

Plane ⊥ to Bent

\[
\frac{kL}{r} = \frac{2.1(212\text{in})}{9\text{in}} = 49.5 < 100 \quad \text{O.K but > 22}
\]

∴ Consider slenderness

Moment Magnification

Plane ⊥ to Bent:

\[
M_{C \perp} = \delta M_{b \perp 2b} + \delta M_{s \perp 2s}
\]
Columns: Compression Members

Column Example:

Moment Magnification

where

the δ’s are moment magnifying factors

δ_b = braced magnifier
δ_s = sway magnifier

\[ \delta_b = \frac{CM}{P} \left( 1 - \frac{u}{\phi KP e} \right) \]

\[ \delta_s = \frac{1}{\Sigma P} \left( 1 - \frac{u}{\phi K \Sigma P e} \right) \]
Column Example:

Moment Magnification

where:

\[ C_m = \text{Equivalent moment correction factor} \]
\[ C_m = 1.0 \text{ (for all other cases)} \]
\[ P_u = 950 \text{ k} \]
\[ \phi_K = 0.75 \]

\[
P = \frac{\pi^2 E I}{(kL_u)^2}
\]
**Column Example:**

**Moment Magnification**

\[ \text{EI max of:} \]

\[ \frac{E_c I_g}{5} + E_s I_s \left( \frac{1 + \beta_d}{1 + \beta_d} \right) = \frac{E_c I_g}{2.5} \]

where:

- \( E_c \) = Modulus of elasticity of concrete
- \( I_g \) = Moment of inertia of the gross section
- \( E_s \) = Modulus of elasticity of steel
- \( I_s \) = Moment of inertia of the steel section
**Columns: Compression Members**

**Column Example:**

**Moment Magnification**

\[ E_c = 1,820 \sqrt{f'_c} \]

\[ = 1,820 \sqrt{4} \text{ksi} = 3,640 \text{ksi} \]

\[ l_g = \frac{\pi r^4}{4} \]

\[ = \frac{\pi (18 \text{ in})^4}{4} = 82,448 \text{ in}^4 \]

where

- \( r \) = radius of column
- \( E_s = 29,000 \text{ksi} \)
- \( I_s \) ➔ difficult to find and depends on position of bars relative to axis of concern
Column Example:

Moment Magnification

\[ D = 36 \text{ in} - 3 \text{ in} - 3 \text{ in} = 30 \text{ in} \quad \Rightarrow \quad r = 15 \text{ in} \]
Columns: Compression Members

Column Example:

Moment Magnification

Assume $I_{\text{Parallel}} = I_s(\text{Centroid}) + Ad^2$

$I_s = 0$ since steel bar is small (1.128 in diameter)

$r = 15''$

$d_1 = r \sin(30) = 15'' \sin(30) = 7.5$ in

$l_1 = (1\text{in}^2)(7.5 \text{in})^2 = 56.25 \text{ in}^4$
Column Example:
Moment Magnification

\[ d_2 = r \sin(60) = 15'' \sin(60) = 13 \text{ in} \]

\[ I_2 = (1 \text{ in}^2)(13 \text{ in})^2 = 169 \text{ in}^4 \]

\[ I_3 = (1 \text{ in}^2)(15 \text{ in})^2 = 225 \text{ in}^4 \]
Column Example:

Moment Magnification

\[
I = 4(56.25\text{in}^4) + 4(169\text{in}^4) + 2(225\text{in}^4) = 1,351 \text{ in}^4
\]

\[
I_s = 0.125 A_s d^2 = 1,350 \text{ in}^4 \text{ (from MacGregor’s text)}
\]

\[
\beta_d = \frac{M_{\text{Permanent}}}{M_{\text{Total}}} = \frac{113}{750} = 0.15
\]

\[
E I = \text{Maximum of the following values:}
\]

\[
E I = \frac{E I}{5} \left(1 + \beta \right) + E I \left(1 + \frac{E I}{5} \right)
\]

\[
E I = \frac{E I}{2.5} \left(1 + \beta \right)
\]
Columns: Compression Members

Column Example:

Moment Magnification

\[ EI = \text{Maximum of the following values:} \]

\[ \frac{3,640 \text{ ksi} \times (82,448 \text{ in}^4)}{5} + 29,000 \text{ ksi} \times (1,351\text{ in}^4) \]

\[ EI = \frac{3,640 \text{ ksi} \times (82,448 \text{ in}^4)}{2.5} \]

\[ EI = 86,261,864 \text{ k-in}^2 \]

\[ EI = 104,386,337 \text{ k-in}^2 \text{ (Controls)} \]
Columns: Compression Members

Column Example:
Moment Magnification

\[ P_e = \frac{\pi^2 (104,386,337 \text{ k-in}^2)}{(2.1(212 \text{ in}))^2} = 5,198 \text{ kips} \]

\[ \delta_b = \frac{1}{1 - \frac{950 \text{ k}}{0.75 (5,198 \text{ k})}} = 1.32 \]

Due to symmetry of bridge and columns

\[ \sum P_u = P_u \text{ and } \sum P_e = P_e \]

\[ \therefore \delta_s = \delta_b = 1.32 \]

\[ M_{\perp} = 1.32 (750 \text{ k-ft}) = 990 \text{ k-ft} \]
Column Example:

Moment Magnification

Plane of Bent:

\[ M_{c//} = \delta_b M_{2b} + \delta_s M_{2s} \]

\[ \delta_b = \frac{C_m}{P} \left( 1 - \frac{u}{\phi P} \right) \]

where: \( C_m = 1.0 \), \( P_u = 950k \), \( \phi = 0.75 \)

\[ P_e = \frac{\pi^2 E I}{(k L_u)^2} \]
Columns: Compression Members

**Column Example:**

Moment Magnification

\[
EI = \frac{104,386,337 \text{ k-in}^2 (1.15)}{(1 + \beta_d)}
\]

where 1.15 is from previous \((1 + \beta_d)\)

\[
\beta_d = \frac{83}{260} = 0.32
\]

\[
\therefore \ EI = \frac{104,386,337 \text{ k-in}^2 (1.15)}{(1.32)} = 90,942,642 \text{ k-in}^2
\]
Columns: Compression Members

**Column Example:**

**Moment Magnification**

\[
P_e = \pi^2 \left( \frac{90,942,642 \text{ k-in}^2}{(1.2 \text{ (212 in)})^2} \right) = 13,869 \text{ k}
\]

\[
\delta_b = \frac{1}{1 - \frac{950 \text{ k}}{0.75 (13,869 \text{ k})}} = 1.10
\]

As before, let \( P_u = \Sigma P_u \) and \( P_e = \Sigma P_e \)

\[
\therefore \delta_s = \delta_b = 1.10
\]

\[
M_// = 260 \text{ k-ft}(1.10) = 286 \text{ k-ft}
\]

\[
M_u = \sqrt{M_\perp^2 + M_//^2} = \sqrt{(990 \text{ k-ft})^2 + (286 \text{ k-ft})^2} = 1,030 \text{ k-ft}
\]

where \( M_u \) = Factored Moment and \( P_u = 950 \text{ kips} \)
## Columns: Compression Members

Table 1: Column Axial & Flexural Capacity

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\phi P_n$ (kips)</th>
<th>$\phi M_n$ (k-ft)</th>
<th>$P_u$ (k)</th>
<th>$M_u$ (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>2,603</td>
<td>0</td>
<td>950</td>
<td>1030</td>
</tr>
<tr>
<td>0.75</td>
<td>2,603</td>
<td>324</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.75</td>
<td>2,443</td>
<td>610</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.75</td>
<td>2,060</td>
<td>859</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.75</td>
<td>1,617</td>
<td>1,030</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.763</td>
<td>1,142</td>
<td>1,131</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.832</td>
<td>799</td>
<td>1,159</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.9</td>
<td>416</td>
<td>1,032</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.9</td>
<td>-23</td>
<td>690</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Columns: Compression Members

Column Interaction Diagram

\[ \phi P_n (k) \]

\[ \phi M_n (k-ft) \]

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Column Example:

Shear (5.8.3.3-3)

\[
V_c = 0.0316 \beta \sqrt{f'_c b_v d_v}
\]

where

- \(V_c\) = nominal concrete shear strength
- \(b_v\) = web width (the same as \(b_w\) in the ACI Code)
- \(d_v\) = effective depth in shear (taken as flexural lever arm)

Not subject to axial tension and will include minimum transverse steel

\[
\therefore \beta = 2.0
\]

By C5.8.2.9

\[
b_v = 36 \text{ in.}
\]
Columns: Compression Members

Column Example:
Shear (C5.8.2.9-1)

\[
d_v = \frac{M_n}{A_f y + A_{ps} f_{ps}}
\]

\[
= \frac{690 \text{ k} \cdot \text{ft}(12 \text{ in/ft})}{0.9}
\]

\[
= 25.56 \text{ in}
\]

Note: \(A_{ps} f_{ps} = 0\)
where:
\(A_{ps}\) = Area of prestressed reinforcement
\(f_{ps}\) = Stress in prestressed reinforcement

Also assuming \(\frac{1}{2}\) of the steel is actually in tension (6 in\(^2\) instead of 12 in\(^2\))
Column Example:

Shear

Alternatively:

\[ d_v = 0.9 d_e \]

\[ d_e = \frac{D}{2} + \frac{D_r}{\pi} \]

\[ D = 36 \text{ in} \]

\[ D_r = 36 \text{ in} -3 \text{ in} -3 \text{ in} - 2(\frac{1}{2} \text{ in}) - 1.128 \text{ in} = 27.872 \text{ in} \]

where \( \frac{1}{2} \text{ in} \) is from #4 tie/spiral and 1.128” is from # 9 rebar

\[
\frac{d_e}{\pi} = \frac{36 \text{ in}}{2} + \frac{27.872 \text{ in}}{\pi} = 26.87 \text{ in}
\]

\[ d_v = 0.9(26.87 \text{ in}) = 24.2 \text{ in} \quad \text{(Use this)} \]
Columns: Compression Members

**Column Example:**

**Shear**

\[ V_c = 0.0316 \left( 2 \sqrt{4 \text{ ksi}} \right) (36 \text{ in}) (24.2 \text{ in}) = 110.2 \text{ kips} \]

Say: \( V_u = 60 \text{ kips} = \text{Factored shear force} \)

\[ 0.5 \phi V_c = 0.5(0.9)(110.2 \text{ kips}) = 49.5 \text{ kips} \]

**Note:**

\[ V_n = 0.25(f'c)b_v d_v \text{ where } V_n = \text{nominal shear resistance} \]

\[ = 0.25 \times (4 \text{ ksi})(36 \text{ in})(24.2 \text{ in}) = 871.2 \text{ k} > V_c \text{ O.K.} \]

\[ 0.5 \phi V_c < V_u \]

[minimum transverse steel]
Columns: Compression Members

Column Example:

Shear

Assume a # 3 spiral with 4 in pitch

\[
A_{\text{Min}} = 0.0316 \sqrt{\frac{f'}{c}} \frac{b}{y} \frac{s}{f}
\]

\[
= 0.0316 \sqrt{4 \text{ ksi} \frac{36 \text{ in}(4 \text{ in})}{60 \text{ ksi}}} = 0.15 \text{ in}^2
\]

\[
< 2 (0.11) = 0.22 \text{ in}^2 \quad \text{O.K.}
\]
Decks
Decks

Limit States: (9.5)

- Concrete appurtenances (curb, parapets, railing, barriers, dividers) to the deck can be considered for service and fatigue, but not for strength or extreme event limit states

**ODOT - Designers shall ignore the structural contribution of concrete appurtenances for all limit states**
Decks

Service Limit States (9.5.2)

- Deflection (local dishing, not overall superstructure deformation) caused by live load plus dynamic load allowance shall not exceed:
  - L/800 for decks w/o pedestrian traffic
  - L/1000 for decks w/ limited pedestrian traffic
  - L/1200 for decks w/ significant pedestrian traffic

where L = span length from center-center of supports
Decks

**Fatigue and Fracture (9.5.3)**

- Fatigue need not be investigated for concrete decks in multi-girder systems. For other decks see 5.5.3.

**Strength (9.5.4)**

- Decks and deck systems analyzed as either elastic or inelastic structures and designed and detailed in accordance w/ Section 5.

**Extreme Events (9.5.5)**

- Decks shall be designed for force effects transmitted by traffic and combination railings using loads, analysis procedure, and limit states in Section 13.
Decks

Analysis Methods (9.6)

Following methods may be used for various limit states as permitted in 9.5

- Approximate elastic method (4.6.2.1)
- Refined methods (4.6.3.2)
- Empirical design (9.7)

**ODOT** - Approximate elastic method of analysis specified in Article 4.6.2.1 shall be used
Approximate Methods of Analysis (4.6.2)

General (4.6.2.1.1)

- Deck is subdivided into strips perpendicular to supporting components considered acceptable for decks
- Extreme positive and negative moments taken to exist at all positive and negative moment regions, respectively

Applicability (4.6.2.1.2)

- For slab bridges and concrete spans spanning > 15 feet and primarily parallel to traffic, provisions of 4.6.2.3 shall apply
Decks

Approximate Methods of Analysis (4.6.2)

*Width of Equivalent Interior Strips (4.6.2.13)*

- Decks primarily spanning parallel to traffic:
  - \( \leq 144" \) for decks where multilane loading is being investigated
  - where applicable, 3.6.1.3.4 may be used in lieu of the strip width
    specified for deck overhang \( \leq 6' \)

*ODOT – 3.6.1.3.4 does not apply. Design deck overhangs in accordance with BMD 302.2.2.*

- Decks spanning primarily in transverse direction not subjected to width limits
- Width of equivalent strip may be taken as specified in Table 1
Decks

Approximate Methods of Analysis (4.6.2)

*Width of Equivalent Interior Strips (4.6.2.1.3)*

Table 4.6.2.1.3 - 1

<table>
<thead>
<tr>
<th>Type of Deck</th>
<th>Direction of Primary Strip Relative to Traffic</th>
<th>Width of Primary Strip (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cast-in-place</td>
<td>Overhang</td>
<td>45.0 + 10.0 X</td>
</tr>
<tr>
<td></td>
<td>Either Parallel or Perpendicular</td>
<td>+M: 26.0 + 6.6 S</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-M: 48.0 + 3.0 S</td>
</tr>
</tbody>
</table>

where  

\[ S = \text{spacing of supporting components (ft)} \]

\[ X = \text{distance from load to point of support (ft)} \]
Decks

Approximate Methods of Analysis (4.6.2)

Width of Equivalent Strips at Edges of Slabs (4.6.2.1.4)

- For design, notional load edge beam taken as a reduced deck strip width specified herein. Any additional integral local thickening or similar protrusion that is located within the reduced deck strip width can be assumed to act with the reduced deck strip width.
Approximate Methods of Analysis (4.6.2)

Width of Equivalent Strips at Edges of Slabs (4.6.2.1.4)

Longitudinal Edges (4.6.2.1.4b)

• Edge beams assumed to support one line of wheels and where appropriate, a tributary portion of the design lane

• For decks spanning primarily in direction of traffic, effective width of strip, w/ or w/o an edge beam, may be taken as the sum of:
  • distance between edge of deck and inside face of barrier
  • 12”
  • ¼ of strip interior width
  but not exceeding either ½ of the full interior strip width or 72”
Approximate Methods of Analysis (4.6.2)

Width of Equivalent Strips at Slab Edges (4.6.2.1.4) Transverse Edges (4.6.2.1.4c)

- Transverse edge beams assumed to support 1 axle of design truck in \( \geq 1 \) design lanes, positioned to produce maximum load effects
- Multiple presence factors and dynamic load allowance apply
- Effective width of strip, w/ or w/o an edge beam, taken as sum of:
  - distance between transverse edge of deck and centerline of the first line of support for the deck (girder web)
  - \( \frac{1}{2} \) of the interior strip width

but not exceeding the full interior strip width
Decks

Approximate Methods of Analysis (4.6.2)

Distribution of Wheel Loads 4.6.2.1.5

• If spacing of supporting components in secondary direction:
  
  > 1.5 * spacing in primary direction:
  
  • All wheel loads applied in primary strip
  
  • 9.7.3.2 applied to secondary direction
Approximate Methods of Analysis (4.6.2)

*Distribution of Wheel Loads 4.6.2.1.5*

- If spacing of supporting components in secondary direction:
  - < 1.5 * spacing in primary direction:
    - Deck modeled as system of intersecting strips
    - Width of equivalent strips in both directions from Table 4.6.2.1.3-1
    - Each wheel load distributed between intersecting strips by ratio of strip stiffness to sum of strip stiffnesses (ratio = \( k/\sum k \)). Strip stiffness taken as:
      \[
      k_s = \frac{EI_s}{S^3}
      \]
      where
      - \( I_s \) = I of the equivalent strip (in\(^4\))
      - S = spacing of supporting components (in)

Decks

Approximate Methods of Analysis (4.6.2)

Calculation of Force Effects 4.6.2.1.6

• Strips treated as continuous or simply supported beams, as appropriate.
• Span lengths taken from center-to-center of supports
• Supporting components assumed infinitely rigid
• Wheel loads can be modeled as concentrated loads or patch loads with lengths of tire contact area as in 3.6.1.2.5 plus deck depth
• Strips analyzed by classical beam theory
• Appendix A4.1 for unfactored live load moments for typical concrete decks, in lieu of more precise calculations
Approximate Methods of Analysis (4.6.2)

Calculation of Force Effects 4.6.2.1.6

• Negative moments and shears taken at:

  • Face of support for monolithic construction, closed steel boxes, closed concrete boxes, open concrete boxes without top flanges, and stemmed precast sections

  • ¼ the flange width from centerline of support for steel I-beams and steel tub girders.

  • 1/3 the flange width (not exceeding 15”) from centerline of support for precast I-shaped concrete beams and open concrete boxes with top flanges
Decks

A4 DECK SLAB DESIGN TABLE 1 (Equivalent strip method)

Assumptions and limitations:

• Concrete slabs supported on parallel girders
• Multiple presence factors and dynamic load allowance included
• For negative moment design sections, interpolate for distances not listed
• Decks supported on $\geq 3$ girders and having a width of $\geq 14.0$ ft between centerlines of exterior girders
• Moments represent upper bound for moments in the interior regions of the slab
## Decks

### Table A4-1 Maximum Live Load Moments Per Unit Width, kip-ft./ft.

<table>
<thead>
<tr>
<th>S</th>
<th>+ M</th>
<th>- M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Distance from Girder CL to Design Section for -M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0 in</td>
</tr>
<tr>
<td>4’- 0”</td>
<td>4.68</td>
<td>2.68</td>
</tr>
<tr>
<td>4’- 3”</td>
<td>4.66</td>
<td>2.73</td>
</tr>
<tr>
<td>4’- 6”</td>
<td>4.63</td>
<td>3.00</td>
</tr>
<tr>
<td>14’-6”</td>
<td>9.25</td>
<td>12.67</td>
</tr>
<tr>
<td>14’-9”</td>
<td>9.36</td>
<td>12.88</td>
</tr>
<tr>
<td>15’-0”</td>
<td>9.47</td>
<td>13.09</td>
</tr>
</tbody>
</table>
Decks

Approximate Methods of Analysis (4.6.2)
Equivalent Strip Widths for Slab-Type Bridges 4.6.2.3

Equivalent width of longitudinal strips per lane for V and M
w/ one lane loaded (i.e. two lines of wheels) may be
determined as:

\[ E = 10.0 + 5.0 \sqrt{L_1 W_1} \quad (4.6.2.3-1) \]

w/ > one lane loaded may be determined as:

\[ E = 84.0 + 1.44 \sqrt{L_1 W_1} \leq \frac{12.0 W}{N_L} \quad (4.6.2.3-2) \]
Approximate Methods of Analysis (4.6.2)

Equivalent Strip Widths for Slab-Type Bridges 4.6.2.3

where:

\[ E = \text{equivalent width (in.)} \]

\[ L_1 = \text{modified span length} = \text{lesser of actual span or 60.0(ft.)} \]

\[ W_1 = \text{modified edge-to-edge width} = \text{to lesser of actual width or 60.0 for multilane loading, 30.0 for single-lane loading (ft.)} \]

\[ W = \text{physical edge-to-edge width of bridge (ft.)} \]

\[ N_L = \text{number of design lanes per 3.6.1.1.1} \]
Approximate Methods of Analysis (4.6.2)

Equivalent Strip Widths for Slab-Type Bridges 4.6.2.3

For skewed bridges, longitudinal force effects may be reduced by factor $r$:

$$ r = 1.05 - 0.25 \tan(\theta) \leq 1.00 $$

(4.6.2.3-3)

where:

$\theta = \text{Skew angle \degree}$
Decks

**General (9.7.1)**

- Depth of deck $\geq 7\"$ excluding provisions for
  - grinding
  - grooving
  - sacrificial surface

**ODOT**

- *Concrete deck $\geq 8.5$ inches as specified in BDM 302.2.1*
- *Minimum cover shall be in accordance with BDM 301.5.7*
Decks

General (9.7.1)

- If deck skew $\leq 25^\circ$
  - primary reinforcement may be placed in direction of skew
  - Otherwise, primary reinforcement placed perpendicular to main supporting elements.

ODOT

- BDM Section 302.2.4.2 covers this
  - For steel beam/girder bridges w/ skew < 15°, transverse steel may be shown placed $\parallel$ to abutments. For skew > 15° or where reinforcing would interfere w/ shear studs, transverse steel placed $\perp$ to centerline of bridge.
  - For P/C I beams, transverse steel placed $\perp$ to centerline of bridge
  - For composite box beam decks, transverse steel placed $\parallel$ to abutment
Decks

**General (9.7.1)**

- Overhanging portion of deck designed:
  - for railing impact loads
  - and
  - in accordance with 3.6.1.3.4 *(ODOT – 3.6.1.3.4 does not apply)*
  - Punching shear effects of outside toe of railing post or barrier due to vehicle collision loads shall be investigated
Decks

Deck Overhang Design (A13.4)

Design Cases

– Case 1: Extreme Event Load Combination II - transverse and longitudinal forces
– Case 2: Extreme Event Load Combination II - vertical forces
– Case 3: Strength I Load Combination – loads that occupy overhang

**ODOT - For Design Cases 1 and 2:**

*For bridges with cantilever overhangs $\geq 7\text{'}-0\text{”}$ measured to face of the traffic barrier, live load factor including dynamic load allowance = 0.50

*Otherwise live load factor, including dynamic load allowance = 0.0*
Decks

Rail Section w/ Loads

Sidewalk

Deck

©Ohio University (July 2007)
Decks

Longitudinal View of Rail w/ Loads
Decks

Plan View of Rail w/ Loads

\[ R, R_1, R_2 \]

\[ F_t \]

\[ L_t \]
Deck Overhang Design (A13.4)

Concrete Parapet (A13.4.2)

- For Design Case 1, deck overhang designed to provide flexural resistance, $M_s$ in kip-ft/ft acting coincident with the tensile force, $T$, > $M_c$ of the parapet at its base.

\[
T = \frac{R_w}{\frac{L}{C} + 2H}
\]

where:

- $R_w$ = Parapet resistance to lateral impact force (kips)
- $H$ = Height of the parapet (ft.)
- $L_c$ = Critical length of yield line failure pattern (ft.)
Deck Overhang Design (A13.4)

Concrete Parapet (A13.4.2)

**ODOT Exception**

- For Design Case 1, deck overhang designed to resist vehicular impact moment, $M_{CT}$, and coincidental axial tension force, $T_{CT}$, as follows:

$$M_{CT} = \frac{R \cdot H}{L_c + 2H + 2X}$$

$$T_{CT} = \frac{R}{L_c + 2H + 2X}$$

where

$X = \text{Lateral distance from toe of barrier to deck design section (ft.)}$
Decks

Deck Overhang Design (A13.4)

Concrete Parapet (A13.4.2)

ODOT Exception

• Transverse force selected for design corresponds to barrier’s crash tested acceptance level (i.e. Test Level). The following table provides design overhang data for standard ODOT barrier types:
# Decks

<table>
<thead>
<tr>
<th>Barrier System</th>
<th>L&lt;sub&gt;c&lt;/sub&gt; (ft.)</th>
<th>R (kip)</th>
<th>H (ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBR-1-99</td>
<td>12.7</td>
<td>165.0</td>
<td>3.5</td>
</tr>
<tr>
<td>42” BR-1</td>
<td>12.4</td>
<td>165.0</td>
<td>3.5</td>
</tr>
<tr>
<td>36” BR-1</td>
<td>8.8</td>
<td>72.0</td>
<td>3.0</td>
</tr>
<tr>
<td>BR-2-98</td>
<td>10.0</td>
<td>72.0</td>
<td>2.5&lt;sup&gt;(1)&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>(1)</sup> For BR-2-98, this height represents the maximum effective height of the railing resistance (Y). Refer to *Article A13.3.3* for more information.
Decks

Deck Overhang Design (A13.4)

Post and Beam Railings (A13.4.3.1)

- For Design Case 1, moment per ft., $M_d$, and thrust per ft., $T$ taken as

$$M_d = \frac{M_{\text{Post}}}{W_b + D}$$

$$T = \frac{P}{W_b + D}$$

where:

$M_{\text{Post}} = \text{flex. resistance of railing post (ft. - kips)}$

$P_p = \text{shear corresponding to } M_{\text{post}} \text{ (kips)}$

$W_b = \text{width of base plate (ft.)}$

$D = \text{dist from outer edge of base plate to innermost row of bolts (ft.)}$
Deck Overhang Design (A13.4)

Post and Beam Railings (A13.4.3.1)

- For Design Case 2, punching shear, $P_v$, and overhang moment, $M_d$, taken as

$$P_v = \frac{F_L}{L_v}$$

$$M_d = \frac{P_X}{b}$$

where:

$F_v$ = vertical force from vehicle laying on rail (kips)
$L$ = post spacing (ft)
$L_v$ = longitudinal distribution of $F_v$ (ft.)
$X$ = dist from outer edge of base plate to section under investigation (ft.)

$$b = 2X + W_b \leq L$$
Decks

Deck Overhang Design (A13.4)

Post and Beam Railings (A13.4.3.1)

ODOT exception

- For TST-1-99, following design data shall be assumed:

\[ M_{post} = 79.2 \text{ kip} \cdot \text{ft} \quad Y = 1.79 \text{ ft.} \]
\[ P_p = 44.2 \text{ kip} \]
\[ W_b = 2.0 \text{ ft.} \]
\[ D = 0.0 \text{ ft.} \]
Deck Overhang Design (A13.4)

Post and Beam Railings - Punching Shear Resistance (A13.4.3.2)

- For Design Case 1, factored shear taken as:

\[ V_{u} = A_{f} F_{y} \]

- Factored resistance to punching shear taken as

\[ V_{r} = \phi V_{n} = \phi V_{c} \left[ W_{b} + h + 2 \left( \frac{E + B}{2} + \frac{h}{2} \right) \right] h \]
Decks

Deck Overhang Design (A13.4)

Post and Beam Railings - Punching Shear Resistance (A13.4.3.2)

\[ V_c = \left( 0.0633 + \frac{0.1265}{\beta_c} \right) \sqrt{f'_c} \leq \phi \ 0.1265 \sqrt{f'_c} \]

where

\[ \frac{B}{2} + \frac{h}{2} \leq B \]

\[ \beta_c = \frac{W}{b} \]

\[ \frac{D}{2} \]
Decks

Deck Overhang Design (A13.4)

Post and Beam Railings - Punching Shear Resistance (A13.4.3.2)

where

- \( A_f \) = area of post compression flange (in\(^2\))
- \( F_y \) = yield strength of post compression flange (ksi)
- \( b \) = length of deck resisting shear = \( h + W_b \) (in.)
- \( h \) = slab depth (in.)
- \( E \) = distance from edge of slab to centroid of compression stress resultant in post (in.)
Decks

Deck Overhang Design (A13.4)

Post and Beam Railings - Punching Shear Resistance (A13.4.3.2)

where

- \( B \) = dist. between centroids of compression and tensile stress resultants in post (in.)
- \( \beta_c \) = long to short side ratio of concentrated load or reaction area
- \( D \) = depth of base plate (in.)
Decks

- Deck edge
- Punching shear critical section
- Loaded compression area

Symbols:
- $E + B/2 + h/2$
- $W_b$
- $E$
- $B$
- $h/2$
- $B/2$
Decks

Reinforcement Distribution (9.7.3.2)

Reinforcement in secondary direction placed in bottom of slabs as a % of primary reinforcement for positive moment as follows:

– Primary parallel to traffic
  \[ \frac{100}{\sqrt{S}} \leq 50 \% \]

– Primary perpendicular to traffic
  \[ \frac{220}{\sqrt{S}} \leq 67 \% \]

where S = effective span length by 9.7.2.3 (ft)

ODOT (BDM 302.2.4.1) - Distribution reinforcement in top-reinforcing layer of reinforced concrete deck on steel / concrete stringers shall be approximately 1/3 of main reinforcement, uniformly spaced
Decks

Effective span length (9.7.2.3)

- Face-to-face distance for slabs monolithic with beams or walls
- Distances between flange tips + flange overhang
- For nonuniform spacing (see Fig 9.7.2.3-1)
Decks

Slab Bridge Example:

- Length: 22'
- Height: 36'
- Width: 36'
Decks

Minimum thickness (Table 2.5.2.6.3-1)

\[
h_{\text{min}} = \frac{1.2 (S+10)}{30}
\]

\[
= \frac{1.2 (22+10)}{30} = 1.28' = 15.36''
\]

→ Say 15.5”

Note: ODOT standard drawing SB-1-03
\[h = 17.25'' \text{ for } S = 22'
\]
Decks

Equivalent Strips: (4.6.2.3)

Interior:

1-Lane

\[ E = 10 + 5 \sqrt{\frac{L}{1}} W_1 \]

where \( L_1 = \text{Span} \leq 60' \) and \( W_1 = \text{width} \leq 30' \)

\[ = 22' \leq 60' \quad \text{and} \quad = 36' \leq 30' \text{ Controls} \]

\[ E = 10 + 5 \sqrt{22 (30)} = 138.5" \]

Note: Loads on this strip do not have to include multiple presence factor, \( m = 1.2 \) for single lane loading since already included
Decks

> 1 Lane:

\[ E = 84 + 1.44 \sqrt{L \frac{W}{W_1}} \leq 12 \frac{W}{N_L} \]

where:

- \( W = \text{Width} = 36' \)
- \( N_L = \# \text{of Lanes} = \frac{W}{12} = \frac{36}{12} = 3 \)
- \( W_1 = \text{Width} \leq 60' \)
  - \( = 36' \leq 60' \)

\[ E = 84 + 1.44 \sqrt{22(36)} \leq 12 \left( \frac{36}{3} \right) \]

\[ = 124.5'' < 144'' \quad \text{O.K} \]

Controls
Decks

Exterior:

Edge Strip

Edge of Deck to inside face of barrier

+ 12”

+ ¼ strip width per 4.6.2.3

≤ ½ strip width or 72”

\[
0 + 12 + \frac{1}{4} (124.5) = 43.125” < \frac{124.5}{2} = 62.25
\]

or 72”

Note: For live loads, lane + truck or tandem apply. M & V divided by strip width to find M & V per 1’ wide section. 1 wheel line for edge strip because of its limited width (< 72” = ½ lane).
Deck - Example

• Example done by Burgess and Niple for ODOT

• Slight modifications to further explanation
# Deck - Example

## NOTES/ASSUMPTIONS:

1. **Design Specifications:**
   - 2004 AASHTO LRFD including 2006 Interims
   - ODOT Bridge Design Manual

2. **Material Strengths:**
   - Reinforcing Steel: \( f_y = 60 \text{ ksi} \)
   - Concrete: \( f'_c = 4.5 \text{ KSI} \)

3. **Future Wearing Surface Load:** 60 PSF
4. Overhang barrier designs valid for:
   • BR-1
   • BR-2-98
   • SBR-1-99

   If TST or other barriers shapes used, check capacity of overhang reinforcement per LRFD section 13

   Overhang designs based on 42” BR-1 barrier
NOTES/ASSUMPTIONS:

5. Design Controls:
   - Integral wearing surface = 1"
   - Minimum transverse steel spacing = 5"
   - Crack control factor, $\gamma_e = 0.75$ (Class 2)
   - Minimum overhang deck thickness = interior deck thickness + 2” (overhang deck thickness shall be clearly shown on plans, i.e. on typical section).
   - $\frac{1}{4}”$ increments for bar spacing
   - Primary reinforcement $\perp$ to traffic
   - 2 $\frac{1}{2}”$ top cover
   - 1 $\frac{1}{2}”$ bottom cover
Deck - Example

NOTES/ASSUMPTIONS:

6. Decks supported on 4 or more beam lines
   - Deck design moments as follows:
     \[ +M_{DL} = 0.0772 \, w \, S_{eff.}^2 \]
     \[ +M_{LL+I} = \text{LRFD Table A4-1} \]
     \[ -M_{DL} = -0.1071 \, w \, S_{eff.}^2 \]
     \[ -M_{LL+I} = \text{LRFD Table A4-1} \]
Deck - Example

Typical Section

5 - W36x170 Spa. @ 10'-0" = 50'-0"

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**Deck - Example**

**Design Information:**

- Live Load = HL-93 [LRFD 3.6.1.2]
- Impact = 33% [LRFD 3.6.1.2]
- F.W.S. = 0.06 k / ft² [BDM 301.4]
- M.W.S. = 1” [BDM 302.1.3.1 & 302.2.1]
- $f'_c = 4.5$ ksi [BDM 302.1.1]
- $f_Y = 60$ ksi [BDM 301.5]
- 42” BR-1 Barriers

**Cover:**

- Top Layer: 2.5”
- Bottom Layer: 1.5”

**References**

- [LRFD 3.6.1.2]
- [BDM 301.4]
- [BDM 302.1.3.1 & 302.2.1]
- [BDM 302.1.1]
- [BDM 301.5]
Deck - Example

Deck Thickness:
Calculate Effective Span Length:………[LRFD 9.7.2.3 & 9.7.3.2]

\[ S_{\text{EFF}} = 10.0' - (12.03'' / 12) + ((12.03'' - .68'') / 12 / 2) = 9.47' \]

See Design Aid Table, based on \( b_f \) and \( b_w \)

\[ T_{\text{MIN}} = (9.47+17)*(12) / 36 = 8.823'' \geq 8.5'' \] [BDM 302.2.1]

Round up to nearest 1/4'' inch
Use \( T = 9.0'' \)

Overhang:.......................................................[LRFD 13.7.3.1.2]
\[ T_{\text{MIN}} = 9.0'' + 2.0'' = 11.0'' \] (Assumes 2'' min haunch)
# Deck - Example

## Equivalent Strip Width:

<table>
<thead>
<tr>
<th>Interior Bay</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[LRFD 4.6.2.1.3]</td>
</tr>
<tr>
<td>+M: 26.0 + 6.6 S</td>
<td>S = 10.0’</td>
</tr>
<tr>
<td>+M: 26.0 + 6.6 (10) = 92” (information only)</td>
<td></td>
</tr>
<tr>
<td>-M: 48.0 + 3.0 S</td>
<td></td>
</tr>
<tr>
<td>-M: 48.0 + 3.0 (10) = 78” (information only)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Overhang</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[LRFD 4.6.2.1.3]</td>
</tr>
<tr>
<td>45.0 + 10.0 X</td>
<td></td>
</tr>
<tr>
<td>X = 3.5’ – 1.5’ – 1.0’ = 1.0’</td>
<td></td>
</tr>
<tr>
<td>(Applied load 1’-0” from barrier face [LRFD 3.6.1.3.1])</td>
<td></td>
</tr>
<tr>
<td>45.0 + 10.0 (1.0) = 55”</td>
<td></td>
</tr>
</tbody>
</table>
Deck - Example

Load Effects:

Dead Loads:

Deck slab self-weight (interior bay – 9”)

\[ = 0.75 \text{ ft} \times (1.0 \text{ ft}) \times (0.150 \text{ k/ft}^3) = 0.113 \text{ k/ft} \]

Deck Slab Self-weight (overhang 11”)

\[ = 0.92 \text{ ft} \times (1.0 \text{ ft}) \times (0.150 \text{ k/ft}^3) = 0.138 \text{ k/ft} \]

Barrier

\[ = 0.150 \text{ k/ft}^3 \times (1.0 \text{ ft}) \times (500.5 \text{ in}^2)/144 \text{ in}^2/\text{ft}^2 = 0.521 \text{ kip/ft} \]

F.W.S.

\[ = 0.06 \text{ k/ft}^2 \times (1.0 \text{ ft}) = 0.06 \text{ k/ft} \]
Deck - Example

Live Load:

- HL-93.................................................................[LRFD 3.6.1.2]
- No design lane load (beam spacing < 15’) [LRFD 3.6.1.3.3]
- Dynamic Load Allowance = 1.33....................[LRFD 3.6.2]
- Multiple Presence Factor (m):.......................[LRFD 3.6.1.1.2]
  - m = 1.2 (Single lane loaded)
  - m = 1.0 (Two lanes loaded)

-M critical section:
  - Located 1/4 of flange width from support centerline......................[LRFD 4.6.2.1.6]
  - Location of Critical Section (for interior bay design only) = 12.03” / 4 = 3.008”
Moments:

Designer has option of generating moments based on:

- Continuity analysis
- Closed formed formula such as the 4-span continuous moments equations presented below
- Table A4-1 for live loads

If continuity analysis is performed, consideration shall be given to part width construction or other project specific design cases.
Deck - Example

Positive Moment (based on 4-span continuous):

\[ M_{DC} = 0.0772 \times w_{DC} \times S_{EFF}^2 \]

\[ = 0.0772 \times 0.113 \text{ k/ft} \times (9.47')^2 = 0.78 \text{ k-ft/ft} \]

\[ M_{DW} = 0.0772 \times w_{DW} \times S_{EFF}^2 \]

\[ = 0.0772 \times 0.06 \text{ k/ft} \times (9.47')^2 = 0.42 \text{ k-ft/ft} \]

\[ M_{LL+I} \text{ (from LRFD Table A4-1)} = 6.89 \text{ k-ft/ft} \]

Strength I Design Moment (+\(M_U\))

\[ = 1.25(0.78) + 1.50(0.42) + 1.75(6.89) = 13.65 \text{ kip-ft/ft} \]

Service I Design Moment (+\(M_W\))

\[ = 0.78 + 0.42 + 6.89 = 8.08 \text{ kip-ft/ft} \]
### Deck Example

**Table A4-1 Maximum Live Load Moments Per Unit Width, kip-ft./ft.**

| S  | + M | - M  
|----|-----|------
|    |     | Distance from Girder CL to Design Section for -M |
|    |     | 0 in | 3 in | 6 in | 9 in | 12 in | 18 in | 24 in |
| 4’- 0” | 4.68 | 2.68 | 2.07 | 1.74 | 1.60 | 1.50 | 1.34 | 1.25 |
| 4’- 3” | 4.66 | 2.73 | 2.25 | 1.95 | 1.74 | 1.57 | 1.33 | 1.20 |
| 4’- 6” | 4.63 | 3.00 | 2.58 | 2.17 | 1.90 | 1.65 | 1.32 | 1.18 |
| 10’-0” | 6.89 | 7.85 | 6.99 | 6.13 | 5.26 | 4.41 | 4.09 | 3.77 |
| 10’-3” | 7.03 | 8.19 | 7.32 | 6.45 | 5.58 | 4.71 | 4.29 | 3.96 |
Negative Moment (based on 4-span continuous):

\[ M_{DC} = -0.1071 \times w_{DC} \times S_{EFF}^2 \]
\[ = -0.1071 \times 0.113 \text{ k/ft} \times 9.47^2 = -1.08 \text{ k-ft/ft} \]

\[ M_{DW} = -0.1071 \times w_{DW} \times S_{EFF}^2 \]
\[ = -0.1071 \times 0.06 \text{ k/ft} \times 9.47^2 = -0.58 \text{ k-ft/ft} \]

\[ M_{LL+I} \text{ (from LRFD Table A4-1)} = -6.99 \text{ k-ft/ft} \]

Strength I Design Moment \((-M_U)\)
\[ = 1.25(-1.08) + 1.50(-0.58) + 1.75(-6.99) = -14.44 \text{ kip-ft/ft} \]

Service I Design Moment \((-M_W)\)
\[ = -1.08 - 0.58 - 6.99 = -8.64 \text{ kip-ft/ft} \]
Deck - Example

Interior Bay Deck Section, Looking Along Bridge
Reinforced Concrete Design:

\[ T = 9.0'' \]

\[ d_b = 6.1875'' \]

\[ d_t = 5.6875'' \]

\[ \phi = 0.90 \text{ (flexure)} \] \[ \ldots \text{ [LRFD 5.5.4.2.1]} \]

\[ n = 8 \]

Negative Moment:

Strength I:

\[ R = \frac{-M}{\phi b d_t^2} = \frac{14.44 (12)}{0.9 (12) (5.6875)^2} = 0.496 \text{ ksi} \]

Needs to be checked
Deck - Example

\[
\rho_{\text{req}} = 0.85 \left( \frac{f'}{f_y} \right) \left[ 1 - \sqrt{1- \frac{2R}{0.85 f'_c}} \right]
\]

\[
= 0.85 \left( \frac{4.5}{60} \right) \left[ 1 - \sqrt{1- \frac{2(0.496)}{0.85 (4.5)}} \right] = 0.00889
\]

\[
A_s \geq \rho b d_t = 0.00889 (12) \times 5.6875 = 0.607 \text{ in}^2/\text{ft}
\]

Max. Bar Spacing \( \leq (0.31/0.607) \times 12 = 6.13'' \) say 5\(3/4'' \) (\(A_s = 0.647 \text{ in}^2/\text{ft}; \ \rho_{\text{act}} = 0.00948\))

**Note:** 6" spacing adequate for strength, however, cracking check below requires 5 \(3/4'' \) spacing
Deck - Example

\[ \rho_{\text{req}} = 0.85 \left( \frac{f'_c}{f_y} \right) \left[ 1 - \sqrt{1 - \frac{2R}{0.85 f'_c}} \right] = 0.85 \left( \frac{4.5}{60} \right) \left[ 1 - \sqrt{1 - \frac{2 (0.496)}{0.85 (4.5)}} \right] = 0.00889 \]

\[ A_s \geq \rho_b d_t = 0.00889 \times 12 \times 5.6875 = 0.607 \text{ in}^2/\text{ft} \]

or

\[ A_s = \frac{M_u}{\phi f_y j d} = \frac{14.44(12)}{0.9(60)(0.95)5.6875} = 0.5939 \text{ in}^2/\text{ft} \]

Max. Bar Spacing \( \leq (0.31/0.607) \times 12 = 6.13” \) say 5\( \frac{3}{4} ” \) (\( A_s = 0.647 \text{ in}^2/\text{ft}; \ \rho_{\text{act}} = 0.00948 \))

**Note:** 6” spacing adequate for strength, however, cracking check below requires 5 \( \frac{3}{4} ” \) spacing
Deck - Example

Check minimum reinforcement........................[LRFD 5.7.3.3.2]

\[
1.2 \frac{M_{cr}}{S_{cr}} = 1.2 \frac{S_{cr}}{f_{cr}}
\]

\[
= 1.2 \frac{(12'')(9'')^2}{6} \frac{(0.37)\sqrt{4.5}}{12} = 12.71 \text{ k-ft/ft}
\]

\[
\phi M_n = 0.9 (60)(0.647)(5.6875 - \frac{0.846}{2}) / 12
\]

\[
= 15.33 \text{ k-ft/ft} > 12.71 \text{ O.K.}
\]

(If failed, provide reinforcement for 1.33*M_{u})
Deck - Example

Check for cracking under Service I limit state……[LRFD 5.7.3.4]

Check if stress at extreme fiber is less than 0.8* f_r:

\[ 0.8 \frac{f_r}{M} = 0.8 \times 0.24 \sqrt{4.5} = 0.407 \text{ ksi} \]

\[ \frac{f_{act}}{12 (9)^2} = \frac{8.64 (12)}{6} = 0.640 \text{ ksi} > 0.407 \text{ NG, check spacing limit} \]

\[ s \leq \frac{700 \gamma}{\beta \frac{f_s}{s}} - 2 \frac{d_c}{s} \]
\[ \gamma_e = 0.75 \]

\[ \beta_S = 1 + \frac{d}{0.7(h - d)} \]

\[ f_s = \frac{-M}{A_s j d_t} \]

\[ k = \sqrt{2\rho_n + (\rho_n)^2} - \rho_n \]

\[ = \sqrt{2(0.00948 \times 8) + (0.00948 \times 8)^2} - (0.00948 \times 8) = 0.3209 \]
Deck - Example

\[ j = 1 - \frac{0.3209}{3} = 0.893 \]

\[ f = \frac{8.64 \times 12}{0.647 (0.893) 5.6875} = 31.57 \text{ ksi} \]

\[ s_{\text{max}} = \frac{700 (0.75)}{1.5808 (31.57)} - 2 (2.313) = 5.89" > 5.75" \text{ O.K.} \]
Deck - Example

\[ 12” \]

\[ \begin{align*}
\text{nA}_s &= 8(0.647) = 5.176 \\
12 \times (x/2) &= 5.176 (d_t - x) \\
6x^2 + 5.176x - 29.4385 &= 0 \\
x &= 1.825” \\
I &= (12*(1.825^3))/12 + 12(1.825)(1.825/2)^2 + 5.176(5.6875-1.825)^2 \\
&= 101.5 \\
f &= n \frac{M_y}{I} \\
&= 8 \frac{8.64(12)(5.6875 - 1.825)}{101.5} = 31.55 \text{ ksi}
\end{align*} \]
Deck - Example

Check tension controlled ($\phi = 0.9$)..........................[LRFD 5.7.2.1]

\[
\text{Max. } A_s = \frac{0.32 f' c \beta b d}{f y} = \frac{0.32 (4.5) 0.825 (12) 5.6875}{60} = 1.351 \text{in}^2/\text{ft}
\]

\[
\frac{c}{d} \leq 0.375 \quad \Rightarrow \quad \frac{A}{s y} \leq 0.375
\]

\[
A_s \leq \frac{0.375 (0.85) \beta f' b d}{f y} = \frac{0.32 \beta f' b d}{f y}
\]

Actual $A_s = 0.647 \text{in}^2/\text{ft} < 1.351$  OK

Use #5’s at 5 3/4" c/c spacing in top transverse layer
Positive Moment:

Strength I:

\[ R = \frac{+M}{\phi b d^2} \]

\[ = \frac{13.65}{0.9 (1.0)(6.1875)^2} = 0.396 \text{ ksi} \]

\[ \rho_{\text{req}} = 0.85 \left( \frac{f'}{f} \right) \left[ 1 - \sqrt{1 - \frac{2R}{0.85 f' c}} \right] \]

\[ = 0.85 \left( \frac{4.5}{60} \right) \left[ 1 - \sqrt{1 - \frac{2(0.396)}{0.85(4.5)}} \right] = 0.00699 \]
### Deck - Example

\[
A_s \geq \rho b d_b = 0.00699 \times (12) \times 6.1875 = 0.519 \text{ in}^2/\text{ft}
\]

Max. Bar Spacing

\[
\leq \frac{0.31}{0.519} (12) = 7.17''
\]

Try 5.75" to match top transverse spacing

\(A_s = 0.647 \text{ in}^2/\text{ft}; \rho_{\text{act}} = 0.00871\) ...........[BDM 302.2.4.2]
Check minimum reinforcement……………………….[LRFD 5.7.3.3.2]

\[ 1.2 \frac{M_{cr}}{S_{cr}} f_{c, r} = 1.2 \frac{(12”)(9”)^2}{6} \cdot \frac{(0.37) \sqrt{4.5}}{12} = 12.71 \text{ k-ft/ft} \]

\[ \phi M_n = 0.9 (60) \cdot 0.647 (6.1875 - \frac{0.846}{2}) / 12 \]
\[ = 16.78 \text{ k-ft/ft} > 12.71 \text{ O.K.} \]

(If fails, provide reinforcement for 1.33 \( M_u \))
Deck - Example

Check cracking under Service I limit state...........[LRFD 5.7.3.4]

Check if stress at extreme fiber is less than $0.8 \times f_r$:

$$0.8 f_r = 0.8 (0.24) \sqrt{4.5} = 0.407 \text{ ksi}$$

$$f_{act} = \frac{8.08 (12)}{12 (9)^2} = \frac{8.08 \times 12}{12 \times 81} = \frac{96.96}{972} = 0.0999 \text{ ksi} > 0.407 \text{ NG, check spacing limit}$$

$$s \leq \frac{700 \gamma_e}{\beta f_s} - 2 d_c$$
\( \gamma_e = 0.75 \) ..........................[BDM 1005]

\[
\beta_s = 1 + \frac{d}{0.7(h-d_c)} = 1 + \frac{1.813}{0.7(8-1.813)} = 1.4185
\]

\[
f_s = \frac{+M}{A_s \frac{j d_b}{w}}\]

\[j = 1 - \frac{k}{3}\]

\[k = \sqrt{2\rho n + (\rho n)^2} - \rho n\]

\[= \sqrt{2(0.00871 \times 8 + (0.00871 \times 8)^2} - (0.00871 \times 8) = 0.310\]
Deck - Example

\[ j = 1 \frac{0.310}{3} = 0.897 \]

\[ f_s = \frac{8.08 \times 12}{0.647 \times 0.897 \times 6.1875} = 27.03 \text{ ksi} \]

\[ s_{\text{max}} = \frac{700 \times 0.75}{1.4185 \times 27.03} - 2 \times 1.813 = 10.07" > 5.75" \text{ O.K.} \]

Check if tension controlled \((\phi = 0.9)\)\ldots \ldots [LRFD 5.7.2.1]

\[ \text{Max. } A_s = \frac{0.32 f_y^\beta b d}{f_c^1} = \frac{0.32 (4.5) 0.825 (12) 6.1875}{60} = 1.470 \text{ in}^2/\text{ft} \]

Actual \(A_s = 0.647 \text{ in}^2/\text{ft} < 1.470 \text{ OK} \)

**Use #5's at 5 3/4" c/c spacing in bottom transverse layer**
Deck - Example

Distributional Reinforcement:

Top: .................................................................[BDM 302.2.4.1]

\[ A_{S_{\text{Dist}}} \geq \frac{1}{3} A_s \text{ Primary} = \frac{1}{3} (0.647 \text{ in}^2/\text{ft}) = 0.216 \text{ in}^2/\text{ft} \]

Max. Bar Spacing \[ \leq \frac{0.20}{0.216} \times 12 = 11.13'' \text{ Say 11'' spacing} \]

Use #4’s @ 11'' c/c spacing \( (A_s = 0.218 \text{ in}^2/\text{ft}) \) in top longitudinal mat

Bottom: .................................................................[LRFD 9.7.3.2]

\[ A_{S_{\text{Dist}}} \geq \text{ Lesser of } \begin{cases} \frac{2.20}{\sqrt{S}} A_{s \text{ Primary}} \\ 0.67 A_{s \text{ Primary}} \end{cases} \]

Note: Use required bottom reinforcement in lieu of provided
Deck - Example

\( S_{\text{eff}} = 9.47' \) ................................................................. [LRFD 9.7.2.3]

\[
A_{s \text{ Dist}} \geq \text{Lesser of} \begin{cases} 
\frac{2.20}{\sqrt{9.47}} (0.519) = 0.371 \text{ in}^2/\text{ft} \\
0.67(0.519) = 0.348 \text{ in}^2/\text{ft}
\end{cases}
\]

\[
\text{Max. Bar Spacing} \leq \frac{0.31}{0.348} \times 12 = 10.69'' \quad \text{Say 10.5” spacing}
\]

Use #5’s @ 10 ½” c/c spacing \((A_s = 0.356 \text{ in}^2/\text{ft})\) in bottom longitudinal mat

Note: Modify (tighten) spacing as needed to avoid girder top flanges while providing even spacing throughout bay
Deck - Example

Interior Bay Reinforcement Summary

**TRANSVERSE REINFORCEMENT:** Use #5 bars @ 5 ¾” c/c spacing Top and Bottom

**LONGITUDINAL REINFORCEMENT:** Use #4 bars at 11” c/c spacing Top and #5 bars at 10 ½” c/c spacing Bottom

*Note:* Negative moment (over pier) reinforcement designed in accordance with LRFD 6.10.1.7 (Steel) or 5.7.3.2 (Prestressed). See 6.10.1.7 for cutoff points.
Overhang Load Effects

For simplicity, assume critical sections occur at CL of exterior beam and @ barrier toe

Note: Barrier centroid is approx. 5 11/16” (0.474’) from deck edge

Load Effects at CL Exterior Beam:

\[ M_{DC} = M_{SLAB} + M_{BARRIER} \]

\[ M_{SLAB} = -0.5(0.138 \text{ k/ft})(3.5\text{'})^2 = -0.845 \text{ k-ft/ft} \]

\[ M_{BARRIER} = -0.521 \text{ kip}(3.5\text{' } - 0.474\text{'}) = -1.577 \text{ k-ft/ft} \]

\[ M_{DC} = -0.845 - 1.577 = -2.422 \text{ k-ft/ft} \]

\[ M_{DW} = M_{FWS} = -0.5(0.06 \text{ k/ft})(3.5\text{' } - 1.5\text{'})^2 = -0.120 \text{ k-ft/ft} \]
**Deck - Example**

\[
M_{LL+I} = -\frac{16 \text{kip} \times (3.5' - 1.5' - 1.0') \times (1.33) \times (1.2) \times 12}{55"} = -5.571 \text{ k-ft/ft}
\]

**Note:** Ignore vertical LL+I for Design Cases 1 & 2 since overhang < 7 ………………………………………..[BDM 1013, A13.4.1]

\[
M_{CT} = -\frac{R \times H}{L + 2H + 2X}
\] .................................[BDM 1013]

- \( R = 165.0 \text{ kip} \)
- \( L_C = 12.4' \)
- \( H = 3.5' \)
- \( X = 3.5' - 1.5' = 2.0' \)
Deck - Example

\[
M_{CT_H} = - \frac{165.0 \times (3.5)}{12.4 + 2 \times (3.5) + 2 \times (2.0)} = -24.679 \text{ k-ft}
\]

\[
T_{CT} = \frac{R}{L + 2H + 2X} = \frac{165.0}{12.4 + 2 \times (3.5) + 2 \times (2.0)} = 7.051 \text{ kip}
\]

Load Effects at Toe of Barrier:

\[
M_{DC} = M_{SLAB} + M_{BARRIER}
\]

\[
M_{SLAB} = -0.5 \times (0.138 \text{ k/ft}) \times (1.5')^2 = -0.155 \text{ k-ft}
\]

\[
M_{BARRIER} = -0.521 \text{ kip} \times (1.5' - 0.474') = -0.535 \text{ k-ft}
\]

\[
M_{DC} = -0.155 - 0.535 = -0.690 \text{ k-ft}
\]

\[
M_{DW} = 0.0 \text{ k-ft} \quad \text{and} \quad M_{LL+I} = 0.0 \text{ k-ft}
\]
Deck - Example

\[ M_{CTH} = - \frac{RH}{L_C + 2H + 2X} \]

- \( R = 165.0 \text{ kip} \)
- \( L_C = 12.4' \)
- \( H = 3.5' \)
- \( X = 0.0' \)

\[ M_{CTH} = - \frac{165.0 \times 3.5}{12.4 + 2 \times 3.5 + 2 \times 0.0} = -29.768 \text{ k} - \text{ft/ft} \]

\[ T_{CT} = \frac{R}{L_C + 2H + 2X} \]

\[ T_{CT} = \frac{165.0}{12.4 + 2 \times 3.5 + 2 \times 0.0} = 8.505 \text{ kip/ft} \]
Design Case 1: Transverse and longitudinal vehicle impact forces for Extreme Event II limit state

Determine controlling location:

At CL Exterior Beam:

\[ M_u = \eta \left[ 1.0 M_{DC} + 1.0 M_{DW} + M_{CT} \right] \]

\[ M_u = 1.0 \left[ 1.0 \left( -2.422 \right) + 1.0 \left( -0.120 \right) + \left( -24.679 \right) \right] \]

\[ = -27.22 \text{ kips-ft} \]

\[ T_u = \eta \left[ T_{CT} \right] = 1.0 \left[ 7.051 \right] = 7.051 \text{ kips} \]
At Toe of Barrier:

\[ M_u = \eta \left[ 1.0 M_{DC} + 1.0 M_{DW} + M_{CT} \right] \]

\[ M_u = 1.0 \left[ 1.0 (-0.155 - 0.535) + 1.0 (0) + (-29.768) \right] \]
\[ = -30.46 \text{ k} - \text{ft} \]

\[ T_u = \eta \left[ T_{CT} \right] = 1.0 \left[ 8.505 \right] = 8.505 \text{ kip} \]

Therefore, based on larger design moment, Toe of Barrier location controls overhang design for Design Case 1.
Overhang Deck Section, Looking Along Bridge
Reinforced Concrete Design, Overhang:

\( T = 11.0'' \)

\( d_t = 7.6875'' \) (Assume #5 bars)

\( \phi = 1.0 \) \[LRFD 1.3.2.1 \) (Extreme Event)\]

\( n = 8 \)

Design options for top transverse overhang reinforcement:

1. Check if interior bay top transverse reinforcement is sufficient

2. Bundle a #4, #5, or #6 bar w/ top interior transverse reinforcement

3. Upsize all top transverse reinforcement w/ same spacing as interior bay
By inspection Option 1 will not work. Calcs for Option 2 shown below:

\[ R = \frac{-M}{\phi b d^2 t} = \frac{30.46}{1.0 (1.0) (7.6875)^2} = 0.515 \text{ ksi} \]

\[ \rho = 0.85 \left( \frac{f'}{c} \right) \left[ 1 - \sqrt{1 - \frac{2R}{0.85f'}} \right] = 0.85 \left( \frac{4.5}{60} \right) \left[ 1 - \sqrt{1 - \frac{2(0.515)}{0.85(4.5)}} \right] = 0.00926 \]

\[ A_S \geq \rho b d \]

\[ = 0.00926(12)7.6875 = 0.854 \text{ in}^2 / \text{ft} \]

Try bundling #4 bars with #5 bars spaced at 5 3/4” c/c

\[ (A_S = 1.064 \text{ in}^2 / \text{ft}; \rho_{act} = 0.01153) \]
Verify that reinforcement can carry additional tension force, $T_u$:

$$ M_u \leq \phi M_n \left( 1.0 - \frac{P_u}{\phi P_n} \right) $$

$$ M_n = A_{fy} \left( d - \frac{a}{2} \right) $$

$$ a = A_{fy} \frac{s}{0.85 f'c} b = \frac{1.064 (60)}{0.85 (4.5) 12} = 1.391" $$

$$ M_n = 1.064 (60) \left( 7.6875 - \frac{1.391}{2} \right) = 446.4 \text{ k-in} = 37.20 \text{ k-ft} $$
Deck - Example

\[ P_u = T_u = 8.505 \text{ kip/ft} \]

\[ P_n = A_s f_y \text{ (Use top overhang & bottom trans. reinforcement for } A_s) \]

\[ = (1.064 + 0.647) \times 60 = 102.66 \text{ kip/ft} \]

\[ M_u = 30.46 \text{ k-ft} < 1.0 \times (37.20) \left( 1.0 - \frac{8.505}{1.0 \times (102.66)} \right) = 34.12 \text{ k-ft} \quad \text{O.K.} \]

Check minimum reinforcement……………………………………[LRFD 5.7.3.3.2]

\[ 1.2 \frac{M}{cr} = 1.2 \frac{S}{cr} \]

\[ = 1.2 \times \frac{(12”)(11”)^2}{6} \times \frac{(.37)\sqrt{4.5}}{12} = 18.99 \text{ k-ft/ft} \]

\[ \phi M_n \left( 1.0 - \frac{P_u}{\phi P_n} \right) = 34.12 \text{ k-ft/ft} > 18.99 \quad \text{O.K.} \]
Deck - Example

Check concrete assumptions........................................[LRFD 5.7.2.1]

\[
\text{Max. } A_s = \frac{0.32 \ f'_c \ \beta \ b \ d}{f_y}
\]

\[
= \frac{0.32(4.5)0.825(12)7.6875}{60} = 1.827 \text{ in}^2/\text{ft}
\]

Actual As = 1.064 in\(^2/\text{ft}\) < 1.827 OK \ (\phi = 0.9)

Check development length at toe of barrier
(Check #5 bar of bundle): ........................................[LRFD 5.11.2.1]

\[
L_{db} = \frac{1.25 A_y}{b_y} \sqrt{\frac{f'_c}{f_y}}
\]

\[
= \frac{1.25 (0.31) 60}{\sqrt{4.5}} = 11.0''
\]

\[L_{db} = \text{Greater of}\]

\[
0.4d_y \frac{f'_c}{b_y}
\]

\[= 0.4(0.625)60 = 15'' \ (\text{controls})\]
Modification Factors:

- $3\cdot d_b \text{ cover} = 1.0$
- $6\cdot d_b \text{ clear} = 1.0$
- $A_{\text{req'd}} / A_{\text{sprov}} = 30.63 / 34.12 = 0.898 \text{ (based on moments)}$
- Epoxy-coated = 1.2

$$l_h = 15'' \cdot 0.898 \cdot 1.2 = 16.16'' < (18'' - 2'') = 16'' \text{ SAY OK}$$

Note: Use hooked bar if straight bar cannot be developed. Check LRFD 5.11.2.4.1
Deck - Example

Design Case 2:
Vertical vehicle impact force for Extreme Event II limit state (only applies to post and beam railing systems). Not required here

Design Case 3:
Strength I limit state at CL Exterior Beam

\[
M_u = \eta \left[ 1.25 M_{DC} + 1.50 M_{DW} + 1.75 M_{LL} + I \right]
\]

\[
= 1.0 \left[ 1.25 (-2.422) + 1.5 (-0.12) + 1.75 (-5.571) \right] = -12.96 \text{ kip} \cdot \text{ft}
\]

\[
\phi M_n = 0.9 \times (37.20 \text{ kip} \cdot \text{ft}) = 33.48 \text{ kip} \cdot \text{ft} > M_u \text{ OK}
\]
Check minimum reinforcement \[ \phi M_n = 33.48 \text{k-ft/ft} > 18.99 \text{ OK} \]

(If fails, provide reinforcement for 1.33*Mu)

Check cracking under Service I @ CL Exterior Beam...\[ \text{LRFD 5.7.3.4} \]

Calculate stress at extreme fiber:

\[ 0.8 f_r = 0.8 (0.24) \sqrt{4.5} = 0.407 \text{ ksi} \]

\[ f_{act} = 8.113 \text{ k-ft} (12) / (12” (11”)^2 / 6) = 0.402 \text{ ksi} < 0.407 \]

OK, Do not check spacing limit
Calculate cutoff point for top overhang reinforcement beyond CL Exterior Beam:

Use 49” beyond CL Exterior Beam. Distance calculated by finding point in first interior bay where typical top interior transverse reinforcement sufficient and extending additional overhang reinforcement a development length beyond that point.
OVERHANG REINFORCEMENT SUMMARY

TRANSVERSE REINFORCEMENT:

– Use bundled #4 and #5 bars @ 5 3/4” c/c spacing
– Extend #4 overhang bars 49” beyond CL Exterior Beam
– Straight bars sufficient for this example, but standard 180° hooks on the fascia bar end may be necessary for other overhang designs

TOP LONGITUDINAL REINFORCEMENT:

– Use #4 bars at 11” c/c spacing (same as interior bay)

BOTTOM REINFORCEMENT:

– Use same as interior bay in both directions
Piers
Piers

Loads/Design (11.7.1)

- Transmit loads from superstructure and itself to foundation
- Loads/load combinations per Section 3
- Design per appropriate material section
Piers

• Protection (11.7.2)

• Collision (11.7.2.1)
  – Risk analysis made for possible traffic collision from highway or waterway to determine degree of impact resistance and/or appropriate protection system
  – Collision loads per 3.6.5 (vehicle) and 3.14 (vessel)

**ODOT - 3.6.5 applies only to nonredundant piers. Clear zone requirements and roadside barrier warrants specified in ODOT Location and Design Manual, Section 600 provide adequate protection for redundant piers. BDM Section 204.5 specifies design considerations and restrictions for cap and column piers for highway grade separation bridges and railway overpass bridges.**
Piers

• **Vessel Collision: CV (3.14)**
  
  – In navigable waterways (≥2’ water depth) where vessel collision is anticipated, structures shall be:
    
    • Designed to resist vessel collision forces and/or
    
    • Adequately protected by fenders, dolphins, berms, islands or sacrificial devices

*ODOT – Apply only if specified in scope*

*(Vessel forces skipped due to infrequency and time limitations)*
• **Collision Walls (11.7.2.2)**
  – Railroads may be require collision walls for piers close to rail

  ODOT (BDM 209.8) - Piers < 25’-0” from track centerline require a crash wall unless T-type or wall type pier used. Crash wall height ≥ 10 feet above top of rail. If pier located < 12’ track centerline, height ≥ 12 feet above the top of rail. Crash wall at least 2’-6” thick. For cap and column pier, face of wall shall extend 12” beyond column faces on track side. Crash wall anchored to footings and columns.

• **Scour (11.7.2.3)**
  – Scour potential be determined and designed for per 2.6.4.4.2

• **Facing (11.7.2.4)**
  – Pier nose designed to break-up or deflect floating ice or drift
Strut and Tie
Strut and Tie

References


http://cee.uiuc.edu/kuchma/strut_and_tie/STM/EXAMPLES/DBeam/dbeam(1).htm

Strut and Tie

Background

In design of R/C and P/C, two regions
- B-Regions (flexural or bending regions)
- D-Regions (regions of discontinuities)

B-Regions

- Plane sections before bending remain plane after bending
- Shear stresses distributed relatively uniform over region
- Designed by sectional methods
D-Regions

- Plane sections before bending do not remain plane after bending
- Shear stresses not uniformly distributed over region
- Abrupt changes in X-sect., concentrated loads, reactions (St. Venant’s Principle $\sigma = P/A$ at 2$d$ from point of loading)
- Designed by Strut and Tie

General (5.6.3.1)

- Used to design pile caps, deep footings, and other situations where distance between centers of applied load and reaction is $< \text{approximately } 2 \times \text{member thickness}$
Strut and Tie

Procedure

- Visualize flow of stresses
- Sketch strut and tie model (truss)
- Select area of ties
- Check strut strengths
- Check nodal zones
- Assure anchorage of ties
- Crack control reinforcement
Figure C5.6.3.2-1 Strut-and-Tie Model for a Deep Beam
Structural Modeling (5.6.3.2)

- Factored resistance of the struts and ties shall be:

\[ P_r = \Phi \, P_n \]

- where

- \( \Phi \) = tension or compression resistance factor, as appropriate.

- \( P_n \) = Nominal resistance of strut or tie per 5.6.3.3 or 5.6.3.4.
Compressive Struts (5.6.3.3)

• The nominal resistance of a compressive strut is:

\[ P_n = f_{cu} A_{cs} + f_y A_{ss} \]

– where

• \( A_{cs} \) = Effective cross-sectional area of strut (see Figs on following slides)
• \( A_{ss} \) = Area of reinforcement in strut
• \( f_y \) = yield strength of strut reinforcement
• \( f_{cu} \) = limiting compressive stress taken as:
Compressive Struts (5.6.3.3)

\[ f' \leq 0.85 f' \]
\[ \frac{f}{c_{tu}} = \frac{c}{0.8 + 170 \varepsilon_1} \]

- in which:

\[ \varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \alpha_s \]

- where

- \( \varepsilon_s \) = tensile strain in the concrete in direction of tension tie
- \( \alpha_s \) = smallest angle between compressive strut and adjoining tension ties
Strut and Tie

a) Strut Anchored by reinforcement
Strut and Tie

b) Strut anchored by bearing and reinforcement

c) Strut anchored by bearing and strut

/bsinθs + ha cosθs

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Tension Ties (5.6.3.4)

- The nominal resistance of a tension tie is:

\[ P_n = f_y A_{st} + A_{ps} (f_{pe} + f_y) \]

- where

  - \( A_{st} \) = area of longitudinal mild steel
  - \( A_{ps} \) = area of prestressing steel
  - \( f_y \) = yield strength of mild reinforcement
  - \( f_{pe} \) = stress in prestressing steel after losses

Note: Tension steel must be properly anchored per 5.11
Node Regions (5.6.3.5)

- Compression stress in node regions cannot exceed:
  - $\Phi \ 0.85 f'_c$ if bounded by compressive struts and bearing areas
  - $\Phi \ 0.75 f'_c$ if anchoring a one-direction tension tie
  - $\Phi \ 0.65 f'_c$ if anchoring tension ties in more than one direction

(Note: higher stresses allowed if confining reinforcement provided and its effect supported by analysis or experimentation)

- $\Phi = \text{resistance factor for bearing (0.70)}$

- Tension tie reinforcement shall be uniformly distributed over an area $\geq$ the tension tie force divided by the stress limits listed above
Node Regions (5.6.3.5)

- Figure C5.6.3.2-1 Strut-and-Tie Model for a Deep Beam
Crack Control Reinforcement (5.6.3.6)

- Orthogonal grid of reinforcement must be placed near each face of the component.
- Bar spacing in grid ≤ 12”
- Ratio of reinforcement area to gross concrete area ≥ 0.003 in each direction
T-Type Pier – Strut and Tie Model

2.5'  9'  9'  9'  9'  9'  9'  2.5'

3'

4'

30'
Pile Cap – Strut and Tie Model
Strut and Tie - Example

Parameters:

- $f'_c = 4$ ksi
- Cap thickness = 36”
- $f_y = 60$ ksi
- 36” dia. Columns
- Loads include wt. of cap
- Bearing plates 27” x 21”

![Diagram of strut and tie example with loads and dimensions]
Strut and Tie - Example

Sketch Truss Model

- 446 k
- 481 k
- 481 k
- 446 k
- 3.33'
- 2.72'
- 1' 6"
- CL
Location of forces (reactions) into columns

Assume forces create uniform stress in cols.

This requires left force to be 0.72’ from edge (0.78’ from center) and right force to be 0.78 from edge (0.72’ from center).

\[
\begin{align*}
\text{Left force:} & \quad 0.72' \\
\text{Right force:} & \quad 0.78'
\end{align*}
\]

\[
\begin{align*}
446 \text{ k} & \quad 481 \text{ k} \\
0.72' & \quad 0.78' & \quad 0.72' & \quad 0.78'
\end{align*}
\]

\[
446 \times 0.78 = 481 \times 0.72
\]
Strut and Tie - Example

Check Bearing on Plate

\[ A_{\text{bearing}} = \frac{P_u}{0.65 \phi f'_c} \]

\[ = \frac{481k}{0.65 (0.7)(4)} \]

\[ = 264 \text{ in}^2 \]

Plate = 21 \times 27

\[ = 567 \text{ in}^2 \]

\[ > 264 \text{ in}^2 \]

ok
Strut and Tie - Example

Forces within Model

446 k

50.8

364 k

34.9

481 k

-326 k

0 k

0 k

3.33'

-576 k

-841 k

326 k

326 k

446 k

481 k

2.72' 1' 6'' 4.78'

CL
Strut and Tie - Example

Tension Ties

Top (1-4)

\[ A_{st} = \frac{P_u}{\phi f_y} = \frac{364}{0.9 \times (60)} = 6.7 \text{ in}^2 \]

Say 7 #9’s

Bottom (3-5)

\[ A_{st} = \frac{P_u}{\phi f_y} = \frac{326}{0.9 \times (60)} = 6.0 \text{ in}^2 \]

Say 6 #9’s
Stirrups

No vertical tension members. However, 5.6.3.6 requires a grid w/ a spacing ≤ 12” and Steel Area = 0.003 A_g

Assume 12” spacing

\[ A_{st} = 0.003 \times (12) \times (36) = 1.30 \text{ in}^2 / \text{ft} \]

Use #5 w/ 4 legs

\[ s = \frac{0.31 \times (4) \times (12)}{1.30} = 11.4" \quad \text{say } s = 10" \]
Compression Strut 3-4

Node 4:

\[ \varepsilon_s = \frac{P_{ue}}{A_{est}E_s} \]

\[ = \frac{364}{(7)(29,000)} = 0.00179 \]

Strain in 1-4
Compression Strut 3-4

Strain @ center of node:
Conservatively assume strain from member 4-6 \( \approx 0 \)

\[
\varepsilon_s = \frac{0.00179 + 0}{2} = 0.0009
\]

Therefore

\[
\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2\alpha_s
\]

\[
= 0.0009 + (0.0009 + 0.002)\cot^2(34.9)
\]

\[
= 0.0069
\]
Compression Strut 3-4

Reduced strength

\[
f_{cu} = \frac{f'}{c} = \frac{4}{0.8 + 170 \varepsilon_1} \]

\[
= 2.03 \text{ ksi} \leq 0.85 f' = 3.4 \text{ ksi}
\]

\( f' \) is the reduced strength.
Strut and Tie - Example

Compression Strut 3-4

Size (Fig. 5.6.3.3.2-1b)

width

\[ w = l \sin(\theta) + h \cos(\theta) \]

plate is 27” wide \((l_b = 27”)\)
assume rebar 4” from surface \((h_a = 2(4) = 8”)\)

\[ w = 27 \sin(34.9) + 8 \cos(34.9) = 22” \]

depth

\[ d = \text{width of cap} = 36” \]

\[ 2 \times 6 \times 1.128 \times 4 = 54” > 36” \text{ width} \]
Compression Strut 3-4

Strength

\[ P_{n} = f_{cu} A_{cs} = 2.03 \times (22)(36) = 1,608 \text{ k} \]

\[ \phi P_{n} = 0.7 \times (1,608) = 1,125 \text{ k} > P_{u} = 841 \text{ k} \quad \text{O.K.} \]
Compression Strut 3-4

Node 3:

Strain in 3-5

\[
\varepsilon_s = \frac{P_u}{A_{st,E_s}} = \frac{326}{(6)(29,000)} = 0.00187
\]
Compression Strut 3-4

Strain @ center of node:

Strain in member 2-3 = 0

\[ \varepsilon_s = \frac{0.00187 + 0}{2} = 0.0009 \]

Therefore, as before

\[ \varepsilon_1 = 0.0069 \quad \& \quad f_{cu} = 2.03 \text{ ksi} \]
**Strut and Tie - Example**

**Compression Strut 3-4**

Size (Fig. 5.6.3.3.2-1b)

\[ w = l_b \sin(\theta) + h_a \cos(\theta) \]

\[ w = 18 \sin(34.9) + 8 \cos(34.9) = 16.9'' \]

**Part of column** = 1.5’ = 18” \((l_b = 18’’\))

**Assume rebar 4” from surface** \((h_a = 2(4) = 8’’\))

**Width**

\[ \text{d = Width of cap} = 36” \]
Compression Strut 3-4

Strength

\[ P = f_n A_{cu,cs} = 2.03 \times 16.9 \times 36 = 1,235 \text{k} \]

\[ \phi P_n = 0.7 \times 1,235 = 864 \text{k} > P_u = 841 \text{k} \text{ O.K.} \]
Check Nodal Zone 3

\[ f_c = 0.75 \Phi f'_c \]

\[ f_c = 0.75 \times 0.7 \times 4 = 2.1 \text{ ksi} \]

\[ f = \frac{P}{A_g} = \frac{326}{2 \times 4 \times 36} = 1.1 \text{ ksi} \]

OK
Check Nodal Zone 3

\[ f_c = \frac{841}{16.9 \ (36)} = 1.38 \text{ ksi} < 2.1 \text{ ksi} \quad \text{OK} \]

\[ f_c = \frac{481}{\pi \left[1.5 \ (12)\right]^2} = 0.945 \text{ ksi} < 2.1 \text{ ksi} \quad \text{OK} \]

\[ f_c = \frac{364}{8 \ (36)} = 1.26 \text{ ksi} < 2.1 \text{ ksi} \quad \text{OK} \]
Strut and Tie - Example

Tension Tie Anchorage
(Critical Tie is top)

Available Distance, $X = 2' + \left(\frac{27''}{2}\right) - 2''$

$X = 35.5''$

Critical Location for tie
Development of #9 in Tension (5.11.2)

\[ l_{db} = \frac{1.25A_f}{\sqrt{fy}} \cdot \frac{1.25(1)(60)}{\sqrt{4}} = 37.5" \]

Factors:
- 1.4 for top reinforcement w/ > 12” concrete below
- 1.5 for epoxy coated bars w/ cover < 3d_b or spacing < 6d_b

1.4 (1.5) = 2.1 > 1.7

use 1.7
\[ l_d = l_{db} (1.7) = 37.5(1.7) = 63.75" > X \]

\[ \therefore \quad \text{hook bars} \]
Strut and Tie - Example

Hooked Development Length

\[ l_{hb} = \frac{38d}{\sqrt[3]{f_c}} = \frac{38(1.128)}{\sqrt{4}} = 21.4'' \]

Factor:

1.2 for epoxy coated

\[ l_{dh} = 21.4 (1.2) = 25.7'' < X \quad \text{O.K.} \]

35.5’’
Crack Control Reinforcement

5.6.3.6 Requires orthogonal grid spaced ≤ 12” w/

\[
\frac{A_s}{A_g} \geq 0.003
\]

\[
A_s = 0.003 \times (12) \times (36) = 1.3 \text{ in}^2 / 1’
\]

Vertical:

Covered w/ stirrups
Strut and Tie - Example

Horizontal:

Say 1 bar each face

\[ \frac{1.3}{2} = 0.65 \text{ in}^2 / \text{face} / 12" \]

Available height:

Top & Bottom:
- Cover: 2.5"
- #5 Stirrups: 0.625"
- #9: 1.128"
- 4.25"

\[ 48" - 2 (4.25") = 39.5" \]
Strut and Tie - Example

Horizontal:

\[
\frac{0.65}{12} (48) = 2.6 \text{ in}^2
\]

Say 6 # 6’s \quad \text{As} = 6 (0.44) = 2.64 \text{ in}^2

\[
39.5 / 7 = 5.6" < 12" \quad \text{O.K.}
\]
Strut and Tie - Example

7 # 9's

6 # 6's

6 # 9's

6 # 6's

# 5's @ 10”
RETAINING WALLS
LIMIT STATES AND RESISTANCE FACTORS (11.5)

General 11.5.1

Design of abutments, piers and walls shall satisfy service limit state (11.5.2) and strength limit state (11.5.3) criteria.

Abutments, piers and retaining walls shall be designed to withstand:

- lateral earth and water pressures
- any live and dead load surcharge
- wall self weight
- temperature and shrinkage effects
- earthquake loads
LIMIT STATES AND RESISTANCE FACTORS (11.5)

General 11.5.1

Earth retaining structures shall consider potential long-term effects of:

- material deterioration
- seepage
- stray currents
- other potentially deleterious environmental factors

Service Life

- 75 years - permanent retaining walls
- 36 months or less - temporary retaining walls
- 100 years – greater level of safety (i.e., may be appropriate for walls where poor performance or failure would cause severe consequences) ODOT – MSE Walls

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LIMIT STATES AND RESISTANCE FACTORS (11.5)

General 11.5.1

Permanent structures shall be designed to:
- retain an aesthetically pleasing appearance
- essentially be maintenance free throughout design service life

Service Limit States 11.5.2

Abutments, piers, and walls shall be investigated at service limit state for:
- excessive vertical and lateral displacement (tolerable deformation criteria for retaining walls based on function and type of wall, anticipated service life, and consequences of unacceptable movements)
- overall stability (evaluated using limit equilibrium methods of analysis)
LIMIT STATES AND RESISTANCE FACTORS (11.5)

Service Limit States 11.5.2

Articles 10.6.2.2, 10.7.2.2, and 10.8.2.2 apply for investigation of vertical wall movements.

For anchored walls, deflections estimated in accordance with 11.9.3.1.

For MSE walls, deflections estimated in accordance with 11.10.4.
LIMIT STATES AND RESISTANCE FACTORS (11.5)

Strength Limit State 11.5.3

Abutment and walls investigated at the strength limit states for:

- Bearing resistance failure
- Lateral Sliding
- Excessive loss of base contact
- Pullout failure of anchors or soil reinforcements
- Structural failure
LIMIT STATES AND RESISTANCE FACTORS 11.5

Resistance Requirement 11.5.4
Abutments, piers and retaining structures and their foundations shall be proportioned by 11.6 - 11.11 so that their resistance satisfies 11.5.5

Factored resistance, $R_R$, calculated for each applicable limit state = nominal resistance, $R_n$, times appropriate resistance factor, $\Phi$, specified in Table 11.5.6-1

Load Combination and Load Factors 11.5.5
Abutments, piers and retaining structures and their foundations shall be proportioned for all applicable load combinations per 3.4.1
LIMIT STATES AND RESISTANCE FACTORS 11.5

Resistance Factors 11.5.6
Vertical elements, such as soldier piles, tangent-piles and slurry trench concrete walls shall be treated as either shallow or deep foundations, as appropriate, for purposes of estimating bearing resistance, using procedures described in Articles 10.6, 10.7, and 10.8.

Some increase in the prescribed resistance factors may be appropriate for design of temporary walls consistent with increased allowable stresses for temporary structures in allowable stress design.
### RETAINING WALLS

**Table: 11.5.6-1 Resistance Factors for Permanent Retaining Walls.**

<table>
<thead>
<tr>
<th>WALL-TYPE &amp; CONDITION</th>
<th>RESISTANCE FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nongravity Cantilevered &amp; anchored Walls</td>
<td>Article 10.5 applies</td>
</tr>
<tr>
<td>Bearing resistance of vertical elements</td>
<td>0.75</td>
</tr>
<tr>
<td>Passive resistance of vertical elements</td>
<td>0.70(1)</td>
</tr>
<tr>
<td>Pullout resistance of anchors(2)</td>
<td>0.65(1)</td>
</tr>
<tr>
<td>• Cohesionless (granular) Soils</td>
<td></td>
</tr>
<tr>
<td>• Cohesive Soils</td>
<td>0.70(1)</td>
</tr>
<tr>
<td>• Rocks</td>
<td>0.50(1)</td>
</tr>
<tr>
<td>Pullout resistance of anchors(2)</td>
<td>1.0(2)</td>
</tr>
<tr>
<td>• Where proof test are conducted</td>
<td></td>
</tr>
<tr>
<td>Flexural Capacity of Vertical elements</td>
<td>0.90</td>
</tr>
</tbody>
</table>
LIMIT STATES AND RESISTANCE FACTORS 11.5

Extreme Event Limit State 11.5.7

Applicable load combinations and load factors specified in Table 3.4.1-1 shall be investigated.

Unless otherwise specified, all $\phi$ factors shall be taken as 1.0 when investigating the extreme event limit state.
ABUTMENT AND CONVENTIONAL RETAINING WALLS 11.6

General 11.6.1.1

Rigid gravity and semigravity retaining walls may be used for bridge substructures or grade separation and are generally for permanent applications.

Rigid gravity and semigravity walls shall not be used without deep foundation support where bearing soil/rock prone to excessive total or differential settlement.
ABUTMENT AND CONVENTIONAL RETAINING WALLS 11.6

Loading 11.6.1.2
Abutment and retaining walls shall be investigated for:
- Lateral earth and water pressures, including live and dead load surcharge
- Abutment/wall self weight
- Temperature and shrinkage deformation effects
- Loads applied to bridge superstructure
ABUTMENT AND CONVENTIONAL RETAINING WALLS 11.6

Loading 11.6.1.2
Provisions of 3.11.5 (earth pressure) and 11.5.5 shall apply

For stability computations, earth loads shall be multiplied by maximum and/or minimum load factors given in Table 3.4.1-2, as appropriate.

Design shall be investigated combinations of forces which may produce the most severe loading condition.
Loading 11.6.1.2
For computing load effects in abutments, weight of fill material over inclined or stepped rear face, or over the base of a reinforced concrete spread footing may be considered part of the effective weight of the abutment

Where spread footings used, rear projection shall be designed as a cantilever supported at the abutment stem and loaded with the full weight of the superimposed material, unless a more exact method used
ABUTMENT AND CONVENTIONAL RETAINING WALLS 11.6

Reinforcement 11.6.1.5
Conventional Walls and Abutments 11.6.1.5.1
Reinforcement to resist formation of temperature and shrinkage cracks shall be as specified in 5.10.8

Safety Against Structural Failure 11.6.4
Structural design of individual wall elements and wall foundations shall comply with provisions of sections 5 - 8

Provisions of Article 10.6.1.3 shall be used to determine distribution of contact pressure for structural design (triangular/trapezoidal stress distribution)
RETAINING WALLS - Example

- \( f'_c = 4 \text{ ksi} \)
- \( \gamma_{\text{Soil}} = 115 \text{ pcf} \)
- \( \gamma_{\text{Conc.}} = 150 \text{ pcf} \)
Horizontal Earth Forces

\[ P_1 = 0.08625 \times (16.5\) ') \times (1') = 1.42 \text{ kips/ft} \]

\[ P_2 = \frac{1}{2} \times (16.5\) ') \times 0.621 \times (1') = 5.12 \text{ kips/ft} \]

Conservatively neglect soil above toe

Shear at base of wall

\[ V_u = 1.50 \times (1.42 + 5.12) = 9.81 \text{ k} \]

where 1.50 is a load factor for EH (Active)
RETAINING WALLS - Example

\[ V_c = 0.0316 \beta \sqrt{f_c' b_d} \]

where:

\[ \beta = 2.0 \]
\[ b_v = 12" \]

\[ d_v \Rightarrow 0.72 h = 0.72 (18") = 12.96" \]

or

\[ 0.9 de = (18 - 2 - \frac{1}{2} " ) 0.9 = 13.95" \quad \text{Say 13.95"} \]

\[ V_c = 0.0316(2)\sqrt{4(12)(13.95)} = 21.2 \text{ k} \]

\[ \phi V_c = 0.9(21.2) = 19.0 \text{ k} > V_u = 9.8 \text{ k} \quad \text{O.K} \]

\[ \phi V = 0.25 \sqrt{f_c' b_d} \]
\[ = 0.25 \sqrt{4(12)(13.95)} = 83.7 \text{ k} \quad \text{OK} \]
Moment at Base of Wall

\[ M_1 = P_1 \left( \frac{16.5}{2} \right) = 1.42 \left( \frac{16.5}{2} \right) = 11.72 \text{ k-ft} \]

\[ M_2 = P_2 \left( \frac{16.5}{3} \right) = 5.12 \left( \frac{16.5}{3} \right) = 28.16 \text{ k-ft} \]

\[ \phi M_n = \phi A \frac{f}{s} \left( d - \frac{a}{2} \right) = \phi A \frac{f}{s} jd = M_u \]

\[ M_u = 1.50 \left( 11.72 + 28.16 \right) = 59.82 \text{ k-ft/ft} \]
$$A_s = \frac{M}{\phi f_y j d}$$

where:

- \( j \) = Portion of \( d \) that is moment arm (assume 0.9)
- \( d = 18 - 2'' - \frac{1}{2}'' = 15.5'' \)
- \( f_y = 60 \text{ ksi} \)
- \( \phi = 0.9 \)

\[
A_s = \frac{59.82(12)}{0.9(60)(0.9)15.5} = 0.95 \text{ in}^2
\]
RETAINING WALLS - Example

Try # 8’s

\[
n = \frac{A_{\text{required}}}{A_{\text{bar}}} = \frac{0.95}{0.79} = 1.21 \text{ Bars}
\]

or

\[
\text{spacing} = \frac{12''}{1.21} = 9.9''
\]

Say #8’s @ 9”
RETAINING WALLS - Example

\[ A_s = \frac{12}{9} \times (0.79) = 1.05 \text{ in}^2/\text{ft} \]

\[
a = \frac{A_f}{s_y} \frac{f_y}{0.85 \beta_c b} = \frac{1.05(60)}{0.85(4)(12)} = 1.54''
\]

\[
c = \frac{a}{\beta_1} = \frac{1.54}{0.85} = 1.82''
\]

The diagram illustrates the forces and dimensions involved in the calculation, with labeled distances and angles.
\[ \varepsilon_s = \frac{0.003}{d-c} \]

\[ \varepsilon_s = \frac{0.003}{1.82} (18 - 2 - 1/2 - 1.82) = 0.0225 > 0.005 \]

Tensioned Controlled \( \Rightarrow \phi = 0.9 \)

\[ \phi M_n = \phi A f_y (d - \frac{a}{2}) = 0.9(1.05)(60)(15.5 - \frac{1.54}{2})12 = 69.6 \text{ k-ft} \]

\[ \phi M_n = 69.6 > M_u = 59.8 \text{ O.K} \]

\[ d_v = d - \frac{a}{2} = 15.5 - \frac{1.54}{2} = 14.73 \]

could have been used in \( V_c \) calc
RETAINING WALLS-EXAMPLE

Check \( \phi M_n > 1.2 M_{cr} \)

\[
1.2 M_{cr} = 1.2 \left( \frac{12 (18)^2}{6} \right) 0.37 \sqrt{\frac{4}{12}} = 47.95 \text{ k - ft / ft}
\]

\[
\phi M_n = 69.6 > 1.2 M_{cr} \quad \text{OK}
\]

Check reinforcement spacing, \( s \)

\[
0.8f_r = 0.8(0.24)\sqrt{4} = 0.384 \text{ ksi}
\]

\[
f_{act} = \frac{(11.72 + 28.16)(12)}{12 (18)^2} = 0.739 \text{ ksi} > 0.8 f_r \quad \therefore \text{check spacing}
\]

\[
s \leq \frac{700 \gamma_e}{\beta_s f_s c} - 2d
\]

\[
\gamma_e = 1.0
\]
\[ y_e = 1.0 \]

\[ d_c = 2 + \frac{1}{2} = 2.5 \]

\[ f_s = \text{Crack Trans. Section} \]

\[ 12x \left( \frac{x}{2} \right) = 8.4(15.5 - x) \]

\[ x = 4.01" \]

\[ I = \frac{12(4.01)^3}{12} + 12(4.01) \left( \frac{4.01}{2} \right)^2 + 8.4(15.5 - 4.01)^2 \]

\[ = 1,367 \text{in}^4 \]
RETAINING WALLS EXAMPLE

\[
f_s = n \frac{M_y}{I} = 8 \frac{(11.72 + 28.16)(12)(15.5 - 4.01)}{1367} = 32.18 \text{ ksi}
\]

\[
\beta_s = 1 + \frac{d}{0.7(h - d/c)} = 1 + \frac{2.5}{0.7(18 - 2.5)} = 1.23
\]

\[
s \leq \frac{700(1.0)}{1.23(32.18)} - 2(2.5) = 12.7'' \quad \text{OK (> 9'')}
\]
RETAINING WALLS - Example

T & S Steel (5.10.8)

\[
A_s \geq \frac{1.3 \, b \, h}{2 \, (b + h) \, f_y} = \frac{1.3 \, (16.5) \, 12 \, (18)}{2 \, [16.5(12) + 18] \, 60} = 0.179 \text{ in}^2/\text{ft in each direction}
\]

\[0.11 \leq A_s \leq 0.6 \quad \text{O.K.}\]

\[\therefore \quad \text{use } #4 \at 12'' \quad (S_{\text{max}} = 18'')\]
RETAINING WALLS - Example

#4’s @ 12”

#8’s @ 9”

#4’s @ 12” T&S

18”
Footings
Footings

**General (5.13.3.1)**

- Provisions herein apply to design of:
  - Isolated footings
  - Combined footings
  - Foundation mats
- For sloped or stepped footings, design requirements shall be satisfied at every section of the slope or steps
- Circular or regular polygon-shaped concrete columns or piers treated as square members with $= \text{area for location of critical } M, V \text{ and } l_d \text{ of reinforcement}$
Footings

**Loads and Reactions (5.13.3.2)**

- Isolated footings supporting a column, pier, or wall assumed to act as a cantilever

- Footing supporting multiple columns, piers, or walls shall be designed for actual conditions of continuity and restraint

- Assume individual driven piles may be out of planned position in a footing by either 6.0 in. or one-quarter of the pile diameter and that the center of a group of piles may be 3.0 in. from its planned position, unless special equipment specified to ensure precision driving.

- For pile bents, the contract documents may require a 2.0 in. tolerance for pile position, in which case that value should be accounted for in the design.
Footings

**Resistance Factors (5.13.3.3)**

- The resistance factors, $\phi$, for soil-bearing pressure and for pile resistance as a function of the soil shall be per Section 10

**Moment in Footings (5.13.3.4)**

- Critical section for flexure at the face of the column, pier, or wall. For non-rectangular columns, the critical section taken at side of concentric rectangle $w/ = \text{area}$
- For footings under masonry walls, critical section halfway between center and edge of wall
- For footings under metallic column bases, critical section halfway between column face and edge of metallic base
Distribution of Moment Reinforcement (5.13.3.5)

- One-way footings and two-way square footings, reinforcement distributed uniformly across the entire width

- Two-way rectangular footings:
  - In the long direction, reinforcement distributed uniformly across the entire width
  - In the short direction, portion of the total reinforcement specified by Eq. 1, shall be distributed uniformly over a band width equal to the length of the short side of footing and centered on the centerline of column or pier. Remainder of reinforcement distributed uniformly outside of the center band width of footing.
Footings

**Distribution of Moment Reinforcement (5.13.3.5)**

\[
A_{s-BW} = A_{s-SD} \left( \frac{2}{\beta + 1} \right) \quad (5.13.3.5-1)
\]

- where:
  - \( \beta \) = ratio of the long side to the short side of footing
  - \( A_{s-BW} \) = area of steel in the band width (in.\(^2\))
  - \( A_{s-SD} \) = total area of steel in short direction (in.\(^2\))
**Footings**

**Shear in Slabs and Footings (5.13.3.6)**

**Critical Sections for Shear (5.13.3.6.1)**

- One-way action critical section
  - extending in a plane across the entire width
  - located per 5.8.3.2 ($d_v$ from column face if compression induced, otherwise at column face)
Shear in Slabs and Footings (5.13.3.6)

Critical Sections for Shear (5.13.3.6.1)

- Two-way action critical section
  - ⊥ to slab plane
  - located so perimeter, $b_o$, is minimum
  - but ≥ $0.5d_v$ to perimeter of concentrated load/reaction area
Footings

Shear in Slabs and Footings (5.13.3.6)

Critical Sections for Shear (5.13.3.6.1)

– For non-constant slab thickness, critical section located $\geq 0.5d_v$ from face of change and such that perimeter, $b_o$, minimized
Footings

Shear in Slabs and Footings (5.13.3.6)

Critical Sections for Shear (5.13.3.6.1)

- For cantilever retaining wall where the downward load on the heel > upward reaction of soil under the heel, the critical section for \( V \) taken at back face of the stem (\( d_v \) is the effective depth for \( V \))

Figure C5.13.3.6.1-1 Example of Critical Section for Shear in Footings
Footings

**Shear in Slabs and Footings (5.13.3.6)**

**Critical Sections for Shear (5.13.3.6.1)**

- If a portion of a pile lies inside critical section, pile load considered uniformly distributed across pile width/diameter.
- Portion of the load outside critical section included in the calculation of shear on the critical section.
Shear in Slabs and Footings (5.13.3.6)

One-Way Action (5.13.3.6.2)

- For one-way action, footing or slab shear resistance shall satisfy requirements of 5.8.3
- Except culverts under $\geq 2.0$ ft. of fill, for which 5.14.5.3 applies
Footings

Shear in Slabs and Footings (5.13.3.6)

Two-Way Action (5.13.3.6.3)

- Nominal two-way shear resistance, $V_n$ (kip), of the concrete w/o transverse reinforcement shall be taken as:

$$V_n = \left(0.063 + \frac{0.126}{\beta_c}\right)\sqrt{f'_c b_o d_v} \leq 0.126 \sqrt{f'_c b_o d_v} \quad (5.13.3.6.3-1)$$

- where:
  - $\beta_c$ = long side to short side ratio of the rectangle through which the concentrated load/reaction force transmitted
  - $b_o$ = perimeter of the critical section (in.)
  - $d_v$ = effective shear depth (in.)
Shear in Slabs and Footings (5.13.3.6)

Two-Way Action (5.13.3.6.3)

- Where $V_u > \phi V_n$, shear reinforcement shall be added per 5.8.3.3 w/ $\theta = 45^\circ$

- For two-way action sections w/ transverse reinforcement, $V_n$ (kip) shall be taken as:

$$V_n = V_c + V_s \leq 0.192 \sqrt{f'_c} b_o d_v \quad (5.13.3.6.3-2)$$

- in which:

$$V_c = 0.0632 \sqrt{f'_c} b_o d_v , \text{ and } (5.13.3.6.3-3)$$

$$V_s = \frac{A_v f_y d_v}{s} \quad (5.13.3.6.3-4)$$
Footings

**Development of Reinforcement (5.13.3.7)**

- Development of reinforcement in slabs and footings per 5.11
- Critical sections for development of reinforcement shall be:
  - At locations per 5.13.3.4
  - All other vertical planes where changes of section or reinforcement occur
Footings

Transfer of Force at Base of Column (5.13.3.8)

- Forces and moments applied at the base of column/pier shall be transferred to the top of footing by bearing and reinforcement.
- Bearing between supporting and supported member shall be \( \leq \) concrete-bearing strength per 5.7.5.
- Lateral forces transferred from pier to footing by shear-transfer per 5.8.4.
- Reinforcement shall be provided across the interface between supporting and supported member. Done w/ main longitudinal column/wall reinforcement, dowels or anchor bolts.
Transfer of Force at Base of Column (5.13.3.8)

- Reinforcement across the interface shall satisfy the following requirements:
  - Force effects > concrete bearing strength shall be transferred by reinforcement
  - If load combinations result in uplift, total tensile force shall be resisted by reinforcement
  - Area of reinforcement ≥ 0.5% gross area of supported member
  - Number of bars ≥ 4

- Diameter of dowels (if used) ≤ 0.15” + longitudinal reinforcement diameter
Footings

Transfer of Force at Base of Column (5.13.3.8)

• At footings, No. 14 and No. 18 main column longitudinal reinforcement that is in compression only may be lap spliced with footing dowels to provide the required area

• Dowels shall be ≤ No. 11 and shall extend
  – Into the column a distance >:
    • Development length of No. 14 or No. 18 bars
    • Splice length of the dowels
  – Into the footing a distance ≥ development length of the dowels
• In the absence of confinement reinforcement in the concrete supporting the bearing device, the factored bearing resistance shall be:

\[ P_r = \phi P_n \quad (5.7.5-1) \]

in which:

\[ P_n = 0.85 f_c' A_1 m \quad (5.7.5-2) \]

– where:

• \( P_n \) = nominal bearing resistance (kip)
• \( A_1 \) = area under bearing device (in.\(^2\))
• \( m \) = modification factor
• Modification factor determined as follows:
  
  – Where the supporting surface is wider on all sides than the loaded area:

\[
m = \sqrt{\frac{A_2}{A_1}} \leq 2.0 \quad (5.7.5-3)
\]

  – Where the loaded area is subjected to nonuniformly distributed bearing stresses:

\[
m = 0.75 \sqrt{\frac{A_2}{A_1}} \leq 1.50 \quad (5.7.5-4)
\]
where $A_2$ = a notional area (in.$^2$) defined by:

- area of the lower base of the largest frustum of a right pyramid, cone, or tapered wedge contained wholly within support
- having its upper base = loaded area
- side slopes 1:2 vertical to horizontal

• When $P_u > P_r$, bursting and spalling forces shall be resisted per 5.10.9

Post-Tensioning
Footing Example

Soil = 115 pcf
\( \gamma_{\text{Soil}} = 115 \text{ pcf} \)
\( \gamma_{\text{Conc.}} = 150 \text{ pcf} \)
### Footing Example

<table>
<thead>
<tr>
<th>Load</th>
<th>Load Factor</th>
<th>Factored Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1 = 0.115(6)(2.5) = 1.725k$</td>
<td>1.75(LS)</td>
<td>3.02k</td>
</tr>
<tr>
<td>$W_2 = 0.115\left(\frac{68}{12}\right)(16.5) = 10.75k$</td>
<td>1.35(EV)</td>
<td>14.51k</td>
</tr>
<tr>
<td>$W_3 = 0.115\left(\frac{1}{2}\right)\left(\frac{4}{12}\right)(16.5) = 0.32k$</td>
<td>1.35(EV)</td>
<td>0.43k</td>
</tr>
<tr>
<td>$W_4 = 0.15\left(\frac{1}{2}\right)\left(\frac{68}{12}\right)(16.5) = 0.41k$</td>
<td>1.25(DC)</td>
<td>0.51k</td>
</tr>
<tr>
<td>$W_5 = 0.15(1)(16.5) = 2.475k$</td>
<td>1.25(DC)</td>
<td>3.09k</td>
</tr>
<tr>
<td>$W_6 = 0.15(10)(1.5) = 2.25k$</td>
<td>1.25(DC)</td>
<td>2.81k</td>
</tr>
<tr>
<td>$W_7 = 0.115(2.5)(3) = 0.86k$</td>
<td>1.35(EV)</td>
<td>1.16k</td>
</tr>
<tr>
<td>$P_1 = 0.08625 (18) = 1.55k$</td>
<td>1.50(EH)</td>
<td>2.33k</td>
</tr>
<tr>
<td>$P_2 = 0.621 (\frac{1}{2}) (18) = 5.59k$</td>
<td>1.50(EH)</td>
<td>8.38k</td>
</tr>
</tbody>
</table>

$\Sigma$ Vertical Forces $= R = 25.53\ k$
# Footing Example

<table>
<thead>
<tr>
<th>Moment Arms from Heel</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1 = 3'$</td>
<td>$3' \times 3.02k = 9.06k'$</td>
</tr>
<tr>
<td>$d_2 = \left( \frac{68}{12} \right) \left( \frac{1}{2} \right) = 2.83'$</td>
<td>$2.83' \times 14.51k = 41.06k'$</td>
</tr>
<tr>
<td>$d_3 = \left( 68 + \frac{4}{3} \right) \left( \frac{1}{12} \right) = 5.78'$</td>
<td>$5.78' \times 0.43k = 2.49k'$</td>
</tr>
<tr>
<td>$d_4 = \left( 68 + \frac{2(4)}{3} \right) \left( \frac{1}{12} \right) = 5.89'$</td>
<td>$5.89' \times 0.51k = 3.00k'$</td>
</tr>
<tr>
<td>$d_5 = 6.5'$</td>
<td>$6.5' \times 3.09k = 20.09k'$</td>
</tr>
<tr>
<td>$d_6 = 5'$</td>
<td>$5' \times 2.81k = 14.05k'$</td>
</tr>
<tr>
<td>$d_7 = 8.5'$</td>
<td>$8.5' \times 1.16k = 9.86k'$</td>
</tr>
<tr>
<td>$d_{p1} = 9'$</td>
<td>$9' \times 2.33k = 20.97k'$</td>
</tr>
<tr>
<td>$d_{p2} = 6'$</td>
<td>$6' \times 8.38k = 50.28k'$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>170.86k'</strong></td>
</tr>
</tbody>
</table>
Footing Example

Location of Resultant from Heel

\[ x = \frac{M}{R} = \frac{170.86\text{k'}}{25.53\text{k}} = 6.69' \]

\[ e = 6.69' - 5' = 1.69' \]

\[ \sigma = \frac{P}{A} \pm \frac{\text{Pey}}{I} = \frac{25.53'}{10(1)} \pm \frac{25.53'(1.69')(5')}{(1)(10)^3(12)} \]

\[ = 2.55 \pm 2.59 \]

\[ = 5.14 \& \approx 0 \]
**Footing Example**

\[ W_s = 1.35 \times (0.115) \times (2.5) = 0.388 \]

\[ W_s = 1.35 \times (0.115) \times (18) = 2.795 \]

\[ W_c = 1.25 \times (0.15) \times (1.5) = 0.281 \]

\[ 5.14 \text{ ksf} \]

\[ \frac{68}{12} = 5.67 \]

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Check Shear at Critical Sections

At Back face of wall (C 5.1.3.3.6.1)

Soil stress \( \frac{5.14}{10} (5.67) = 2.91 \text{ ksf} \)

\( V_u = (0.281 + 2.795) (5.67) - 2.91 \left( \frac{1}{2} \right) (5.67) = 9.2 \text{ k} \)

\( V_c = 0.0316 \beta \sqrt{\frac{f}{c}} b_v d_v \)

where: \( \beta = 2.0 \quad b_v = 12'' \)

\( d_v \Rightarrow 0.72 h = 0.72 (18) = 12.96'' \)

or \( 0.9 d_e = 0.9 (18 - 3 - \frac{1}{2}) = 13.05'' \) (controls)

Cover \( \frac{1}{2} d_b \)
Footing Example

\[ V_c = 0.0316(2)\sqrt{4(12)(13.05)} = 19.79\, k \]

\[ \phi V_c = 0.9(19.79\, k) = 17.8\, k > V_u = 9.2\, k \quad \therefore \text{O.K.} \]

At \( d_v \) from front of the wall

\[ d_v = \frac{13.05''}{12} = 1.09' \]

\[ 3 - 1.09 = 1.91 \]

\[ 0.388 \]

\[ 0.281 \]

\[ 5.14 \]

\[ 5.14 - 0.514(1.91) = 4.16 \]
Footing Example

\[ V_u = 4.16 \times 1.91 + (5.14 - 4.16) \times 0.5 \times 1.91 \]
\[ - (0.388 + 0.281) \times 1.91 = 7.6 \text{ k} \]

\[ \phi V_c = 17.8 > V_u = 7.6 \text{ O.K} \]

Flexure Design
Critical Sections at face of the Wall
Back of Wall:

\[ M_U = \frac{(2.795 + 0.281)(5.67)^2}{2} \]
\[ - \frac{1}{2}(2.91)(5.67)^2 \left( \frac{1}{3} \right) \]
\[ = 33.85 \text{ k-ft} = 406 \text{ k-in} \]

(Tension on top)
Footing Example

Front of Wall:

\[ M_U = 3.6 \left( \frac{3^2}{2} \right) + \left( \frac{5.14 - 3.6}{2} \right) \left( \frac{2}{3} \right) (3) - (0.281 + 0.388)(3) \left( \frac{3}{2} \right) \]

\[ = 17.8 \text{k-ft} = 213.7 \text{k-in} \quad \text{(Tension On Bottom)} \]

Therefore back of wall controls
Footing Example

\[
\phi M_n = \phi A_s f_y (d - \frac{a}{2}) = \phi A_s f_y j d = MU_s
\]

\[
A_s = \frac{MU}{\phi f_y j d}
\]

\[
d = 18" - 3" - \frac{1}{2}" = 14.5"
\]

\[
A_s = \frac{406}{0.9 \times (60) \times 0.9 \times (14.5)} = 0.58 \text{ in}^2/\text{ft}
\]

Try # 6’s (\(A_s = 0.44 \text{ in}^2\))

\[
S = \frac{12}{0.58 \times (0.44)} = 9.1" \quad \text{Say 9"}
\]
Footing Example

\[ a = \frac{A_{fy}}{0.85 f'c} \]
\[ = \frac{0.44 \left( \frac{12}{9} \right)(60)}{0.85 (4)(12)} = 0.86" \]

\[ c = \frac{a}{\beta_1} \]
\[ = \frac{0.86}{0.85} = 1.01" \]

\[ \varepsilon_s = \frac{0.003}{c}(d - c) \]
\[ = \frac{0.003}{1.01} \left( 18 - 3 - \frac{0.75}{2} - 1.01 \right) = 0.04 >> 0.005 \]
(Tension Controlled)

\[ \phi M_n = \phi A_{fy} \left( d - \frac{a}{2} \right) \]
\[ = 0.9(0.44)\left( \frac{12}{9} \right)60 \left( 14.625 - \frac{0.86}{2} \right) \]

\[ \phi M_n = 450 k" > M_u = 406 k" \]
Footing Example

\[
1.2M_{CR} = 1.2 \left( \frac{12(18)^2}{6} \right) 0.37 \sqrt{4} = 575 \text{ "k}
\]

\[
\phi M < 1.2 \frac{M_{CR}}{n} \quad \text{No Good}
\]

\[
\Rightarrow 1.33M_u = 1.33(406) = 540 \text{ "k}
\]

\[
A_s = \frac{540}{0.9(60)(0.9)14.5} = 0.766 \text{ in}^2/\text{ft}
\]

\[
\frac{12}{0.766}(0.44) = 6.9 "
\]

Try # 6 @ 7"
Footing Example

\[
a = \frac{0.44(60)(12/7)}{0.85(4)(12)} = 1.11''
\]
\[
c = \frac{1.11}{0.85} = 1.31''
\]
\[
\varepsilon_s = \frac{0.003}{1.31} \left(14.625 - \frac{1.31}{2}\right) = 0.032 >> 0.005 \quad \Rightarrow \Phi = 0.9
\]

\[
\phi M_n = 0.9(0.44)(12/7)(60) \left(14.625 - \frac{1.11}{2}\right)
\]
\[
= 573 \text{ k"} > M_U = 540 \text{ k"}
\]

Use # 6’s at 7”
Footing Example

T & S Steel: (5.10.8)

\[ A_s \geq \frac{1.30 \cdot b \cdot h}{2 (b + h) f_y} \]

\[ = \frac{1.30 (120) 18}{2 (120 + 18) 60} = 0.17 \text{ in}^2/\text{ft on each face} \]

\[ 0.11 \leq A_s \leq 0.60 \quad \text{OK} \]

Try # 4’s

\[ S = 12/0.17 (0.2) = 14.1” \quad \text{Say 12”} \]

Use # 4’s @ 12” in longitudinal direction top & bottom
Footing Example

# 6's @ 7"

# 4's @ 12"
Footing Example

Note: This was for one case. Other cases w/ various combinations of max and min load factors as shown should be considered.

<table>
<thead>
<tr>
<th>Load</th>
<th>Cases</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>DC</td>
<td>1.25</td>
<td>1.25</td>
<td>0.9</td>
<td>1.25</td>
<td>0.9</td>
<td>1.25</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>EH</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>1.50</td>
</tr>
<tr>
<td>EV</td>
<td>1.35</td>
<td>1.0</td>
<td>1.35</td>
<td>1.35</td>
<td>1.0</td>
<td>1.0</td>
<td>1.35</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Development Length
Deformed Bars in Tension (5.11.2.1)

Tension Development Length (5.11.2.1.1)

- Tension development length, $\ell_d$, > basic tension development length, $\ell_{db}$, * modification factor(s)
- $\ell_{d\text{Min}} = 12.0$ in., except for lap splices (5.11.5.3.1) and development of shear reinforcement (5.11.2.6)
Development Length

Deformed Bars in Tension (5.11.2.1)

Tension Development Length (5.11.2.1.1)

- \( \ell_{db} \) (in.) shall be taken as:
  - For \( \leq \) No. 11 bar: \( \frac{1.25 A_f}{b_y} \frac{f_y}{\sqrt{f_c}} \)
  - No. 14 bars: \( \frac{0.4 d_f}{b_y} \)
  - No. 18 bars: \( \frac{2.70 f_y}{\sqrt{f_c}} \)
  - No. 18 bars: \( \frac{3.5 f_y}{\sqrt{f_c}} \)
Development Length

Deformed Bars in Tension (5.11.2.1)

Tension Development Length (5.11.2.1.1)

• where:
  – \( A_b \) = area of bar (in.\(^2\))
  – \( f_y \) = specified yield strength of reinforcing bars (ksi)
  – \( f'_c \) = specified compressive strength of concrete (ksi)
  – \( d_b \) = diameter of bar (in.)
Deformed Bars in Tension (5.11.2.1)

- **Modification Factors That Increase \( \ell_d \) (5.11.2.1.2)**
  - Top reinforcement w/ > 12.0 in. of concrete cast below reinforcement
  - Lightweight aggregate concrete w/ \( f_{ct} \) (ksi) specified
  - All-lightweight concrete w/ \( f_{ct} \) not specified
  - Sand-lightweight concrete w/ \( f_{ct} \) not specified

\[
0.22 \sqrt{\frac{f'}{f}} \geq 1.0
\]

- \( \frac{f}{f_{ct}} \)

\( \ell_d \) increase factors:

- 1.4
- 1.3
- 1.2

(Linearly interpolate between all-lightweight and sand-lightweight when partial sand replacement used)
Deformed Bars in Tension (5.11.2.1)

• Modification Factors That Increase $\ell_d$ (5.11.2.1.2)
  
  – For epoxy-coated bars with cover less than $3_{db}$ or with clear spacing between bars less than $6_{db}$ ........... 1.5

  – For epoxy-coated bars not covered above........... 1.2

(The factor for top reinforcement multiplied by the applicable epoxy-coated bar factor ≤ 1.7)
Development Length

Deformed Bars in Tension (5.11.2.1)

- **Modification Factors which Decrease $\ell_d$ (5.11.2.1.3)**
  - Reinforcement spaced laterally $\geq 6.0$ in. center-to-center and $\geq 3.0$ in. clear cover in spacing direction .......................... 0.8
  - For members w/ excess flexural reinforcement or where anchorage/development for full yield strength of reinforcement not required.....................
    \[
    \frac{A_s \text{ required}}{A_s \text{ provided}} = 0.8
    \]
  - Reinforcement enclosed within a spiral composed of bars $\geq 0.25$ in. diameter and w/ pitch $\leq 4.0$ in. ................................. 0.75
**Development Length**

**Deformed Bars in Compression (5.11.2.2)**

**Compressive Development Length (5.11.2.2.1)**

- Compression development length, \( \ell_d \), shall be > than basic development length, \( \ell_{db} \), * modification factor(s) or 8.0 in.
- \( \ell_{db} \) determined from:

\[
\ell_{db} \geq 0.63 \frac{d}{b} \frac{f_y}{f'} \\
\ell_{db} \geq 0.3 \frac{d}{b} \frac{f_y}{f'}
\]

where:

- \( f_y \) = specified yield strength (ksi)
- \( f'_c \) = compressive strength (ksi)
- \( d_b \) = diameter of bar (in.)
Deformed Bars in Compression (5.11.2.2)

- **Modification Factors (5.11.2.2.2)**
  - For members w/ excess flexural reinforcement or where anchorage/development for full yield strength of reinforcement not required……………….. 
    \[
    \frac{(A_{\text{required}})}{s} \quad \frac{(A_{\text{provided}})}{s}
    \]
  - Reinforcement enclosed within spirals ≥ 0.25 in. diameter and w/ pitch ≤ 4.0 in. ………………. 0.75
Bundled Bars (5.11.2.3)

- Tension or compression of individual bars within a bundle shall be:
  - $1.20 \times \ell_d$ three-bar bundle
  - $1.33 \times \ell_d$ four-bar bundle
- Modifications factors for bars in tension determined by assuming bundled bars as a single bar w/ diameter determined from an equivalent total area
Development Length

**Standard Hooks in Tension (5.11.2.4)**

**Basic Hook Development Length (5.11.2.4.1)**

- Development length, $\ell_{dh}$, (in.) for a standard hook in tension shall not be less than:
  - The basic development length $\ell_{hb}$, * applicable modification factor(s)*
  - $8.0 \times d_b$
  - 6.0 in.
Standard Hooks in Tension (5.11.2.4)

Basic Hook Development Length (5.11.2.4.1)

- $\ell_{hb}$ for a hooked-bar w/ $f_y \leq 60.0$ ksi shall be:

$$\ell_{hb} = \frac{38 \cdot d_b}{\sqrt{f'_c}}$$

- where:
  - $d_b = \text{diameter of bar (in.)}$
  - $f'_c = \text{compressive strength (ksi)}$
Development Length

Figure C5.11.2.4-1 Hooked-Bar Details for Development of Standard Hooks (ACI)
Development Length

Standard Hooks in Tension (5.11.2.4)

- **Modification Factors (5.11.2.4.2)**
  - Reinforcement w/ $f_y > 60.0$ ksi
    \[
    \frac{f_y}{60.0}
    \]
  - For $\leq$ No. 11 bar w/ side cover normal to plane of hook $\geq 2.5$ in., and for $90^\circ$ hook, cover on bar extension beyond hook $\geq 2.0$ in.
  - For $\leq$ No. 11 bar enclosed vertically or horizontally within ties or stirrup ties spaced $\leq 3d_b$ along the development length, $\ell_{dh}$
    \[
    0.7
    \]
    \[
    0.8
    \]
Development Length

Standard Hooks in Tension (5.11.2.4)

- **Modification Factors (5.11.2.4.2)**
  - Where reinforcement provided exceeds that required or anchorage or development of full yield strength is not required: 
    \[
    \frac{A_{\text{required}}}{A_{\text{provided}}} 
    \]
  - Lightweight aggregate concrete: 1.3
  - Epoxy-coated reinforcement: 1.2
Development Length

Standard Hooks in Tension (5.11.2.4)

Hooked-Bar Tie Requirements (5.11.2.4.3)

- For bars being developed at discontinuous ends of members with both side cover and top or bottom cover < 2.5 in., hooked-bar shall be enclosed within ties / stirrups spaced ≤ 3$d_b$ along the full development length. The modification factor for transverse reinforcement shall not apply.
Development Length

Figure 5.11.2.4.3-1 Hooked-Bar Tie Requirements.
Development Length

Shear Reinforcement (5.11.2.6)

General (5.11.2.6.1)

• Stirrup reinforcement in concrete pipe covered in 12.10.4.2.7 not here

• Shear reinforcement shall be located as close to surfaces of members as cover requirements and other reinforcement permit

• Between anchored ends, each bend in continuous portion of a U-stirrups shall enclose a longitudinal bar
Shear Reinforcement (5.11.2.6)

Anchorage of Deformed Reinforcement (5.11.2.6.2)

- Ends of single-leg, simple U-, or multiple U-stirrups shall be anchored as follows:
  - For ≤ No. 5 bar, and for No. 6 to No. 8 bars w/ $f_y$ of ≤ 40.0 ksi:
    - Standard hook around longitudinal reinforcement
  - For No. 6 to No. 8 stirrups with $f_y > 40.0$ ksi:
    - Standard stirrup hook around a longitudinal bar, plus one embedment length between midheight of member and outside end of the hook, $l_e$ shall satisfy:
      \[
      l_e \geq \frac{0.44 d_b f_y}{\sqrt{f'_c}}
      \]
Development Length

Shear Reinforcement (5.11.2.6)

Closed Stirrups (5.11.2.6.4)

- Pairs of U-stirrups / ties that are placed to form a closed unit shall have length of laps \( \geq 1.7 \ell_d \), where \( \ell_d \) is tension development length

- In members \( \geq 18.0 \) in. deep, closed stirrup splices with tension force from factored loads, \( A_b f_y \), \( \leq 9.0 \) kip per leg, may be considered adequate if the stirrup legs extend full available depth of member

- Transverse torsion reinforcement shall be fully continuous w/ 135° standard hooks around longitudinal reinforcement for anchorage
Development by Mechanical Anchorages (5.11.3)

- Mechanical devices capable of developing strength of reinforcement w/o damage to concrete may be used. Performance shall be verified by laboratory tests.

- Development of reinforcement may consist of a combination of mechanical anchorage and additional embedment length of reinforcement.