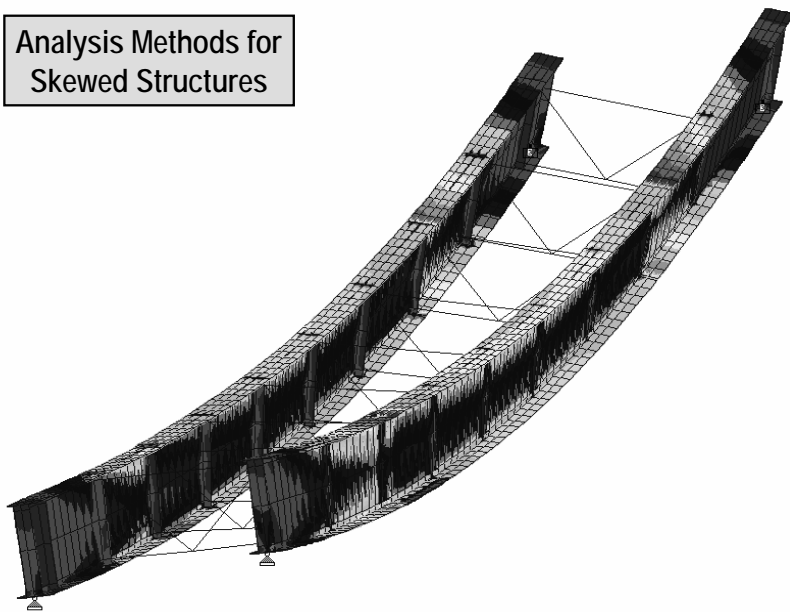


Analysis Methods for Skewed Structures



Analysis Types:

• Line girder model

Crossframe Effects Ignored

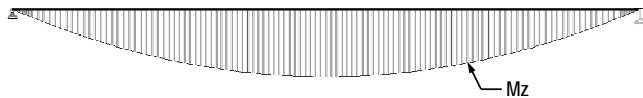
- MDX
- Merlin Dash
- BSDI
- StlBridge
- PC-BARS
- Others...

Refined model

Crossframe Effects Included

- Grid model  
(Model Crossframes w/ beams)  
MDX Grid  
DESCUS
- Grid model  
(Model Crossframes w/trusses)  
Finite Element
- 3D Finite Element model  
BSDI  
Finite Element

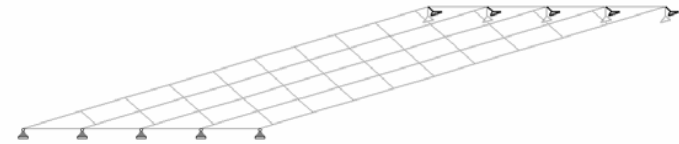
### Line Girder Model



- Most commonly used method for non-complex structures (MDX, Merlin Dash)
- Each girder is modeled independently, with crossframe effects ignored
- Cannot predict skew effects
- Should not be used on structures where skew effects are expected to be significant

### Refined Model:

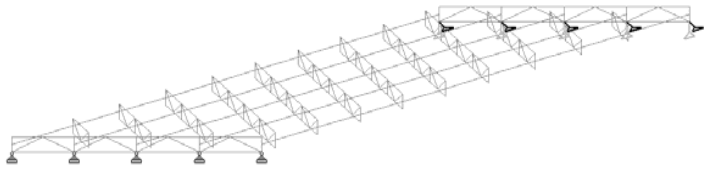
Grid model using crossframe beam elements



- All girders and crossframes are included in one model.
- Both girders and crossframes are modeled using beam elements.
- A standard beam element cannot accurately duplicate crossframe stiffness, so crossframe stiffness in the model is approximate.
- DESCUS and MDX Grid use variations of this method.

Refined Model:

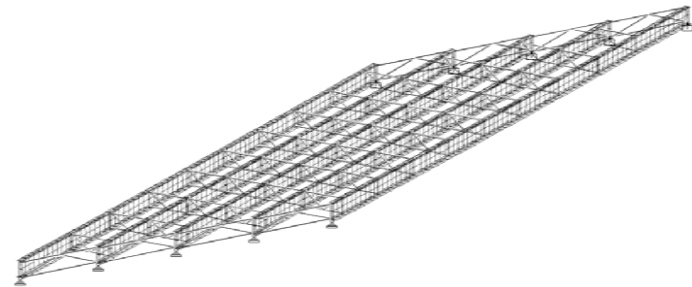
Grid model using crossframe truss elements



- All girders and crossframes are included in one model.
- Girders are modeled as linear beam elements.
- Crossframes are modeled using truss elements.
- Crossframe stiffness can be more accurately modeled than with the beam element model.

Refined Model:

Three dimensional finite element model



- Girder flanges, webs, and stiffeners are modeled as plate elements.
- Crossframe members are modeled as truss elements.
- Most complex model (of those compared)
- Most accurately reflects actual behavior

Model Capabilities:

Model Type	Calculates Twist Due to Intermediate Crossframes:	Predicts End Crossframe Effects:	Includes Girder Warping Stiffness:
Line Girder	No	No	No*
Grillage w/ CF Beam Elements	Yes	Partial	No*
Grillage w/ CF Truss Elements	Yes	Yes	No*
Three Dimensional Finite Element	Yes	Yes	Yes

\* Girder warping stiffness can be approximated in these models.

Parametric Study:

Study of single span structures with variable skews and span lengths.

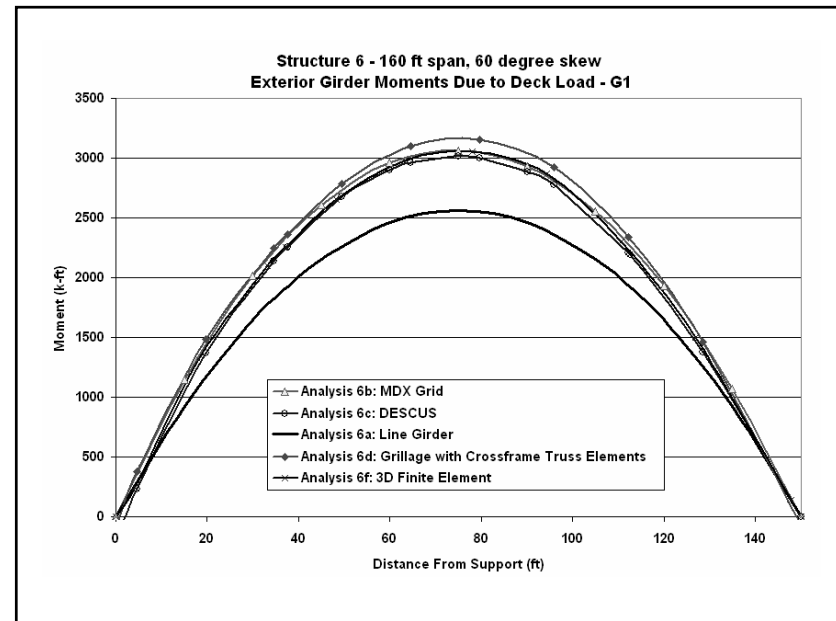
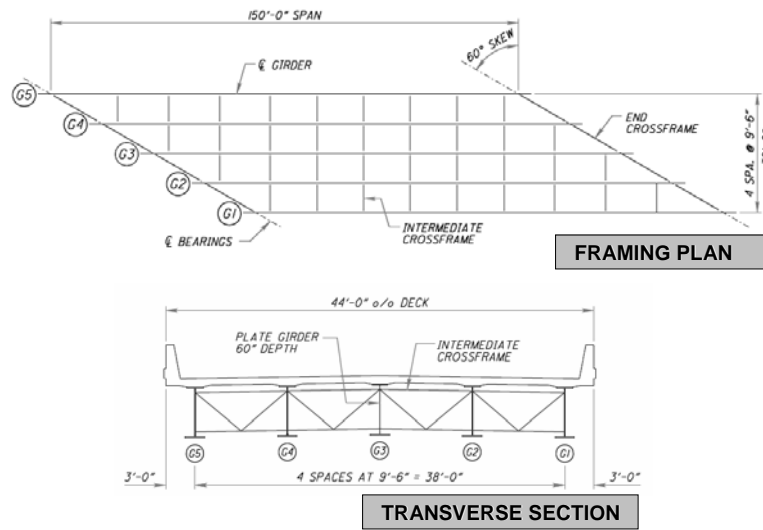
Structures:

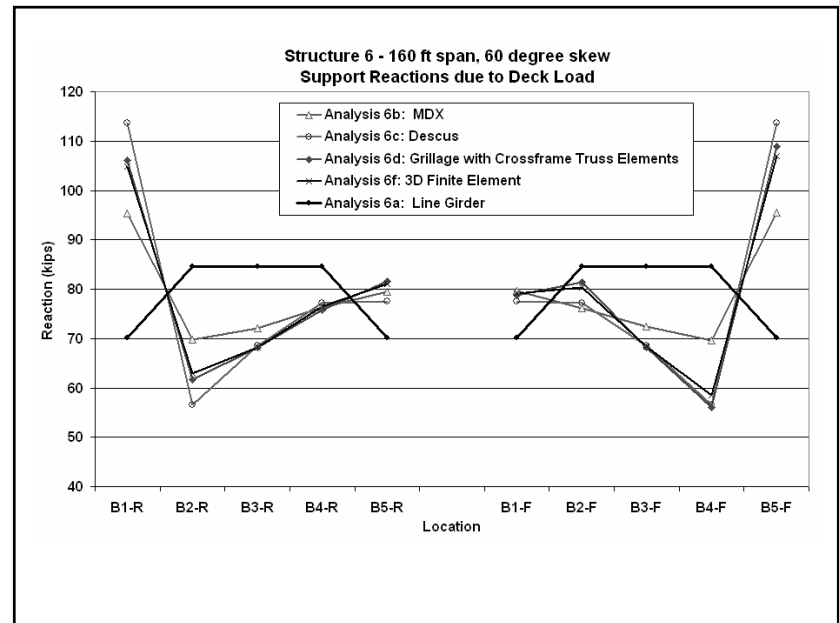
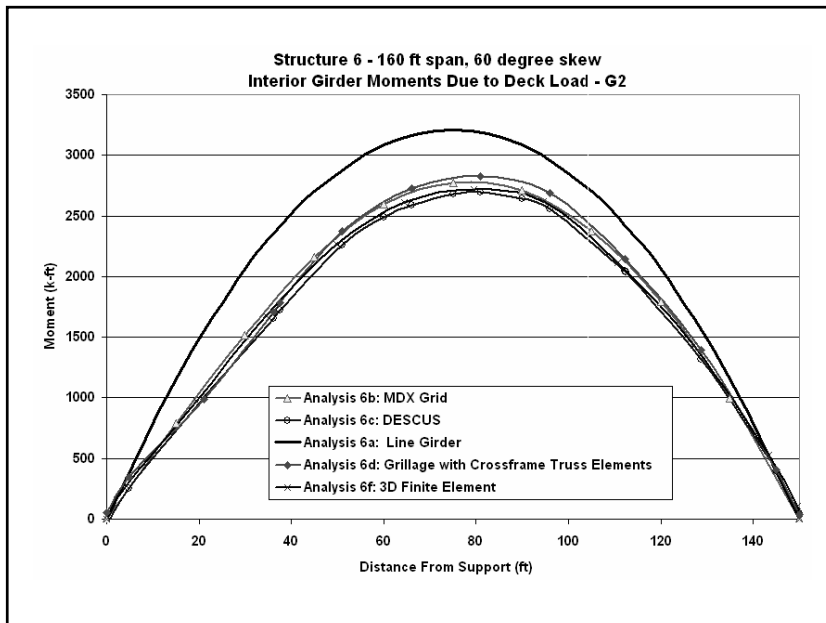
Structure Number	Span	Skew
1	75	30
2	150	30
3	75	45
4	150	45
5	75	60
6	150	60

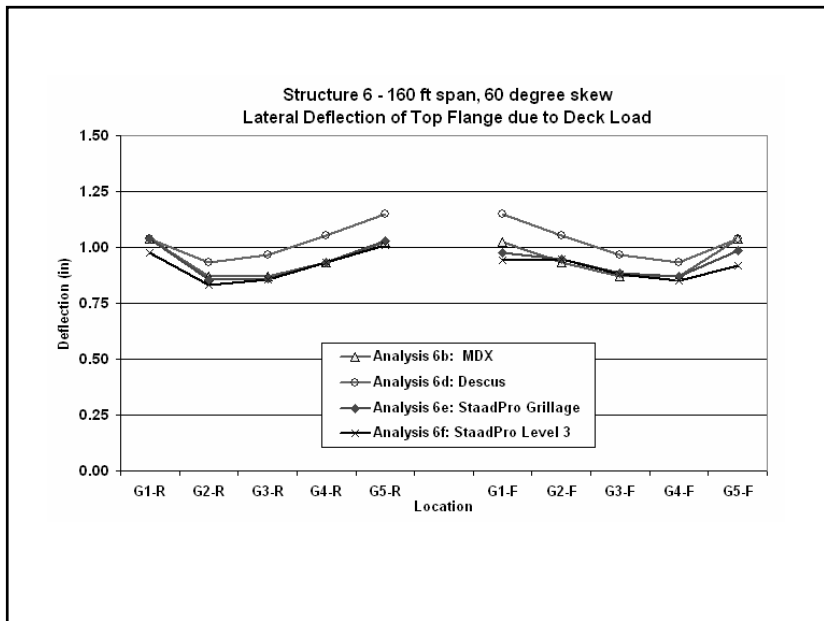
Analyses:

Analysis Number	Model Type	Software	Span	Skew
1a	Line Girder	MDX	75	30
1b	Grillage	MDX	75	30
2a	Line Girder	MDX	150	30
2b	Grillage	MDX	150	30
3a	Line Girder	MDX	75	45
3b	Grillage	MDX	75	45
3d	Grillage	DESCUS	75	45
3e	Grillage	Staad Pro	75	45
3f	Level 3	Staad Pro	75	45
4a	Line Girder	MDX	150	45
4b	Grillage	MDX	150	45
5a	Line Girder	MDX	75	60
5b	Grillage	MDX	75	60
6a	Line Girder	MDX	150	60
6b	Grillage	MDX	150	60
6d	Grillage	DESCUS	150	60
6e	Grillage	Staad Pro	150	60
6f	Level 3	Staad Pro	150	60

### Structure 6:







Study Conclusions:

- For all the bridges studied, results from conventional gird analysis (with beam element crossframes) was sufficiently accurate when considering intermediate crossframe effects.
- Differences in crossframe stiffness do produce differences in results between the different methods, but these differences were small in the cases investigated.
- Truss element crossframes are needed to properly account for end crossframe effects, but these effects do not occur if the end crossframe is not fully connected during the deck pour.
- For complex structures, higher levels of analysis should be considered in order to obtain accurate results.

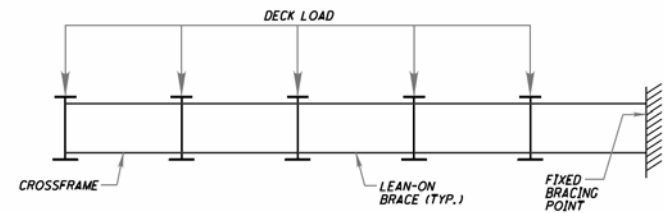
Analysis of Lean-on Construction:

Multiple steps are required to analyze a lean-on system:

- Evaluate girder forces using a global model
- Determine if stiffness of bracing is adequate
- Determine if strength of bracing is adequate

Evaluating Girder Forces, External Lean-on Systems:

- An external lean-on system is a system where no bracing is fully connected in the portion of the structure that will be loaded.
- Lean-on systems that are braced externally only (no girders receiving deck load are directly connected using crossframes) can be analyzed using line-girder analysis.

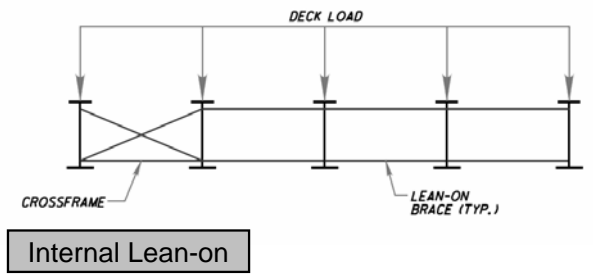


External Lean-on



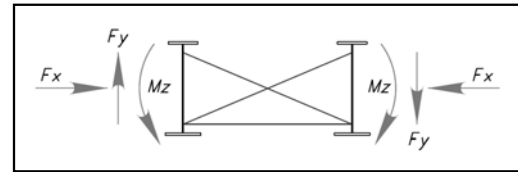
Evaluating Girder Forces, Internal Lean-on Systems:

- An internal lean-on system has both crossframes and lean-on braces within the area being loaded.
- A refined analysis must be used to accurately model the behavior of an internal lean-on system.



Conventional Crossframe Stiffness Matrix:

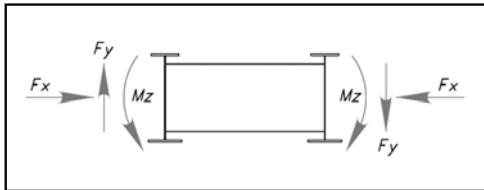
- Conventional crossframes restrain both differential deflection and differential twist between adjacent girders.



$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & K_{23} \\ 0 & K_{32} & K_{33} \end{bmatrix} \times \begin{bmatrix} d_x \\ d_y \\ \phi_z \end{bmatrix}$$

Lean-on Brace Stiffness Matrix:

- Lean-on braces restrain differential twist, but not differential deflection.

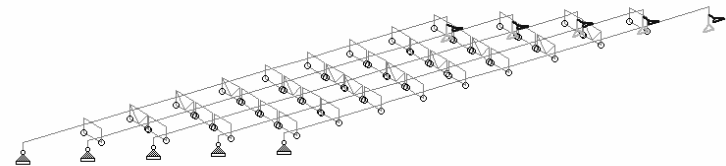


$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_{33} \end{bmatrix} \times \begin{bmatrix} d_x \\ d_y \\ \phi_z \end{bmatrix}$$

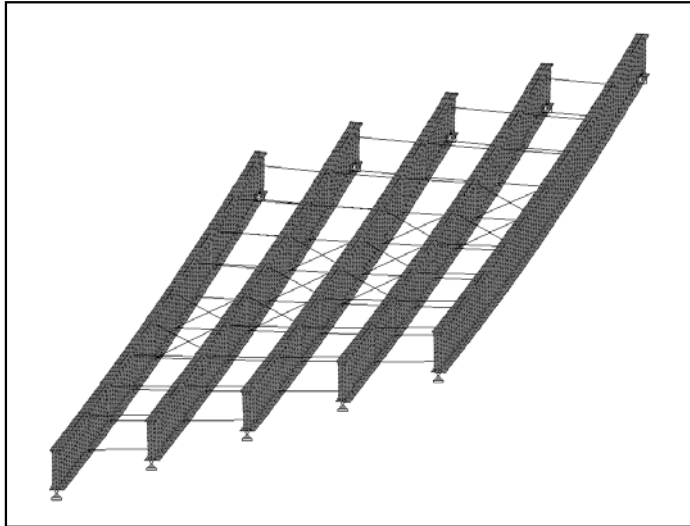
Internal Lean-on System model:

- Not all packaged analysis software is capable of accurately modeling the behavior of a lean-on brace.
- Lean on braces can be modeled by creating a grid model using finite element software.

Internal Lean-on



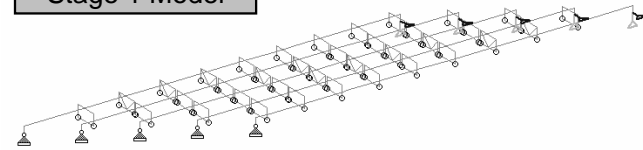
Internal Lean-on System model:



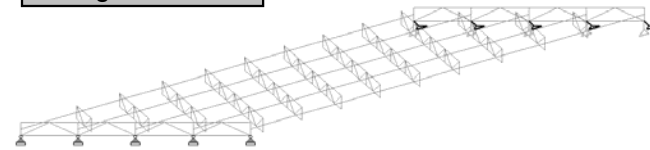
Internal Lean-on System model:

- Diagonal members can be installed at the lean-on brace locations after the deck pour is complete.
- A separate model will need to be created to model the structure after the crossframes are fully connected.

Stage 1 Model



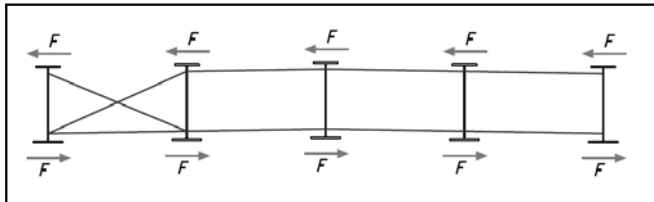
Stage 2 Model



Lean on System: Strength and Stiffness of Girder Bracing

Calculations need to be performed to show that the strength and stiffness of each line of lateral bracing is adequate.

- When I-beams are loaded, out of plane forces are generated due to initial imperfections in the orientation of the structure.
- If these out-of-plane forces exceed the capacity of the bracing, failure of the bracing members can occur.
- If deflections due to these forces become large, buckling of the girders can occur.



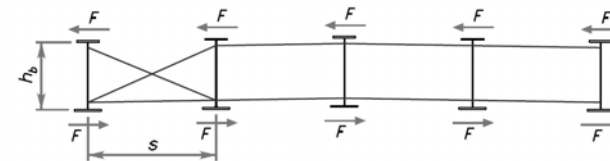
Lean on Sytem: Bracing Strength

- Each girder generates a lateral force  $F$  that can be calculated using the following equation:

$$F = \frac{0.005LL_bM_u^2}{nh_b^2EI_yC_b^2}$$

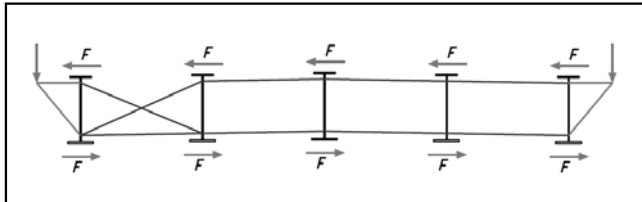
- $L$  = Beam Span
- $M_u$  = Factored Moment at Bracing Location
- $n$  = Number of Braces in Girder Span
- $L_b$  = Spacing Between Torsional Braces
- $h_b$  = Depth of Beam
- $C_b$  = Moment Gradient Factor
- $I_y$  = Moment of inertia for lateral bending

- For design purposes, the lateral force from each girder will be assumed to act in the same direction.



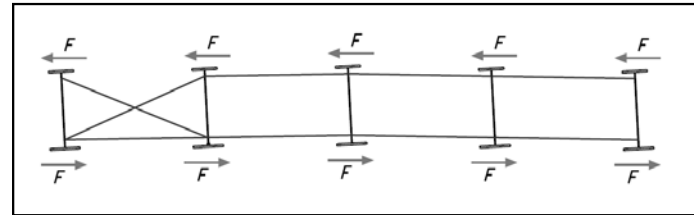
### Lean on System: Bracing Strength

- Other lateral forces, such as forces due to the overhang bracket and forces produced when the deck load is added, may act in conjunction with the lateral force  $F$ .
- All relevant service forces should be included when the capacity of the bracing members is checked.



### Lean-on System: Bracing Stiffness

- If the deflection due to lateral forces becomes too large, lateral torsional buckling of the beams can occur.
- A minimum level of stiffness must be obtained for each line of bracing in order to ensure that buckling does not occur.

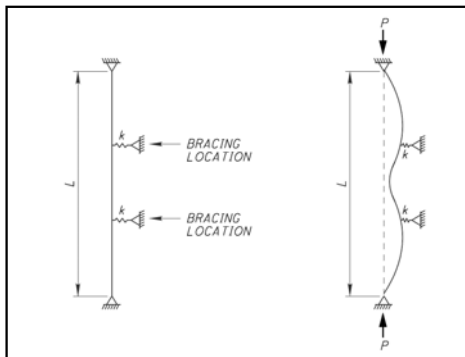


$$\beta_{sys} \geq \beta_T$$

$\beta_{sys}$  = Total system stiffness  
 $\beta_T$  = Minimum Required Stiffness

### Lean-on System: Bracing Stiffness

- The compression flange of the girder can be thought of as a column supported by spring supports at each crossframe location.
- If the stiffness of the supports is below a minimum stiffness ( $k$ ), the column buckles.



### Calculation of bracing stiffness required:

- The minimum required stiffness can be calculated based on the compressive force in the girders, which is proportional to the girder moment.
- $\beta_T$  in the equation below represents the required stiffness for a line of crossframes.

$$\beta_T = \frac{2.4LM_u^2}{nEI_yC_b^2}$$

- $L$  = Span Length  
 $M_u$  = Factored Moment at Bracing Location  
 $n$  = Number of Braces in Girder Span  
 $E$  = Modulus of Elasticity  
 $I_y$  = Weak Axis Moment of Inertia  
 $C_b$  = Moment Gradient Factor

Calculation of bracing stiffness provided:

- The total system stiffness for a braced system is dependent on the stiffness of the braces and the cross-sectional stiffness of the girders.
- Both the in-plane torsional stiffness and the warping stiffness of the girder should be considered.

$$\frac{1}{\beta_{sys}} = \frac{1}{\beta_b} + \frac{1}{\beta_{sec}} + \frac{1}{\beta_g}$$

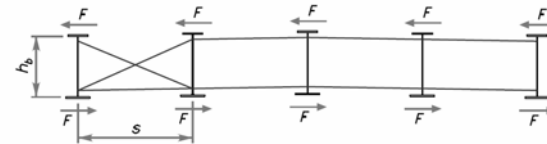
- $\beta_{sys}$  = Total system stiffness
- $\beta_b$  = Brace Stiffness
- $\beta_{sec}$  = Cross-sectional girder distortion
- $\beta_g$  = In-plane girder stiffness

Bracing Stiffness:

- $\beta_b$  accounts for stiffness of the bracing.
- Different formulas are used for different types of crossframes and diaphragms.
- The formula presented here is for an ODOT standard X-brace.

$$\beta_b = \frac{Es^2h_b^2}{\frac{n_{gc}L_d^3}{A_d} + \frac{n_{gc}s^3}{A_s}(n_{gc} - 2)}$$

- $\beta_b$  = In-plane girder stiffness
- $L_d$  = Length of Diagonal
- $n_{gc}$  = Number of girders per crossframe
- $E$  = Modulus of elasticity
- $s$  = Girder spacing
- $h_b$  = Height of girder
- $A_d$  = Cross-sectional area of diagonal
- $A_s$  = Cross-sectional area of strut

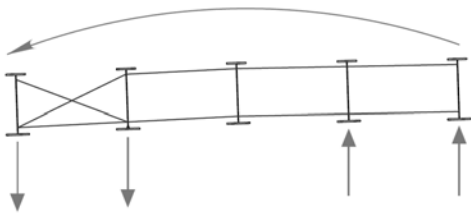


In-plane girder stiffness:

- $\beta_g$  accounts for the strong-axis stiffness of the girders resisting global twisting of the bridge section.

$$\beta_g = \frac{24(n_g - 1)^2 s^2 EI_x}{n_g L^3}$$

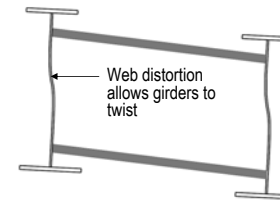
$\beta_g$  = In-plane girder stiffness  
 $L$  = Span length  
 $n_g$  = Number of girders  
 $E$  = Modulus of elasticity  
 $I_x$  = Strong axis moment of inertia  
 $s$  = Girder spacing



Cross Sectional Girder Distortion:

- $\beta_{sec}$  accounts for stiffness of the girder web at the bracing connection

$$\beta_{sec} = 3.3 \frac{E}{h} \left( \frac{(N + 1.5h)t_w^3}{12} + \frac{t_s b_s^3}{12} \right)$$



$\beta_{sec}$  = Cross-sectional girder distortion  
 $h$  = Distance between flange centroids  
 $N$  = Contact length for torsional brace  
 $E$  = Modulus of elasticity  
 $t_w$  = thickness of web  
 $t_s$  = thickness of stiffener  
 $b_s$  = width of stiffener



### Bracing Stiffness Summary:

System stiffness must be greater than minimum required stiffness:  $\beta_{sys} \geq \beta_T$

Calculate system stiffness & required stiffness:

$$\frac{1}{\beta_{sys}} = \frac{1}{\beta_b} + \frac{1}{\beta_{sec}} + \frac{1}{\beta_g} \quad \beta_T = \frac{2.4LM_u^2}{nEI_y C_b^2}$$

Calculate system stiffness components:

$$\beta_b = \frac{Es^2 h_b^2}{\frac{n_{gc} L_d^3}{A_d} + \frac{n_{gc} s^3}{A_s} (n_{gc} - 2)} \quad \beta_{sec} = 3.3 \frac{E}{h} \left( \frac{(N + 1.5h)t_w^3}{12} + \frac{t_s b_s^3}{12} \right)$$
$$\beta_g = \frac{24(n_g - 1)^2 s^2 EI_x}{n_g L^3}$$

### Lean on System: Strength and Stiffness of Girder Bracing

A complete explanation of lean-on bracing design is beyond the scope of this presentation. The following references provide guidance on this topic:

Herman, Helwig, Holt, Medlock, Romage, and Zhou, "Lean-on Crossframe Bracing for Steel Girders with Skewed Supports"  
<http://www.steelbridges.org/pdfs/.%5CHerman.pdf>

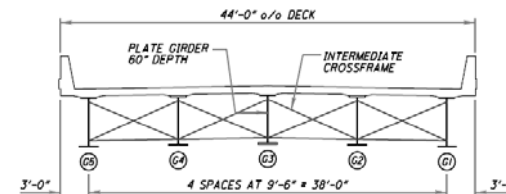
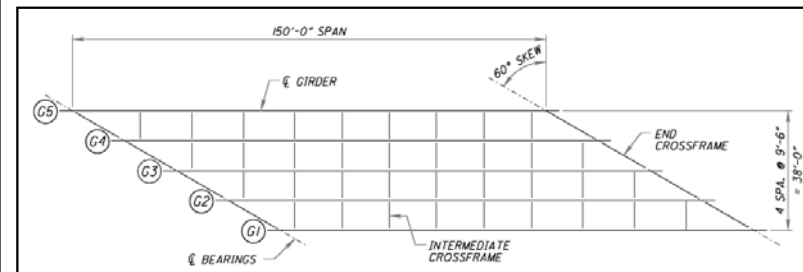
Beckmann and Medlock, "Skewed Bridges and Girder Movements Due to Rotations and Differential Deflections"  
<http://www.steelbridges.org/pdfs/.%5CBeckmann.pdf>

Yura, J. A. (2001), "Fundamentals of Beam Bracing", Engineering Journal, American Institute of Steel Construction, 1st Quarter, pp. 11-26.

### Summary of Lean-on System Design:

1. Perform line girder analysis to determine moments and shears due to girder self weight.
2. Perform grid analysis using non-composite section properties and top and bottom strut braces at appropriate locations to determine moments and shears due to the deck weight.
3. Perform grid analysis using composite section properties and fully connected crossframes to determine moments and shears due to superimposed dead load and live load.
4. Calculate bracing forces occurring during the deck pour to verify that member capacity is adequate.
5. Calculate required stiffness and provided stiffness for each line of bracing during the deck pour to verify that adequate bracing is provided.

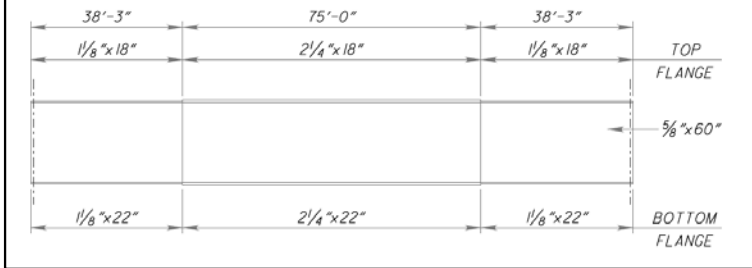
### Design Example:



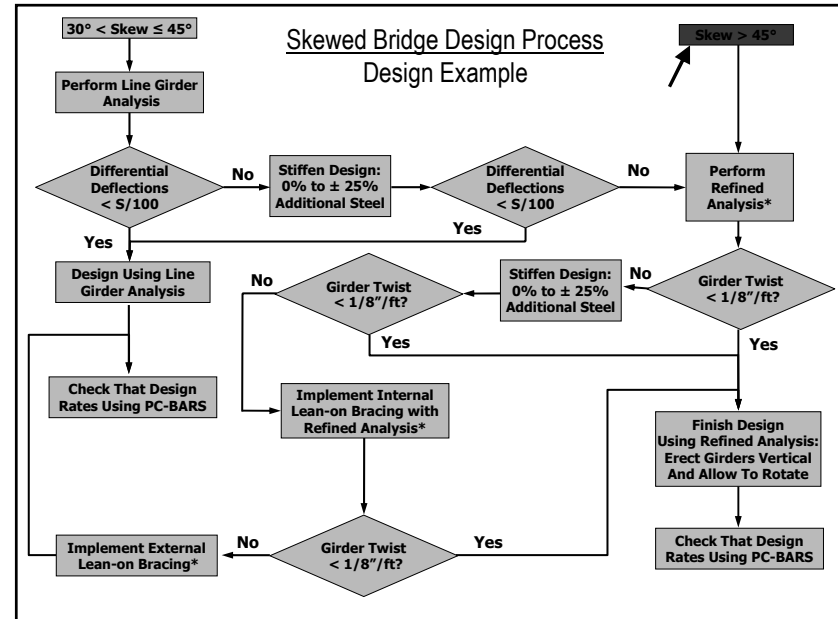
- Single span
- 5 girder lines
- 150 ft span
- 60 degree skew

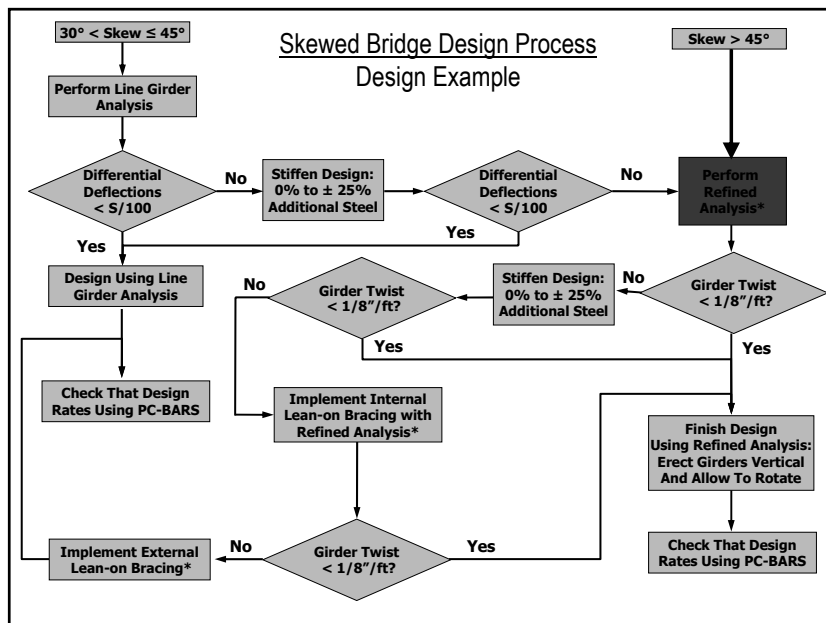
Design Example Continued:

Preliminary girder elevation:



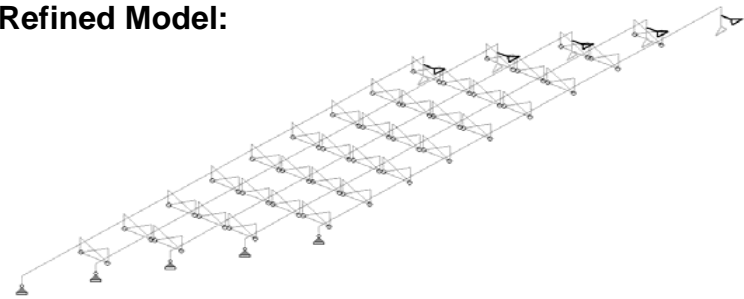
- Initial girder size is determined using line girder analysis.
- In the final design, all girders should meet line girder design requirements at a minimum. Do not reduce the strength of the girders based on refined analysis results.





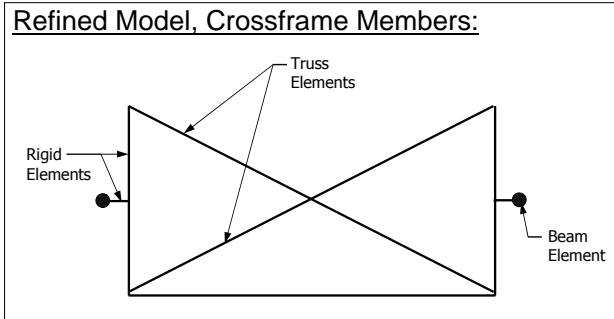
Perform Refined Analysis\*

### Refined Model:

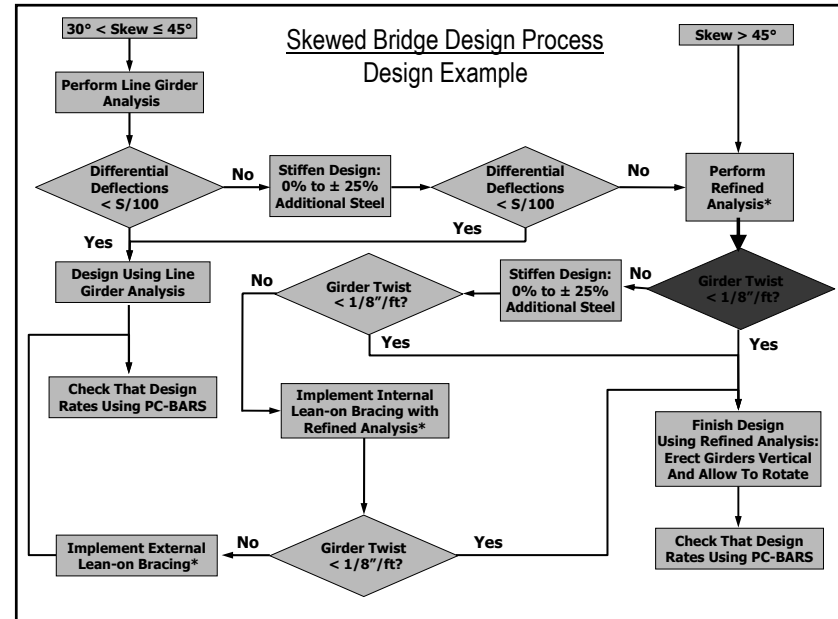


- The initial model is for the deck pour case only.
- Non-composite section properties should be used.
- A grid model with truss-element crossframes is used in this case.
- End crossframes are not included because they will not be connected during the deck pour.

Perform Refined Analysis\*

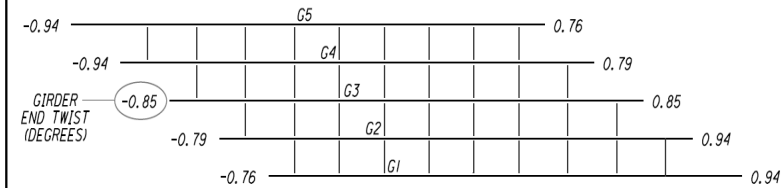


- Crossframes members are modeled using truss elements.
- Rigid elements are used to represent stiffener connections to girders.
- This is one possible method. Other methods can be used to achieve the same result.



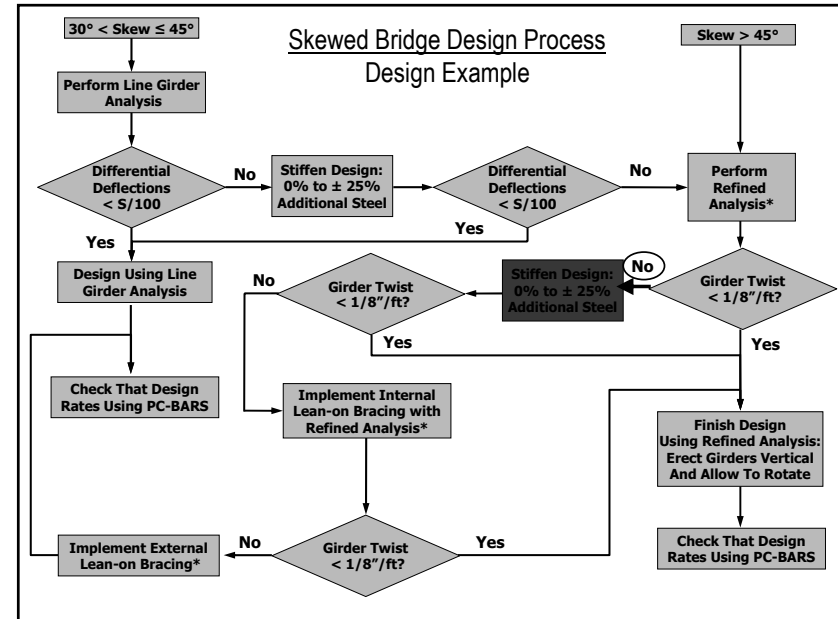
Girder Twist  
< 1/8" /ft?

Output from refined model:



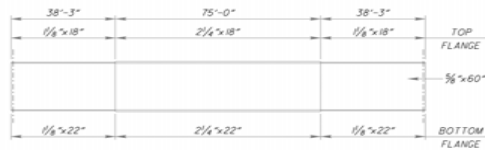
- The highest twists occur at the girder end in this case. Twists along the full length of each girder should be investigated.
- Maximum girder twist = 0.94°
- $\tan(0.94^\circ) \times 12'' = 3/16''$  per foot

3/16" per foot > 1/8" per foot  
Twist is outside of acceptable range



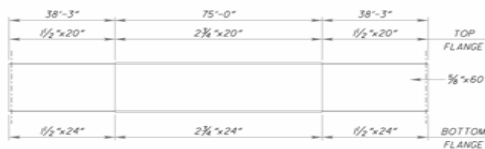
Stiffen Design:  
0% to ± 25%  
Additional Steel

Original Design:



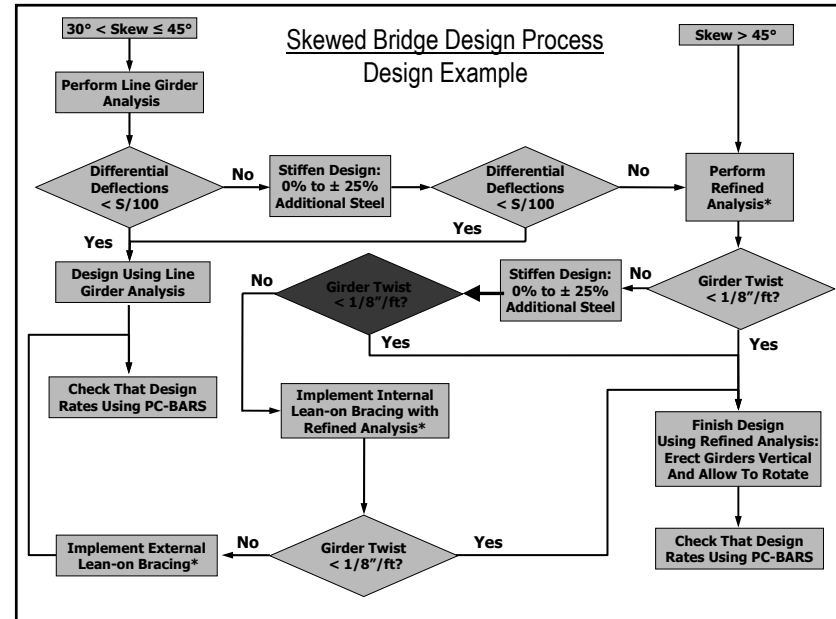
Girder Weight =  
54.0 kips

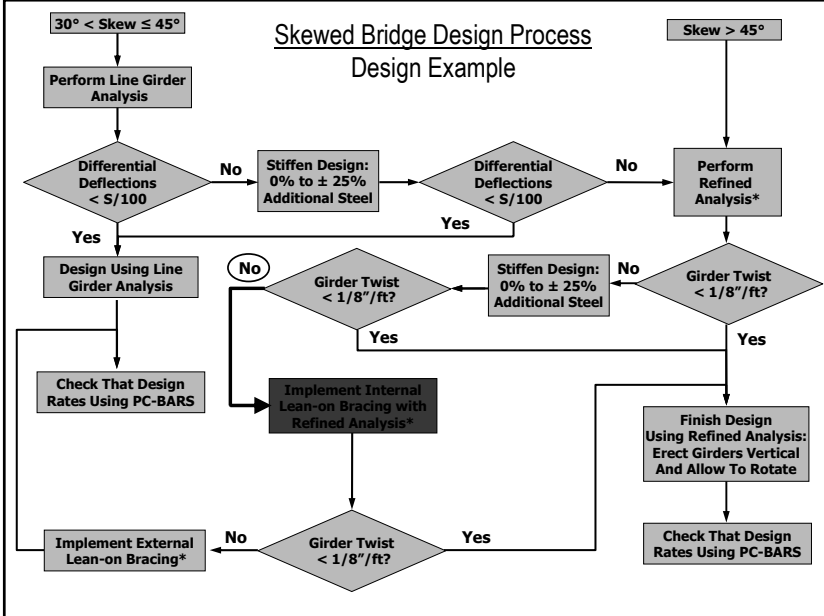
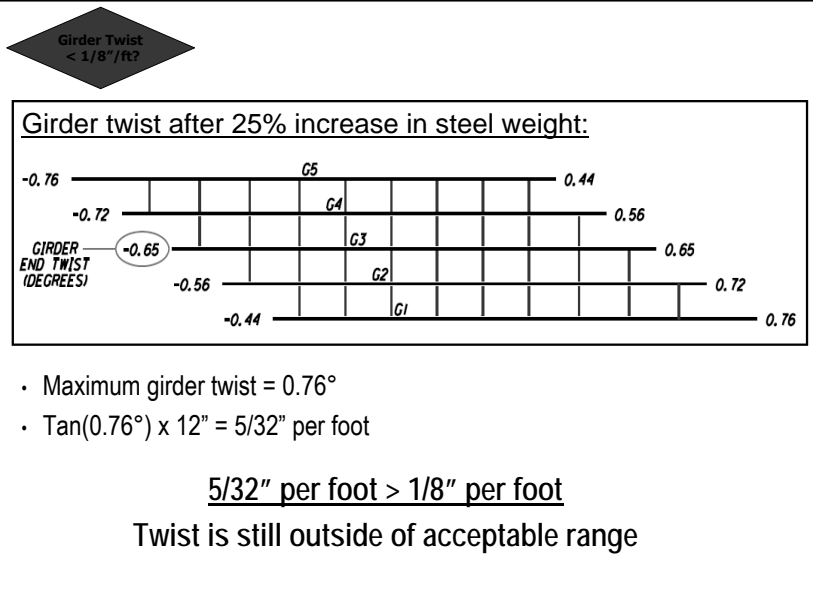
Stiffened Design:



Girder Weight =  
67.4 kips  
(25% increase)

- Increasing girder stiffness reduces deflection and rotations.
- In this case, web depth cannot be increased due to site constraints.



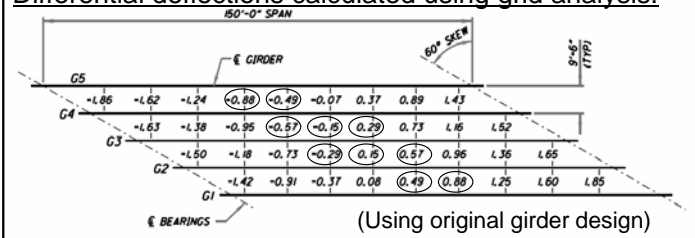




Implement Internal Lean-on Bracing with Refined Analysis\*

- Lean-on designs reduce girder twist by temporarily replacing crossframes where large differential deflections occur with braces consisting of top and bottom struts only.
- In an internal lean-on design, some crossframes are left in place during the deck pour while others are replaced.
- Mapping crossframe differential deflections is helpful in determining which crossframes should be left in place.

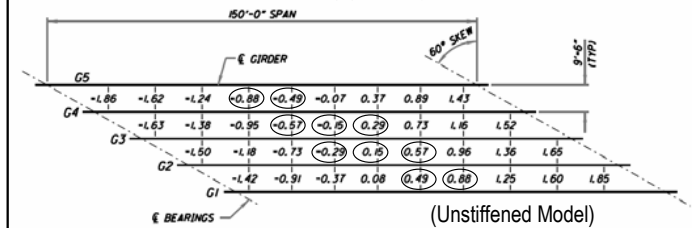
Differential deflections calculated using grid analysis:



(Using original girder design)

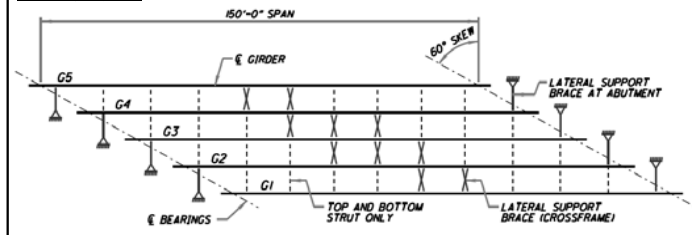
Implement Internal Lean-on Bracing with Refined Analysis\*

Differential deflections calculated using grid analysis:



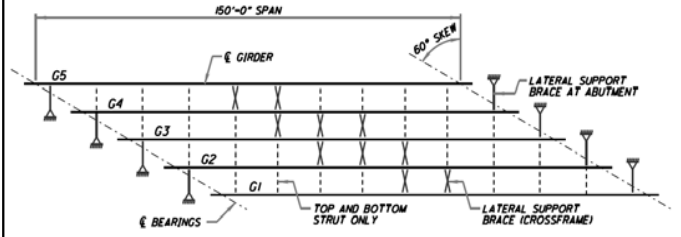
(Unstiffened Model)

Bracing layout:



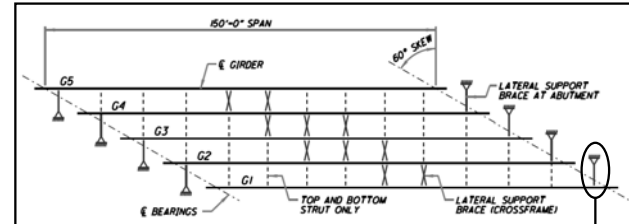
Implement Internal Lean-on Bracing with Refined Analysis\*

Bracing layout:

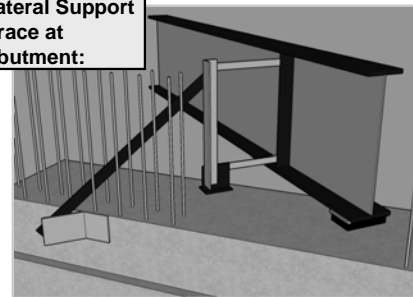


- In order to maintain stability, each line of lateral bracing needs to be restrained by at least one fully connected crossframe or by a lateral support brace at the abutment.
- Calculations need to be performed to show that the strength and stiffness of each line of lateral bracing is adequate.

Implement Internal Lean-on Bracing with Refined Analysis\*

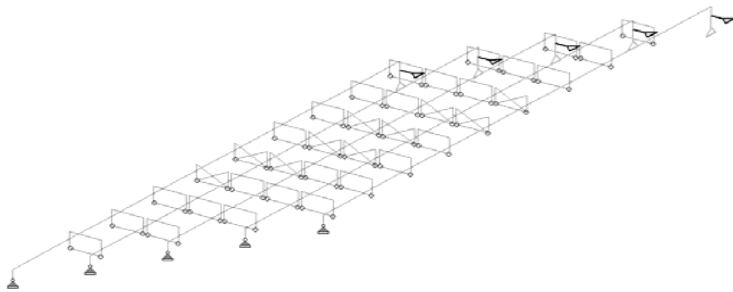


Lateral Support Brace at Abutment:



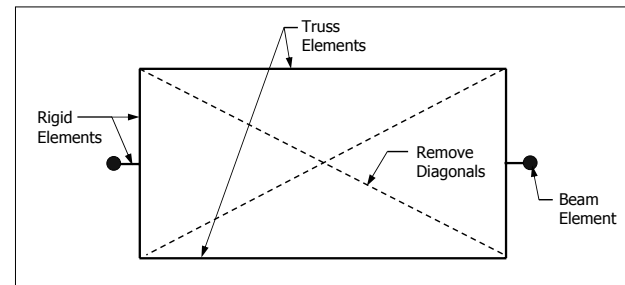
- Abutment braces should be avoided if possible.
- They often become necessary where large differential deflections occur near the abutment.

Implement Internal Lean-on Bracing with Refined Analysis\*

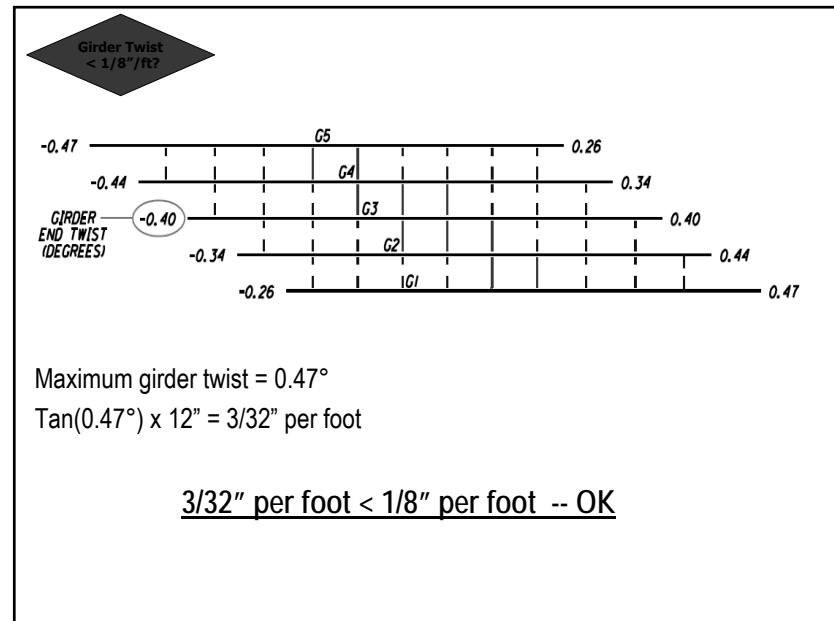
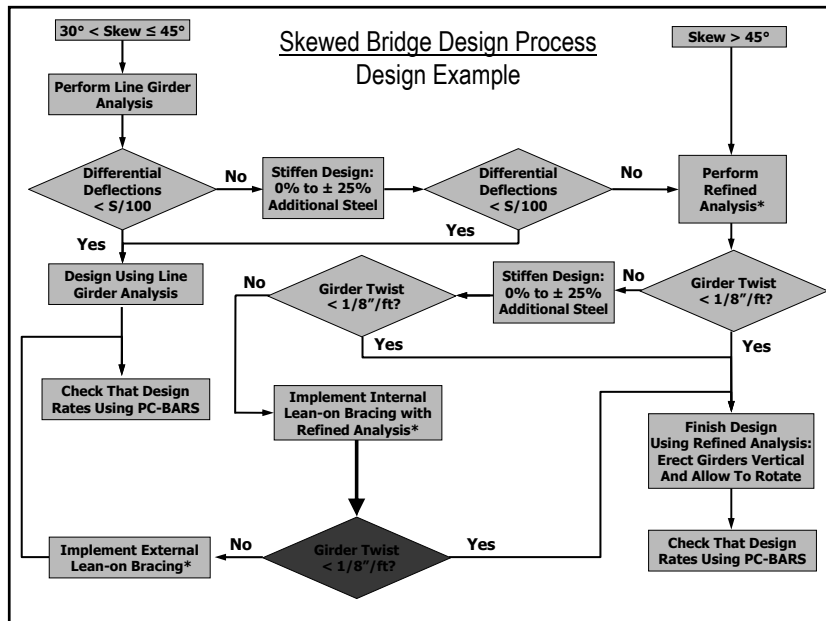


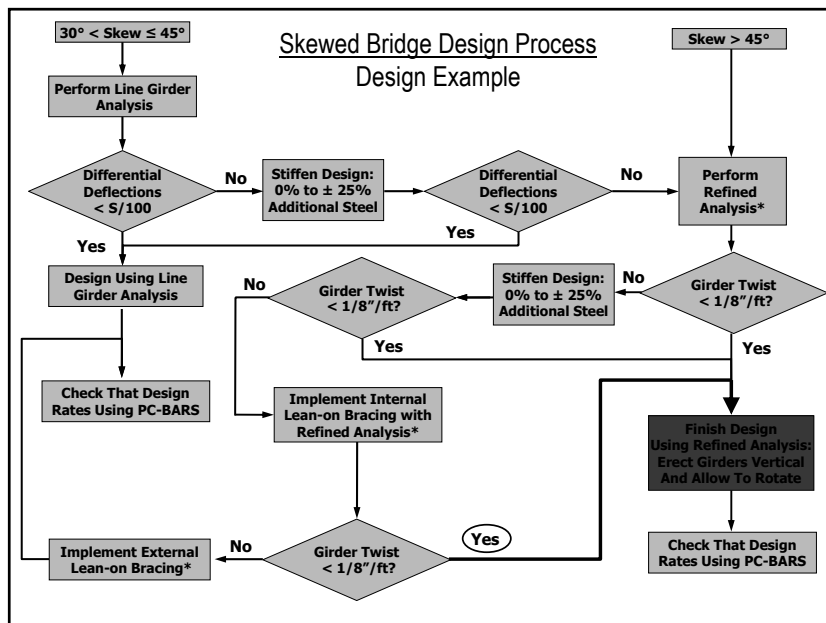
- Crossframe members are modified in the refined model to reflect the lean-on design.
- 25% steel weight increase can be used if necessary. In this case we will use the original design, prior to stiffening.
- Once again, we are looking at loads due to the deck pour only.

Implement Internal Lean-on Bracing with Refined Analysis\*



- In the finite element model truss elements can be used to model top and bottom struts at locations where crossframes are removed.
- If packaged grid analysis software is used, consult technical support before attempting to model this type of brace.





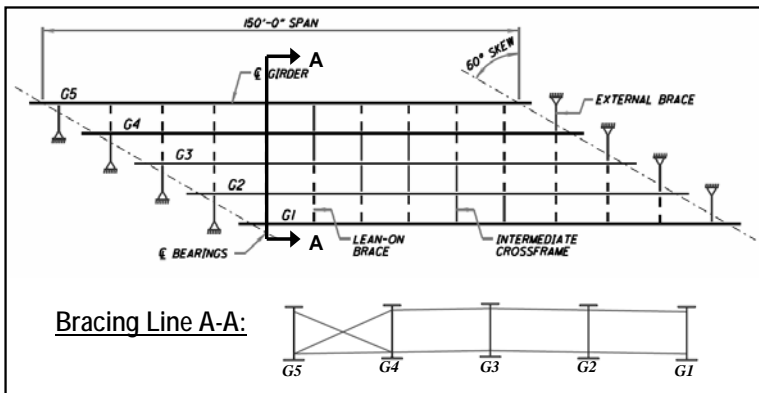
Finish Design  
Using Refined Analysis:  
Erect Girders Vertical  
And Allow To Rotate

#### Remaining Design Tasks:

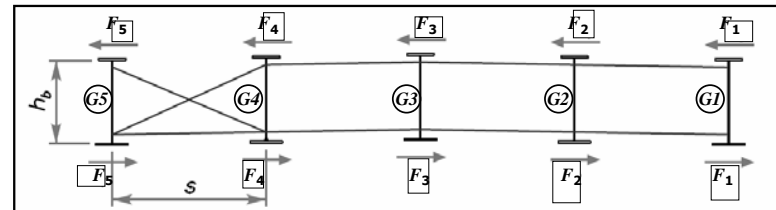
1. Perform calculations to verify that the strength and stiffness of the lean-on bracing system is adequate.
2. Calculate composite dead load and live load forces using a revised model with fully connected crossframes and composite section properties.
3. Perform code checks, revise the design as necessary.

Finish Design  
Using Refined Analysis:  
Erect Girders Vertical  
And Allow To Rotate

Calculations need to be performed to show that the strength and stiffness of each line of lateral bracing is adequate.  
For this example, partial calculations will be performed for one line of bracing.



### Bracing Strength:



Calculate all forces,  $F_1$  through  $F_5$ , based on  $M_u$  at brace location:

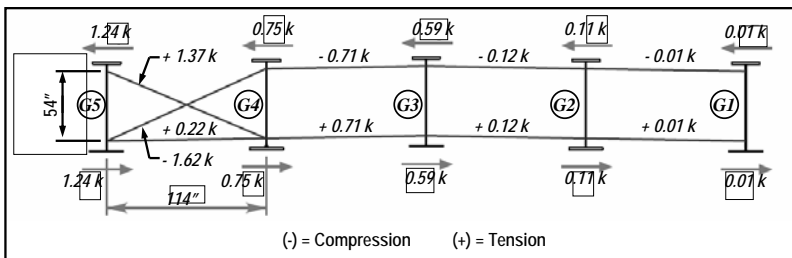
$$F_5 = \frac{0.005LL_b M_u^2}{nh_b^2 EI_y C_b^2} = \frac{0.005(1800')(180')(49,221 \text{ k}')^2}{(9)(61.13'')^2 (29,000 \text{ ksi})(3,091 \text{ in}^4)(1.03)^2} = 1.24 \text{ kips}$$

Calculate using the same method:

$$\left. \begin{array}{l} F_1 = 0.01 \text{ kips} \\ F_2 = 0.11 \text{ kips} \\ F_3 = 0.59 \text{ kips} \\ F_4 = 0.75 \text{ kips} \end{array} \right\}$$

Bracing Strength:

- Sum forces in each bracing member based on calculated lateral forces.
- Forces from stage 1 grid analysis and forces due to overhang bracket loads should also be included (not shown here).



Bracing Strength:

Calculate bracing member capacities and verify that member size is adequate.

Maximum factored load in diagonal = 1.62 kips

Ultimate capacity of diagonal = 42 kips

OK

Maximum factored load in horizontal strut = 0.71 kips

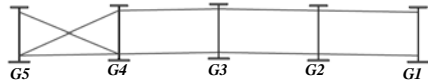
Ultimate capacity of horizontal strut = 51 kips

OK

Bracing stiffness is adequate for line A-A.  
Repeat analysis for each bracing line.

**Bracing Stiffness:**

**Bracing Line A-A:**



Calculate stiffness required ( $\beta_T$ ) for each girder along line A-A:

$$\beta_T (G1) = \frac{2.4LM_u^2}{nEI_y C_b^2} = \frac{2.4 (1800 \text{ in})(49221 \text{ k} \cdot \text{in})^2}{(9 \text{ braces})(29000 \text{ ksi})(3091 \text{ in}^4)(1.03)^2} = 12,324 \text{ k} \cdot \text{in/rad}$$

$$\beta_T (G2) = 7,434 \text{ k} \cdot \text{in/rad}$$

$$\beta_T (G3) = 5,850 \text{ k} \cdot \text{in/rad}$$

$$\beta_T (G4) = 1,055 \text{ k} \cdot \text{in/rad}$$

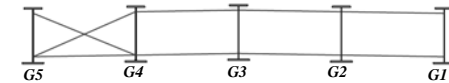
$$\beta_T (G5) = 122 \text{ k} \cdot \text{in/rad}$$

Maximum – Use this value for comparison

It is conservative to use the maximum value of  $\beta_T$  for comparison. In a more detailed analysis, stiffness provided ( $\beta_{sys}$ ) can be calculated separately for each girder.

**Bracing Stiffness:**

**Bracing Line A-A:**



$$\beta_b = \frac{Es^2 h_b^2}{\frac{n_{gc} L_d^3}{A_d} + \frac{n_{gc} S^3}{A_s} (n_{gc} - 2)} = \frac{(29000 \text{ ksi})(114 \text{ in})^2 (61.125 \text{ in})^2}{\frac{(5 \text{ gird/cf})(120.74 \text{ in})^3}{(2.89 \text{ in}^2)} + \frac{(5 \text{ gird/cf})(114 \text{ in})^3}{(2.89 \text{ in}^2)} (5 \text{ gird/cf} - 2)} = 131,173 \text{ k} \cdot \text{in}$$

Minimum Value, Applies to G1 only (Conservative)

$$\beta_{sec} = 3.3 \frac{E}{h} \left( \frac{(N + 1.5h)t_w^3}{12} + \frac{t_s b_s^3}{12} \right) = 3.3 \frac{29000 \text{ ksi}}{61.13 \text{ in}} \left( \frac{(0 + 1.5(61.13))(0.625)^3}{12} + \frac{(.375)(10.625)^3}{12} \right) = 61,606 \text{ k} \cdot \text{in}$$

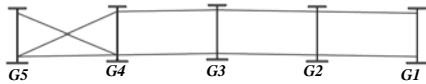
$$\beta_g = \frac{24(n_g - 1)^2 s^2 EI_x}{n_g L^3} = \frac{24(5 \text{ girders} - 1)^2 (114 \text{ in})^2 (29000 \text{ ksi})(53058 \text{ in}^4)}{(5 \text{ girders})(1800 \text{ in})^3} = 263,331 \text{ k} \cdot \text{in}$$

$$\beta_{sys} = \frac{1}{\frac{1}{\beta_b} + \frac{1}{\beta_{sec}} + \frac{1}{\beta_g}} = \frac{1}{\frac{1}{131,173} + \frac{1}{61,606} + \frac{1}{263,331}} = 36,162 \text{ k} \cdot \text{in}$$



Bracing Stiffness:

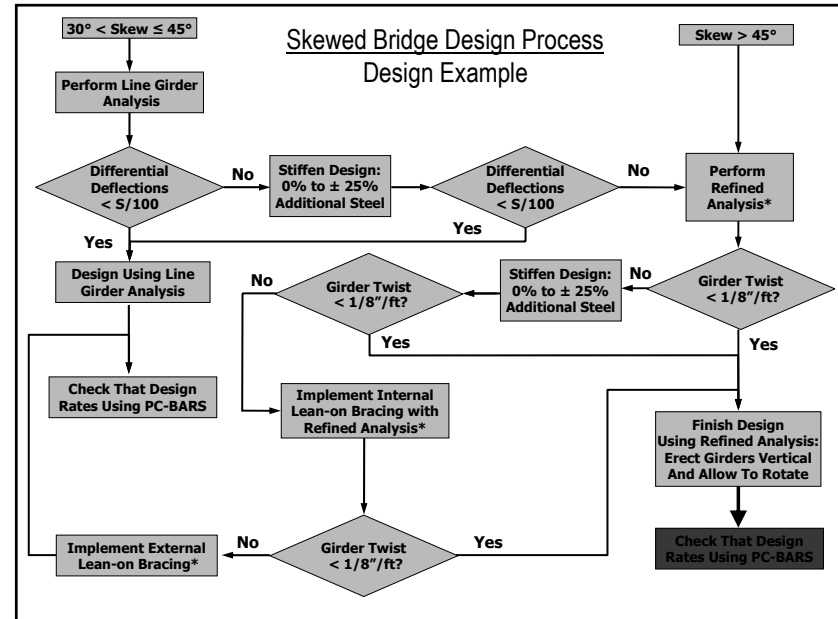
Bracing Line A-A:



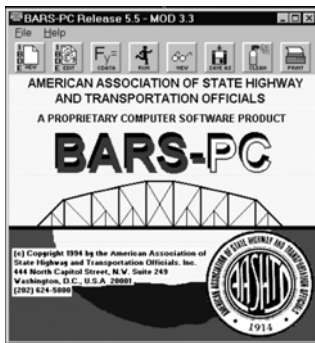
$$\beta_{sys} = 36,162 \text{ k} \cdot \text{in} \geq \beta_T = 12,324 \text{ k} \cdot \text{in}$$

$$\beta_{sys} \geq \beta_T$$

Bracing stiffness is adequate for line A-A.  
Repeat analysis for each bracing line.



Check That Design Rates Using PC-BARS



- All designs must rate to HS-25 when analyzed using BARS-PC, regardless of analysis method.
- Differences do exist between BARS-PC and some widely used steel design software.
- It is a good idea to run BARS-PC early.

QUESTIONS ?

E-mail questions to:  
[ose@dot.state.oh.us](mailto:ose@dot.state.oh.us)