Verification and Calibration of the Design Methods for Rock Socketed Drilled Shafts for Lateral Loads

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for the
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Office of Research and Development

and the
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This project was aimed at: (1) evaluating and developing design methods for laterally loaded drilled shafts socketed in anisotropic rock, (2) Develop a p-y criterion that can be used to analyze the response of drilled shaft socketed into transversely isotropic rock or jointed rock, (3) Develop a new methodology for determining the five elastic constants of a transversely isotropic rock medium using the in-situ pressuremeter test device, (4) Develop a new equation for estimating the transverse isotropic rock shear modulus, \( G' \) using other elastic constants. (5) Develop a new methodology to obtain a p-y criterion using in-situ pressuremeter technique that can be used to analyze the response of drilled shaft socketed into transversely isotropic rock.

The hyperbolic p-y criterion for rock proposed in SJN 134137 based on the field test data and extensive theoretical work was further validated using additional load test data. Validation of the proposed p-y criterion of rock was carried out by comparing the predictions of shaft deflections and bending moments using the hyperbolic p-y criterion against actual lateral load tests results.

Based on the findings of this study, a complete solution for the design of drilled shafts socketed in anisotropic rock or intermediate geomaterials under lateral loads is provided.
## SI Conversion Factors

### Approximate Conversions from SI Units

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February 2011


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CHAPTER I: INTRODUCTION

1.1 STATEMENT OF THE PROBLEM

Rock socketed drilled shafts provide significant benefits for carrying lateral loads. Embedment in rock, in most cases, reduces the lateral displacement substantially compared to a deep foundation in soil. However, the current design practice for laterally loaded drilled shafts embedded in rock profiles requires a challenging effort on the part of the engineer. A substantial cost savings could be realized, while maintaining an acceptable and safe performance, if a rational method for the analysis and design of drilled shafts were developed. ODOT currently does not have an appropriate method for designing rock-socketed drilled shafts and subsurface investigation programs do not provide the necessary geotechnical strength parameters and stress-strain properties to be used for the design of drilled shafts socketed into rock.

A range of analytical tools are available to foundation designers to consider rock sockets under lateral and moment loading. These include simple, closed-form equations requiring a small number of material properties (Carter & Kulhawy 1988, 1992). A more rigorous model that predicts the complete nonlinear response but requires more material properties is also available (Zhang et al. 2000). A serious challenge with regard to applying any analytical method to the response of rock is the dominant role played by the secondary structure of rock. The Canadian Foundation Manual (1978) has addressed secondary structure by basing the behavior of rock, in addition to compressive strength, on the spacing and thickness of soil-filled cracks and joints. All of the currently available
methods suffer from a lack of field data for verification, and are best applied with local experience, through knowledge of the geologic environment, and field load testing.

P-y method of analysis of laterally loaded drilled shaft is the most widely accepted design approach. However, rock has been given little attention in the p-y method of analysis. In many practical designs, rock exists under a stratum or strata of overburden of sufficient thickness that the computed deflection of a pile at the rock is so small that the resistance of the rock is small and the response of the pile is little affected, regardless of the stiffness of the rock. However, the combination of rock near or at the surface and a significant magnitude of lateral loading do occur frequently. In such cases, even though the axial load is substantial, lateral loading may dictate the design penetration length. The research team has developed a hyperbolic p-y criterion for isotropic rock mass based on a limited number of the lateral load test data in Ohio. The mathematic framework and theoretical basis of the proposed p-y criterion has been well established. Nevertheless, the p-y criterion needs further verification by comparing with additional lateral load tests conducted in Ohio rock formation.

Anisotropy is very common in many rocks even when there are no distinctive discontinuous structure (e.g., joints and beddings). This is because of the preferred orientation of mineral grains or the directional stress history. Foliation and schistosity make schists, slates, and many other metamorphic rocks highly directional in their deformability, strength, and other properties. Bedding makes shales, thin-bedded sandstone and limestone and other common sedimentary rocks highly anisotropic (Goodman, 1989). In a recent review by Turner, et al (2006), it was stated that
characterizing the nature of the rock mass and determining its properties becomes one of the main challenges faced by foundation designers because of the rock anisotropy, especially when considering the laterally loaded rock-socketed drilled shafts. Weathered rock, weak rock, soft rock, and fractured rock are some terminology used by many researchers to describe the anisotropic rock. However, all these rocks could be characterized as anisotropic rock. To characterize the rock properties, including the effects of joints, there are different rock mass classifications in use: Rock Mass Quality (Q) system developed by the Norwegian Geotechnical Institute (Barton et al., 1974), Rock Mass Rating (RMR) system developed by the Council for Scientific and Industrial Research, South Africa (Bieniawski, 1976 and 1989), and Geological Strength Index (GSI) system (Hoek, 1994). All these classification systems were developed initially for tunnel or dam applications and later adopted for other engineering applications, such as slopes and foundations. However, using these classification systems for the foundation engineering is still not well established. In particular, one needs to be cognizant about the fact that solely relying on these rock classification systems and the related empirical correlation equations can lead to erroneous designs which in turn usually results in excessive socket length requirements. The influence of the secondary structure and the anisotropy effect on the p-y criterion of rock requires further development.

For the design of drilled shafts under lateral loading in rock, special emphasis is necessary in the geotechnical investigation of the rock formation. The research team has learned from the previous research project that there was a lack of well thought out guidelines and specifications for subsurface exploration in rock, particularly for lateral rock-shaft interaction analysis. The research team has also learned that careful attention is
required to establish procedures and specifications for the fieldwork. For example, the values for RQD, percent of recovery, and compressive strength of the rock can probably be more seriously in error from improper procedures than the corresponding properties of the soil. Methods of investigation should reveal such a condition and designers must address the potential weakness of the rock in a site-specific manner. Currently, ODOT does not have a precise procedure to differentiate between rocks and strong soils. In general, rocks are sampled by coring rather than by pushing a thin-walled tube. There are some intermediate geomaterials (IGM) that the designers may wish to pay special attention as well. ODOT can benefit tremendously by having a well developed field investigation guidelines for rock formation and in IGM in lateral deep foundation applications.

The demand for a more rational, economical design and an attempt to improve the consistency in the overall structural safety has resulted in the re-examination of the entire design process for bridges. Until recently, adoption of the statistical nature of the Load and Resistance Factor Design (LRFD) methods has mainly been directed towards the bridge superstructure - not the foundation elements. The AASHTO LRFD code, up to the 2004 3rd edition, has limited the determination of geotechnical capacities to the calibration of known (presumptive) values obtained in ASD/LFD design codes. Currently, significant changes to Section 10 of the specifications have been adopted by AASHTO LRFD bridge subcommittee and been published in the 2006 interims and the revisions up to the 2010 edition. This includes changes which incorporate the statistical methods to evaluate the nominal resistance of a foundation material. The LRFD methods contained in the AASHTO Guide were developed for
conditions different from those encountered in Ohio. Thus, verification and calibration of the AASHTO LRFD recommendations using Ohio specific data is necessary. The most accurate design method for drilled shafts is to extract design parameters from a load test on test shafts constructed at construction project site. The load tests are expensive and therefore are only considered for relatively large projects. The information derived from the load test could be used: 1) to design production shafts with more confidence, possibly resulting in large cost savings to the project, and 2) as database to improve the accuracy of design methods for predicting drilled shaft capacity.

In-situ pressuremeter test (PMT) has been used widely as an in-situ testing method in connection with the design of drilled shafts in soils subjected to horizontal loads. Indeed some analogy exists between the cylindrical expansion of the PMT probe and the horizontal movement of the drilled shaft segment. However, steps are necessary to extrapolate the pressuremeter expansion curve to the p-y curve for the drilled shaft. Among all existing methods for the design of laterally loaded drilled shafts using the in-situ pressuremeter test results, there is no method developed for the prediction of p-y curves for the drilled shafts socketed into transversely isotropic rock. This study will accomplish the need of establishing p-y curves from pressuremeter test results in the transversely isotropic rock mass.

1.2 OBJECTIVES

The objectives of this research are two folds: (a) to develop pertinent p-y criteria for cohesive IGM and for rock mass with special attention to the effects of secondary features, and (b) to develop an appropriate interpretation method for converting the
pressuremeter expansion curve into the relevant p-y curves for drilled shafts in a rock medium. The specific objectives related to the development of p-y criteria can be enumerated below:

1. Develop a p-y criterion for cohesive IGM using hyperbolic mathematical formulation.

2. Develop an equivalent transversely isotropic homogeneous model to characterize the stiffness of jointed rock masses with parallel discontinuities.

3. Develop a method for the determination of initial modulus of subgrade reaction of a laterally loaded drilled shaft socketed into a transversely isotropic rock mass.

4. Investigate through Finite Element (FE) simulation techniques the mechanisms of the mobilization of side shear resistance of a rock socketed drilled shaft to develop an empirical solution so that it can be used to estimate the ultimate side shear resistance at the rock-drilled shaft interface.

5. Develop a method to estimate the ultimate lateral resistance of transversely isotropic rock and jointed rock due to the lateral deflection of a drilled shaft.

6. Develop a p-y criterion that can be used to analyze the response of drilled shaft socketed into transversely isotropic rock or jointed rock.

7. Identify the appropriate field or laboratory test methods for determining the transversely isotropic rock properties used in the developed p-y criterion. Necessary correlations between these properties were also investigated.
8. Perform sensitivity and parametric analysis to shed insight on the effects of the rock anisotropy on the predicted drilled shaft behaviour under lateral loads.

9. Perform full-scale field lateral load tests on fully instrumented drilled shafts socketed into cohesive IGM to obtain reliable and comprehensive field test data for validating the p-y criterion.

The specific objectives related to the pressuremeter applications can be enumerated as follows.

1. Develop a new methodology for determining the five elastic constants of a transversely isotropic rock medium using the in-situ pressuremeter test device.

2. Develop the interrelationships between the five elastic constants of the transversely elastic rock and the tangent, $K_i$, to the linear portion of the $(\Delta p, \Delta V/V_o)$ curve obtained from the pressuremeter tests.

3. Develop a series of charts for quick estimate of the Young’s modulus $E$ and $E'$ for different Poisson’s ratio $\nu$ and inclination angle $\theta$ between the axis of the borehole and the plane of the symmetry of the transversely isotropic elastic medium.

4. Study statistically the relations among the elastic constants of a transversely isotropic rock based on a comprehensive and well documented data base.

5. Develop a new equation for estimating the transverse isotropic rock shear modulus, $(G')$ using other elastic constants.
6. Develop an expression for estimating the lateral resistance factor for the transversely isotropic rock.

7. Develop a relationship between the limit pressure of pressuremeter $P_l$ for the transversely isotropic rock and the rock identification number.

8. Develop a new methodology to obtain a p-y criterion using in-situ pressuremeter technique that can be used to analyze the response of drilled shaft socketed into transverse isotropic rock.

1.3 WORK PLAN

The work involved in the development of p-y criteria for IGM and rock mass consists of two main parts: one is the theoretical work to develop p-y criteria for cohesive IGM and for the transversely isotropic rock mass; the other one is to evaluate the developed p-y criteria based on one full-scale field lateral load test results. Design examples of the developed p-y criterion are presented to demonstrate the application of the p-y criterion for the laterally loaded drilled shafts in rock. Specifically, the tasks related to p-y criteria development are outlined below and depicted in Figure 1-1.

Two parameters are needed to construct a hyperbolic p-y curve; the initial slope, $K_i$ and the ultimate rock resistance, $P_u$. The 3-D FE parametric study results of a laterally loaded drilled shaft socketed in a transversely isotropic continuum are employed to develop a series of design charts for estimating $K_i$ as a function of the transversely isotropic elastic constants. The same methodology is used to develop an empirical solution for $K_i$ for the cohesive IGM.
A simple practical method to characterize the stiffness of jointed rock masses with parallel discontinuities is proposed by developing a homogeneous transversely isotropic model to replace the jointed rock. The equivalent elastic parameters for this model as a function of the properties of intact rock and joint are derived based on 3D FE parametric study on a rock block, where the effects of the joint spacing, joint thickness, and the Poisson’s effect of the joints filling are considered in the derivations.

The determination of ultimate rock resistance involves performing a parametric study on 3-D FE models of a laterally drilled shaft socketed into jointed rock to identify the possible failure modes of rocks. The results of this study in conjunction with the companion theoretical derivations and the relevant rock strength criteria are utilized to derive the semi-analytical equations to estimate the ultimate rock resistance per unit shaft length, $p_u$, for the transversely isotropic rock.

The ultimate side shear resistance between rock and shaft is an important parameter in determining the ultimate lateral rock resistance. Therefore, a comprehensive FE simulation study is undertaken to study the factors affecting the side resistance mobilization mechanisms and to develop a theoretical solution for predicting the ultimate shaft side resistance.

An extensive literature review to identify the most appropriate field and laboratory test methods for determining the transversely isotropic rock properties and to enhance and simplify characterization procedures for the transversely isotropic rock mass is conducted from which useful empirical correlations are developed for the interrelationship between the transversely isotropic elastic constants.
Finally, based on the ultimate rock reaction and initial slope of p-y curve, a hyperbolic p-y criterion for cohesive IGM and transversely isotropic rock mass are proposed. Two field lateral load test results are used to facilitate the development and validation of the p-y criterion for cohesive IGM.

Regarding the work plan for the development of an interpretation method of pressuremeter data for deriving the p-y curves of rock mass, it consists of two parts: one is the theoretical work to derive site specific p-y curves using the pressuremeter test data of the pressure-volume expansion curve; the other one is an evaluation of the developed interpretation method using a FE model simulation results. Additionally, an evaluation of the possible relations among the five elastic constants of the transversely isotropic rock is statistically examined based on a full and well documented data base collected exclusively from the literature. The development works done are outlined below and depicted in Figure 1-2.

A literature review to identify the most appropriate field and laboratory test methods for determining the transversely isotropic rock, as being the rock type of interest, properties and to enhance and simplify characterization procedures for the transversely isotropic rock mass is identified. Also, a literature review on the design and analysis methods of laterally loaded drilled shafts and piles in rock is performed. To obtain p-y curves of drilled shafts socketed in transversely isotropic rock from pressuremeter test, it is necessary to identify the lateral resistance factor ($\eta$) of such a rock which requires the determination of the elastic constants associated with the transverse isotropy. A new approach for determining the five elastic constants of a transversely isotropic rock
medium using the in-situ indirect pressuremeter test device were developed. The
determination of the five elastic constants of a transversely isotropic rock involves
performing a parametric study on 3-D FE simulations of pressuremeter tests in a
transversely isotropic elastic media. The results of this study are utilized to develop the
interrelationships between the five elastic constants of the transversely elastic rock and
the tangent, $K_{in}$, to the linear portion of the $(\Delta p, \Delta V/V_s)$ curve obtained from the
pressuremeter tests.

The transverse isotropic rock shear modulus $G'$ is very important and laborious to be
determined by the conventional experimental and in-situ tests. Therefore, two different
empirical equations for estimating the shear modulus $G'$ in terms of the other elastic
constants were presented in this study based on a comprehensive well documented
database for experimental measurement of the five elastic constrains of different
transversely isotropic rocks. Relating $G'$ to other elastic constants of transversely
isotropic rocks can be a useful and practical approach to the determination of the
important elastic constants for engineering computations.

Finally, based on the 3-D FE parametric study, a new approach to derive $p$-$y$ curve
form in-situ pressuremeter test was developed, and a hyperbolic $p$-$y$ criterion for the
transversely isotropic rock is proposed. Also, a lateral resistance multiplier factor is
proposed for transversely isotropic rock.
Figure 1-1 Flow chart of the work for developing p-y criteria
Figure 1-2 Flow chart of the work for pressuremeter interpretation
1.4 OUTLINE OF THE REPORT

Chapter II provides a review of the work previously done on the subject relevant to p-y method of analysis for laterally loaded drilled shafts in both cohesive IGM and rock mass. Both the practical and theoretical implications of this review have provided the necessary impetus to the current research work in the development of pertinent p-y criteria for cohesive IGM and for the rock mass exhibiting transversely isotropic behavior. Chapter II also provides a review of the research work done on rock anisotropy and the behavior of transversely isotropic rocks and presents some of the available methods to determine the elastic constants of the transversely isotropic rock medium. A review of the development and applications of pressuremeter test as a field method for analysis of the laterally loaded drilled shafts is also presented in this chapter.

Chapter III presents the 3D FE modeling techniques and results using the ANSYS computer program for elastic behavior of drilled shaft under lateral loads. Based on the FE simulation results and comprehensive statistical regression analyses, a methodology for estimating the initial modulus of subgrade reaction of a laterally loaded drilled shaft socketed into transversely isotropic rock is developed.

Chapter IV presents the 3D FE modeling results for developing an equivalent transversely isotropic homogeneous model to describe the stress-strain behavior of a rock mass with parallel joints.

Chapter V presents a method for estimating the ultimate side shear resistance based on the results of a series of FE simulations. The FE simulations have included a systematic
parametric study of the effects of various influencing factors on the ultimate side shear resistance between the rock and drilled shaft, including the interface strength parameters, the modulus of the drilled shaft and rock mass, and the drilled shaft geometry.

Chapter VI presents the equations for estimating the ultimate lateral resistance of the laterally loaded drilled shaft socketed in rock. The formulation of equations predicting the ultimate lateral resistance in rock was based on the theoretical mechanics using the limit equilibrium analysis method and the results of a series of 3-D FE model simulations of a drilled shaft socketed into jointed rock. The formulated equations are applicable to both shallow and great depth conditions.

Chapter VII presents a hyperbolic p-y criterion for transversely isotropic rock. The evaluation of this criterion is also presented in this chapter.

Chapter VIII presents a unified p-y criterion for cohesive soils and cohesive IGM using the hyperbolic mathematical formulation. Validation of the proposed p-y criterion is also presented in this chapter by comparing the predictions with the full scale lateral load test results.

Chapter IX presents the 3D FE numerical simulation techniques to develop the interrelationship between the five elastic constants of the transversely isotropic rock and the tangent, \( K_i \), to the linear portion of the pressuremeter test data, (i.e., \( (\Delta p, \Delta V/V_i) \) curve). A practical procedure is presented for the determination of the elastic constants from \( K_i \). Moreover, a series of regression charts for a quick estimate of the Young’s modulus \( E \) and \( E' \) for different Poisson’s ratio \( \nu \) and inclination angle \( \theta \)
between the axis of the borehole and the plane of the symmetry of the transversely isotropic elastic medium are presented in this chapter.

Chapter X presents a comprehensive database of experimentally determined five independent elastic constants for the transversely isotropic rocks from a literature review. A statistical correlation study leads to the development of a new empirical equation for estimating $G'$ using other elastic constants.

Chapter XI presents the development of a new methodology for deriving hyperbolic $p$-$y$ curves from pressuremeter test based on a 3D FE study for a transversely isotropic rock media.

Chapter XII presents verification of the $p$-$y$ criterion for weak rock proposed in SJN 134137 recent lateral load test data.

Chapter XIII presents a summary of the work done, conclusions, and recommendations for implementation and future research.
CHAPTER II: LITERATURE REVIEW

The problem of the laterally loaded drilled shaft was originally of particular interest in the offshore platforms. Lateral loads from wind and waves are usually the most critical factor in the design of such structures. Solutions of this design problem also apply to a variety of onshore cases including drilled shaft supported earthquake resistance structures, power poles, and drilled shaft-supported structures which may be subjected to lateral or wind forces.

To date, there are few published analysis methods for the lateral response of rock-socketed drilled shafts. It has been a customary practice to adopt the p-y analysis with p-y criterion developed for soils to solve the problem of rock-socketed drilled shafts (Gabr, 1993). Currently, two categories of analysis methods for laterally loaded rock-socketed drilled shafts have been developed. One category treats rock as a continuum mass (Carter and Kulhawy 1992; and Zhang et al 2000), the other one discretizes the rock mass into a set of non-linear springs (Reese 1997; Gabr et al. 2002; and Ohio Department of Transportation (ODOT) State Job Number (SJN) 134137 by Nusairat et al. 2006).

2.1 ANALYSES METHODS OF LATERALLY LOADED ROCK-SOCKETED DRILLED SHAFTS

Several analytical methods have been proposed that attempt to model laterally loaded rock socketed drilled shaft response; however, none of which can completely account for all factors that influence the lateral soil and drilled shaft interactions. These approaches fall into two general categories: (1) continuum methods (Carter and Kulhawy 1992; and
Zhang et al. 2000) and (2) subgrade reaction approach (Reese 1997; Gabr et al. 2002; and SJN 134137 by Nusairat et al. 2006). The latter approach is the one adopted in this study, thus, a detailed literature review is performed on this method.

2.1.1 Elastic continuum methods

The continuum approach treats the soil or rock as a semi-infinite elastic continuum and the deep foundation as an elastic inclusion. This approach has been extended to rock socketed shafts and to incorporate elasto-plastic response of the soil or rock mass.

The elastic continuum approach for laterally loaded deep foundations was originally developed by Poulos (1971), initially for analysis of a single pile under lateral and moment loading at the pile head. The numerical solution is based on the boundary element method, with the pile modeled as a thin elastic strip and the soil modeled as a homogeneous, isotropic elastic material.

The elastic continuum approach was further developed by Randolph (1981) through the use of the FE method (FEM). Solutions presented by Randolph cover a wide range of conditions for flexible drilled shafts and the results are presented in the form of charts as well as convenient closed-form solutions for a limited range of parameters. The solutions do not adequately cover the full range of parameters applicable to rock socketed shafts used in practice. Extension of this approach by Carter and Kulhawy (1992) to rigid shafts and shafts of intermediate flexibility has led to practical analytical tools based on the continuum approach. Sun (1994) applied the elastic continuum theory to deep foundations using variational calculus to obtain the governing differential equations of
the soil and drilled shaft system, based on the Vlasov model for a beam on elastic foundation. This approach was extended to rock-socketed shafts by Zhang et al. (2000).

2.1.2 Winkler method (Subgrade reaction approach) and p-y method

The Winkler method, or sometimes known as the subgrade reaction method, currently appears to be the most widely used method in the design of laterally loaded drilled shafts. It is based on modeling the shaft as a beam on elastic foundation.

One of the great advantages of this method over the elastic continuum method is that the idea is easy to program in the finite difference or FE methods and that the soil nonlinearity and multiple soil layers can be easily taken into account. The concept can be easily implemented in dynamic analysis as well. In addition, the computational cost is significantly less than the FE method. However, the obvious disadvantage of this method is the lack of consideration of continuity of soils due to its discretization scheme.

The term of subgrade reaction indicates the pressure, \( p \), per unit area of the surface of the contact between a loaded beam or slab and the subgrade on which it rests and on to which it transfers the loads. The coefficient of subgrade reaction, \( k \), is the ratio between the soil pressure, \( p \), at any given point of the surface of contact and the displacement, \( y \), produced by the load application at that point:

\[
k = \frac{p}{y}
\]  

(2-1)

To implement this concept for a laterally loaded drilled shaft, Equation (2-1) has been modified frequently (e.g. Reese and Matlock, 1956) as:
\[ K = \frac{p}{y} \quad (2-2) \]

where \( K \) is the modulus of subgrade reaction (F/L^2) and \( p \) is the soil reaction per unit length of the drilled shaft (F/L).

With the subgrade reaction concept, the lateral drilled shaft response can be obtained by solving the forth order differential equation as:

\[ E_p I_p \frac{d^4 y}{dz^4} + Ky = 0 \quad (2-3) \]

where \( E_p \) is the modulus of elasticity of the drilled shaft, \( I_p \) is the moment of inertia of the drilled shaft, and \( z \) is depth. Solutions of Equation (2-3) can be obtained either analytically or numerically.

The aforementioned solution based on subgrade reaction theory is valid only for a case of linear soil properties. In reality, the relationship between soil pressure per unit drilled shaft length \( p \) and deflection \( y \) is nonlinear. Taking the nonlinearity of soil into account, the linear soil springs are replaced with a series of nonlinear soil springs, which represent the soil resistance-deflection curve or the so-called, “p-y” curve. The p-y curves of the soil have been developed based on the back analysis of the full scale lateral drilled shaft load test.

Several researchers have proposed methods to construct p-y curves for various soil types based upon back-computation from full-scale test results. The p-y method was extended to the analysis of a single rock-socketed drilled shaft under lateral loading by
Reese (1997). Gabr et al. (2002) proposed a p-y criterion for weak rock based on their field test data. During the work on ODOT research project SJN 134137, Nusairat et al. (2006) proposed a hyperbolic p-y criterion for rock. The following sections present a brief description of each p-y curves for rock currently available in the industry.

2.1.2.1 Reese (1997)

Reese (1997), based on two load tests, proposed the most widely used method to construct p-y curves for “weak” rock. The ultimate resistance $P_u$ for weak rock was calculated as follows based on limit equilibrium as a function of depth below ground surface:

$$p_u = \alpha_r \sigma_{ci} D \left(1 + 1.4 \frac{z_r}{D}\right) \quad \text{for } 0 \leq z_r \leq 3D \quad (2-4)$$

$$p_u = 5.2 \alpha_r \sigma_{ci} D \quad \text{for } z_r \geq 3D \quad (2-5)$$

where $\sigma_{ci} =$ uniaxial compressive strength of intact rock; $\alpha_r =$ strength reduction factor, which is used to account for fracturing of rock mass; $D =$ diameter of the drilled shaft; and $z_r =$ depth below rock surface. The value of $\alpha_r$ is assumed to be $1/3$ for RQD of zero and it increases linearly to unity at a RQD of 100%.

The slope of initial portion of p-y curves was given by

$$K_{ir} = k_{ir} E_m \quad (2-6)$$

where $K_{ir} =$ initial tangent to p-y curve; $E_m =$ deformation modulus of rock masses, which may be obtained from a pressuremeter or dilatometer test; and $k_{ir} =$ dimensionless constant.
Equation (2-7) through (2-10) describe the interim p-y criterion for the first, second, and third segment, respectively.

\[ p = K_{ir} y; \quad y \leq y_A \quad (2-7) \]

\[ p = \frac{p_u}{2} \left( \frac{y}{y_{rm}} \right)^{0.25}; \quad y \geq y_A \quad \text{and} \quad p \leq p_u \quad (2-8) \]

\[ p = p_u; \quad p \geq p_u \quad (2-9) \]

\[ y_A = \left[ \frac{p_u}{2(y_{rm})^{0.25} K_{ir}} \right]^{1.333} \quad (2-10) \]

where, \( y_{rm} = k_{rm} D \), \( k_{rm} = \) strain at 50% of ultimate load, ranging from 0.0005 to 0.00005.

2.1.2.2 Gabr et al. (2002)

Gabr et al. (2002) proposed a hyperbolic p-y criterion for weak rock based on field tests on small diameter drilled shafts socketed in weak rock. The following procedure can be used to construct a p-y curve according to Gabr et al. (2002).

Step 1: Calculation of Coefficient of Subgrade Reaction

The coefficient of subgrade reaction can be calculated using Equation (2-56) proposed by (Vesic, 1961).

Step 2: Calculation of Flexibility Factor

A flexibility factor, \( K_R \), is computed as follows (Poulos and Davis, 1972):
where, \( L \) is the embedment length of shaft.

Step 3: Calculation of Point of Rotation

The following equation is used to define the turning point as a function of the embedded shaft length:

\[
T_0 = (1 + 0.18 \log K_R) L
\]  

(2-12)

Step 4: Calculation of \( I_T \) Number

\[
I_T = -28 - 383 \log \left( \frac{T_0}{L} \right) \quad I_T \geq 1
\]  

(2-13)

Step 5: Calculation of the Subgrade Reaction

\[
k_h = n_h D \quad (0 \leq z \leq T_0)
\]  

(2-14)

\[
k_h = I_T n_h D \quad (T_0 < z \leq L)
\]  

(2-15)

Step 6: Calculation of Ultimate Resistance of Rock Mass \( P_u \)

Equation (2-19) proposed by Zhang et al. (2000) was employed to calculate the ultimate resistance of rock.

Step 7: Construction of the \( P-y \) Curve
2.1.2.3 ODOT SJN 134137 – Nusairat et al. (2006) Approach

Work on ODOT research project under SJN 134137 culminated the development of a new p-y curve criterion for weak rock based on the results of FE analysis and the fitting back to the results of two full-scale load tests in Dayton and Pomeroy, OH documented in the final report for SJN 134137 titled “Design of Rock Socketed Drilled Shafts”. Hyperbolic mathematical formulation to construct the p-y curve was adopted. New procedures to determine the initial tangent slope of p-y curves, $K_i$, and the ultimate resistance of the rock mass were proposed.

Equation (2-41) was proposed in SJN 134137 for the initial tangent slope of p-y curves. It is semi-empirical and is derived based on the results of 3D FE study.

Recommendations for modulus values $E_m$ from PMTs for use in Equation (2-41) were documented in the SJN 134137 report, but in the absence of PMT measurements the following correlation equation was suggested to relate $E_m$ to modulus of intact rock and GSI:

$$E_m = \frac{E_i}{100} (e^{\frac{GSI}{21.7}}) \quad (2-16)$$

where $E_i$ = elastic modulus of intact rock obtained during uniaxial compression testing of core samples.
2.2 ANALYSIS METHODS FOR ESTIMATING ULTIMATE LATERAL ROCK REACTION

In addition to the above mentioned analysis methods for solving load-deflection relationship at the drilled shaft head, methods for estimating ultimate rock reaction have also been proposed. Carter and Kulhawy (1992) presented a method to determine the rock capacity by using cohesion and friction angle of rock. Carter and Kulhawy (1992) method, treats rock mass as a homogeneous and elasto-plastic material, without considering secondary structures of rock mass, such as cracks and fractures. Reese (1997) considered the secondary structure of rock mass by using a rock strength reduction factor which can be determined from Rock Quality Designation (RQD). Zhang et al. (2000) proposed a method to estimate the ultimate reaction of rock masses per unit shaft length using Hoek-Brown rock strength criterion (Hoek and Brown 1988), in which the effects of RQD and other secondary rock structures were included. However, simple rock resistance distribution along the shaft circumference under lateral loads was assumed (Carter and Kulhawy 1992). It seems that Zhang et al. (2000) method for estimation of lateral capacity of rock-socketed drilled shaft considered most of the characteristics of rock mass; however, the authors did not investigate possible failure modes of rock mass, especially possible sliding failures along pre-existing joints. Regarding the sliding failure on joints, To et al. (2003) proposed a method to estimate the lateral load capacity of drilled shafts in jointed rock. The block theory (Goodman and Shi 1985) was used to identify the failure block, and the static limit equilibrium was used to obtain the ultimate capacity. The Coulomb failure criterion was utilized to model the sliding failure on joints. Yang (2006) identified two modes of failure based on stress and deformation fields around the
shaft: (i) planner wedge failure mode for rock mass at or near ground surface, and (ii) strength controlled failure mode for rock at great depth.

2.2.1 Carter and Kulhawy (1992)

Carter and Kulhawy (1992) proposed a solution in which they suggested that the lateral resistance was derived from ultimate side shear \( \tau_{\text{ult}} \) between shaft and rock, and frontal compressive strength of rock \( (P_L) \). They further suggested that the magnitude of the horizontal shear resistance at the rock-shaft interface was equal to that in of the vertical shear resistance. The assumed distribution of ultimate resistance along the shaft is shown in Figure 2-1, from which one can see that lateral resistance is equal to \( \tau_{\text{ult}}D \) at the surface of the rock and is increasing linearly with depth to a magnitude of \( (P_L + \tau_{\text{ult}})D \) at a depth of 3D. Below this depth, the ultimate resistance remains constant with depth. \( P_L \) reflects the limit stress developed in rock, which can be calculated according to the expansion theory of a long cylindrical cavity in an elasto-plastic, cohesive-frictional, dilatant material (Carter et al. 1986).

\[
H_u = \left( \frac{P_L}{6} + \tau_{\text{ult}}D \right)L \quad \text{for } L<3D \tag{2-17}
\]

\[
H_u = \left( \frac{P_L}{2} + \tau_{\text{ult}} \right)3D^2 + (P_L + \tau_{\text{ult}})(L-3D)D \quad \text{for } L>3D \tag{2-18}
\]
2.2.2 Zhang et al. (2000)

To compute the ultimate resistance \( p_u \) of rock mass to laterally loaded drilled shafts, Zhang et al. (2000) adopted the assumed resistance distribution by Carter and Kulhawy (1992), shown in Figure 2-2, and Hoek-Brown rock strength criterion (Hoek and Brown 1988). The total resistance of rock mass is assumed to consist of two parts: the side resistance and the front normal resistance. They suggest that the ultimate resistance \( p_u \) can be estimated using Equation (2-19).

\[
p_u = (p_L + \tau_{ult})D \tag{2-19}
\]

where \( \tau_{ult} \) was assumed to be the same as the ultimate side resistance under axial loading and can be given by:

\[
\tau_{ult} = 0.20(\sigma_{ci})^{0.5} \quad \text{(MPa)} \quad \text{for smooth socket} \tag{2-20}
\]

\[
\tau_{ult} = 0.80(\sigma_{ci})^{0.5} \quad \text{(MPa)} \quad \text{for rough socket} \tag{2-21}
\]
The strength criterion for rock mass developed by Hoek and Brown (1980, 1988) was adopted to determine the normal limit stress $p_L$. The Hoek-Brown criterion, which is suitable for intact rock and rock mass, can be given by

$$\sigma'_i = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a$$  \hspace{1cm} (2-22)

where $\sigma_{ci} = \text{the uniaxial unconfined compressive strength of the intact rock}; \ m_b, \ s$ and $a = \text{material constants that depend on the characteristics of rock mass and can be estimated as follows (Hoek, et al., 2002).}$

$$m_b = \exp \left( \frac{\text{GSI} - 100}{28 - 14D_r} \right) m_i$$  \hspace{1cm} (2-23)

$$s = \exp \left( \frac{\text{GSI} - 100}{9 - 3D_r} \right)$$  \hspace{1cm} (2-24)

$$a = \frac{1}{2} + \frac{1}{6} (e^{-\text{GSI}/15} - e^{-20/3})$$  \hspace{1cm} (2-25)
where $D_r$ is a factor depending upon the degree of disturbance to which the rock mass has been subjected due to blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses. For deep foundation analysis, although the excavation releases the horizontal earth pressure on rock masses, the pouring of concrete restores the pressure. Therefore, the value of $D_r$ is selected as 0 for applications in deep foundation analysis.

The material constant $m_i$ used in Equation 2-23 can be determined from triaxial tests (Hoek and Brown, 1997). When no laboratory test data are available, it can also be estimated from Table 2-1. The values shown in Table 2-1 can be varied ±2.

<table>
<thead>
<tr>
<th>Table 2-1</th>
<th>Values of Constant $m_i$ for Intact Rock (After Marinos and Hoek, 2000)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sedimentary Rocks</strong></td>
<td></td>
</tr>
<tr>
<td>Anhydrite</td>
<td>12</td>
</tr>
<tr>
<td>Conglomerates</td>
<td>21</td>
</tr>
<tr>
<td>Marls</td>
<td>7</td>
</tr>
<tr>
<td>Crystalline Limestones</td>
<td>12</td>
</tr>
<tr>
<td><strong>Metamorphic Rocks</strong></td>
<td></td>
</tr>
<tr>
<td>Amphibolites</td>
<td>26</td>
</tr>
<tr>
<td>Metasandstone</td>
<td>19</td>
</tr>
<tr>
<td>Schists</td>
<td>10</td>
</tr>
<tr>
<td><strong>Igneous Rocks</strong></td>
<td></td>
</tr>
<tr>
<td>Agglomerate</td>
<td>19</td>
</tr>
<tr>
<td>Dacite</td>
<td>25</td>
</tr>
<tr>
<td>Gabbro</td>
<td>27</td>
</tr>
<tr>
<td>Obsidian</td>
<td>19</td>
</tr>
<tr>
<td>Tuff</td>
<td>13</td>
</tr>
</tbody>
</table>

$$\sigma_n' = \sigma_3' + \frac{(\sigma_1' - \sigma_3')^2}{2(\sigma_1' - \sigma_3') + 0.5m_b\sigma_{ci}'} \quad (2-26)$$
\[ \tau = \left( \sigma'_n - \sigma'_3 \right) \sqrt{1 + \frac{m_b \sigma_{ci}}{2(\sigma'_1 - \sigma'_3)}} \]

(2-27)

2.2.3 To et al. (2003)

For the drilled shafts socketed into jointed rock, To et al. (2003) assumed a wedge type block failure and Coulomb failure criterion to obtain the lateral capacity of drilled shafts. Goodman and Shi (1985) block theory was used to determine the possible failure block for a rock mass containing two sets of joints. To et al. assumed sliding failure along the joint plane if it is kinematically admissible. When movement of wedge is prevalent then tensile failure on the rock mass is assumed. Figure 2-3 shows the typical forces on the wedge, where \( W \) = weight of the wedge, \( P \) = axial load on shaft, \( F \) = lateral force, \( T \) = tensile force due to the fracture in a blocks that is initially irremovable, but could be removable if it breaks due to the lateral force exerted by the shaft; \( N_1 \) = normal force on joint, and \( R_1 \) = tangential force on joint. The principle of static limit equilibrium for the forces on the wedge can be used to solve for ultimate lateral force \( F \). The determination of removable wedge required by this method is a very tedious procedure. The drawback of To et al. work is that only two sets of joints can be considered and the joints in each set should be parallel. The failure mode is restricted to failure at the top portion of rock mass with a free surface at the ground-line.
2.2.4 SJN 134137 (Nusairat et al. 2006)

Work on SJN 134137 culminated the development of two equations for evaluating $p_u$ depending on the ultimate failure modes. The first corresponds to a wedge failure mode depicted in Figure 2-4, for the failure near the ground surface. An assumption of a planner wedge failure was made even though the FE analysis presented in this study shows that actual failure would be more resembling a curved surface on the side of the failure block. The second corresponds to a failure mode due to strength failure of rock mass at great depth below ground surface as described in Figure 2-5. Details of the two equations are presented in the following sections.
2.2.4.1 Ultimate Resistance of Rock Near Surface

For highly fractured rock mass and competent rock, wedge type failure model for top rock layer was identified in Figure 2-3. In Figure 2-4, $F_{\text{net}}$ is the total net rock resistance; $D$ is the diameter of the drilled shaft; $H$ is the height of the wedge; $F_a$ is the active earth force exerted on the drilled shaft; $F_s$ is the friction force on the sides of the wedge; $F_n$ is the normal force applied to the sides of the wedge and is assumed to be equal to the at-rest earth force; $F_{\text{sb}}$ is the friction force on the bottom face of the wedge; $F_{\text{nb}}$ is the normal force on the bottom face of the wedge which is to be determined through force equilibrium on vertical direction; $W$ is the weight of the wedge; $\sigma'_{v_0}$ is effective vertical soil pressure due to overburden soil on the top of rock and it is equal to zero if no overlying soils are present. According to force equilibrium in the loading direction, the net rock reaction can be determined as follows:

$$F_{\text{net}} = 2F_a \cos \theta \sin \beta + F_{\text{sb}} \sin \beta + F_{\text{nb}} \cos \beta - 2F_a \sin \theta - F_a$$  \hspace{1cm} (2-28)

$$F_a = \frac{1}{2} K_s \gamma' D (H - z_0)^2$$  \hspace{1cm} (2-29)
\[ F_n = K_0 \sigma'_{v0} A_s + \frac{1}{6} K_0 \gamma' H^3 \tan \beta \sec \theta \] (2-30)

\[ F_s = c' A_s + K_0 \sigma'_{v0} \tan \phi' A_s + \frac{1}{6} H^3 K_0 \gamma' \tan \phi' \tan \beta \sec \theta \] (2-31)

\[ F_{nb} = \frac{\sigma'_{v0} A_s + W + 2F_s \cos \theta \cos \beta + c' A_b \cos \beta}{\sin \beta - \tan \phi' \cos \beta} \] (2-32)

\[ F_{sh} = c' A_b + F_{nb} \tan \phi' \] (2-33)

in which \( \gamma' \) is the effective unit weight of the rock mass; and

\[ K_a = \tan^2 (45 - \phi'/2) \] (2-34)

\[ K_0 = 1 - \sin \phi' \] (2-35)

\[ z_0 = \frac{2c'}{\gamma' \sqrt{K_a}} - \frac{\sigma'_{v0}}{\gamma'} \] (2-36)

\[ A_s = \frac{1}{2} H^2 \tan \beta \sec \theta \] (2-37)

\[ A_b = (D + H \tan \beta \tan \theta)H \sec \beta \] (2-38)

\[ A_t = (D + H \tan \beta \tan \theta)H \tan \beta \] (2-39)

\[ W = \gamma' \left( \frac{1}{3} H^3 \tan^2 \beta \tan \theta + \frac{1}{2} H^2 D \tan \beta \right) \] (2-40)

The value of \( \beta \) was approximated as \( 45 + \phi'/2 \) by Reese et al. (1974) for their wedge type of failure in sand. Hoek (1983) pointed out that the failure plane of rock is \( 45 + \phi'/2 \) because rock mass also follows Mohr-Coulomb failure criterion. Bowman (1958) suggested values of \( \theta \) from \( \phi'/3 \) to \( \phi'/2 \) for loose sand and up to \( \phi' \) for dense sand for a similar wedge type of failure in soils. However, herein the value of \( \theta \) is taken as \( \phi'/2 \) based on the case studies (as discussed in detail in Chapter VI of SJN 134137, Nusairat et
al. 2006) of actual field test results. The values of $\phi'$ and $c'$ can be obtained from Equations (2-41) and (2-42) by taking the value of $\sigma_3'$ to be the effective overburden pressure at the depth of $1/3H$, since the side surface is triangular in shape.

$$\phi' = 90 - \arcsin\left(\frac{2\tau}{\sigma_1' - \sigma_3'}\right) \quad (2-41)$$

$$c' = \tau - \sigma_n' \tan \phi' \quad (2-42)$$

where $\sigma_1'$ is in-situ vertical effective stress; $\sigma_1'$ can be obtained using Equation (2-22) proposed by Hoek et al (2002).

In this study, the following equations are adopted for $\theta$ and $\beta$.

$$\theta = \frac{\phi'}{2} \quad (2-43)$$

$$\beta = 45 + \frac{\phi'}{2} \quad (2-44)$$

The ultimate resistance of rock mass per unit shaft length $p_u$ (F/L) based on the wedge failure mode identified herein can be calculated as:

$$p_u = \frac{dF_{net}}{dH} = 2\cos\theta \sin\beta\frac{dF}{dH} + \sin\beta\frac{dF_{ab}}{dH} + \cos\beta\frac{dF_{ab}}{dH} - 2\sin\theta\frac{dF_n}{dH} - \frac{dF_a}{dH} \quad (2-45)$$

where

$$\frac{dF_a}{dH} = \gamma'K_a(H - z_0)D \quad (dF_a/dH \geq 0) \quad (2-46)$$

$$\frac{dF_n}{dH} = K_0H\tan\beta\sec\theta(\sigma_{v_0}' + \frac{1}{2}\gamma'H) \quad (2-47)$$

$$\frac{dF_s}{dH} = H\tan\beta\sec\theta(c' + K_0\sigma_{v_0}'\tan\phi' + \frac{H}{2}K_0\gamma'\tan\phi') \quad (2-48)$$

$$\frac{dF_{ab}}{dH} = \frac{D\tan\beta(\sigma_{v_0}' + H\gamma') + H\tan^2\beta\tan\theta(2\sigma_{v_0}' + H\gamma') + c'(D + 2H\tan\beta\tan\theta) + 2\cos\beta\cos\theta\frac{dF_s}{dH}}{\sin\beta - \tan\phi'\cos\beta} \quad (2-49)$$
\[
\frac{dF_{sh}}{dH} = \tan \phi' \frac{dF_{sh}}{dH} + c'(D \sec \beta + 2H \tan \beta \sec \beta \tan \theta)
\] (2-50)

2.2.4.2 Ultimate Rock Resistance at Great Depth

For in-depth jointed rock with a set of parallel weak planes, heavily fractured rock mass and competent rock, the failure model shown in Figure 2-5 is adopted. It is assumed that the ultimate resistance of rock is reached when both the maximum shear resistance between the drilled shaft and rock mass, \(\tau_{\text{max}}\), and the normal limit pressures of rock mass, \(p_L\) are reached. Therefore, the ultimate rock resistance per unit length, \(p_u\), for in-depth rock can be computed as follows.

\[
p_u = 2 \int_0^{\pi/2} p_1 D / 2 \sin^2 \beta d\beta + 2 \int_0^{\pi/2} \tau_{\text{max}} D / 2 \sin(2\alpha) \cos \alpha d\alpha - p_a D
\] (2-51)

\[
p_a = \frac{\pi}{4} D p_L + \frac{2}{3} D \tau_{\text{max}} - p_a D
\] (2-52)

where \(D\) = the diameter of a drilled shaft; \(p_a\) = the active horizontal earth pressure; Rankine’s earth pressure theory can be used to obtain \(p_a\) as follows.

\[
p_a = K_a \sigma'_v - 2c' \sqrt{K_a}
\] (2-53)

\[
K_a = \tan^2 (45 - \Phi'/2)
\] (2-54)

\[
\tau_{\text{max}} = 5.42 \sigma_{ci}^{0.5} \quad \text{where the units are in psi}
\] (2-55)

where \(\sigma'_v\) = effective overburden earth pressure including the pressure induced by possible overlying soils. The normal limit pressure of rock mass, \(p_L\), is the major principal stress at failure, \(\sigma'_1\), which can be calculated using Equation (2-22) in which \(\sigma'_3\) is equal to \(\sigma'_v\). The ultimate shaft-rock interface friction in horizontal direction, \(\tau_{\text{max}}\), can be estimated from Equation (2-55). The effective cohesion \(c'\) needed in Equation (2-53) can be calculated using Equation (2-42) by taking \(\sigma'_3\) equal to \(\sigma'_v\).
Because the ultimate capacity calculated based on the presented in-depth failure model could be smaller than the one calculated based on wedge failure model; therefore, for the top layer of rock mass with or without overlying soils, the ultimate resistance of rock per unit shaft length is determined as the smaller of the values calculated from Equations (2-45) and (2-52).

![Image](image.png)

Figure 2-5 Suggested stress distribution at failure at great depth

2.3 INITIAL MODULUS OF SUBGRADE REACTION

The term initial modulus of subgrade reaction refers to the initial slope of a p-y curve. In literature, many studies have been conducted regarding the determination of modulus of subgrade reaction for rock masses.

Vesic (1961) provided an elastic solution for the modulus of subgrade reaction, as follows.
Bowles (1988) suggested doubling the value of $K$ in Equation (2-58). Based on field test data, Carter (1984) modified Vesic’s equation to account for the effect of drilled shaft diameter as follows.

$$
K = \frac{0.65E}{\nu - (2-56)} \left[ \frac{ED^4}{E_pI_p} \right]^{1/12}
$$

(2-56)

where $E_pI_p = \text{flexural rigidity of drilled shafts}$.  

Reese (1997) proposed an interim p-y curve criterion for weak rock based on field data. In this criterion, the initial slope of p-y curves $K_i$, is given by Equation (2-6).

$$
K = \frac{3\pi G_s}{2} \left\{ 2\eta \left[ \frac{K_i(\eta)}{K_0(\eta)} \right] - \eta^2 \left[ \left( \frac{K_i(\eta)}{K_0(\eta)} \right)^2 - 1 \right] \right\}
$$

(2-58)

where $G_s = \text{shear modulus of soils}; \eta = \text{a load transfer factor} (\text{Guo, 2001});$ and $K_0$ and $K_1 = \text{Bessel functions}$.  

Reese (1997) suggested Equation (2-59) and (2-60) for $k_{ir}$, which were empirically derived from experiments and reflected the assumption that the presence of the rock surface will have a similar effect in $k_{ir}$, as was shown for $p_{ur}$.  

$II-21$
\[ k_{u} = (100 + \frac{400z_{r}}{3D}) \quad \text{for} \quad 0 \leq z_{r} \leq 3D \quad (2-59) \]
\[ k_{ir} = 500 \quad \text{for} \quad z_{r} \geq 3D \quad (2-60) \]

Using 3D FE Analysis of laterally loaded drilled shaft socketed in elastic isotropic continuum using ABAQUS. An empirical correlation equation for estimating the initial modulus of subgrade reaction of rock was developed under SJN 134137 (Nusairat et al. 2006) as follows.

\[ K_{i} = E_{m}(D/D_{ref})e^{-2\nu}\left(\frac{E_{p}I_{p}}{E_{m}D^{3}}\right)^{0.284} \quad (2-61) \]

### 2.4 ULTIMATE SIDE SHEAR RESISTANCE

Predicting the mobilization of side shear resistance has been the subject of investigation for many years and indeed numerous interface shear models have been proposed (Williams and Pells, 1981; Briaud and Smith 1983; Smith et al. 1987; Thorne, 1977; Hooley and Lefroy, 1993; Rowe and Armitage, 1987). However, most of models evoke simplifications and rely mostly on semi-empirical correlations; therefore, the accuracy of these models may be inadequate, particularly when the accuracy of predicting deflection of the drilled shaft under lateral load is important for a given project. Also, a more thorough understanding of the various factors influencing the development of horizontal side resistance may be needed in order to develop a more comprehensive prediction model for side shear resistance.

Empirical correlations, proposed by many researchers, between the shaft resistance and the unconfined compressive strength of rock are most widely used for estimating the
ultimate side shear resistance of a drilled shaft socketed in rock. It may be stated that in general these empirical correlations are expressed as.

$$\tau_{\text{ult}} (\text{MPa}) = \alpha \sigma_c^\beta$$

(2-62)

where,

- $\sigma_c$ = unconfined compression strength for intact rock (MPa)
- $\alpha$ and $\beta$ are factors determined empirically from load tests

The empirical factors proposed by a number of researchers are summarized and shown in Table 2-2. Most of these empirical relationships were developed for specific and limited data sets, which may have correlated well with the proposed equations. However, O’Neill et al. (1995) compared these empirical shaft resistance design methods with an international database of 137 pile load tests in intermediate-strength rock. O’Neill et al. concluded that none of the methods could be considered a satisfactory predictor for the database.

Another significant database study on the shaft resistance of drilled shafts socketed into rock has been conducted by Seol and Jeong (2007). This study included drilled shafts sockets drilled at many sites and in a wide range of rock types. It was found that correlation between $\tau_{\text{ult}}$ and $\sigma_c$ varies widely but that $\tau_{\text{ult}}$ correlates well with $E_m$ and RMR. Seol and Jeong proposed that the average and minimum $\tau_{\text{ult}}$ of rock-socketed drilled shafts in the Korean Peninsula can be estimated as follows:

$$\tau_{\text{ult}} = 0.135p_a \left( \frac{E_m}{p_a} \right)^{0.5}$$

(2-63)
\[ \tau_{ulh} = 0.36p_a \cdot \text{RMR} \geq 0.27p_a \cdot \text{RMR} \]  

(2-64)

where \( p_a \) is the atmospheric pressure in the unit used for \( E_m \).

Table 2-2 Summary of the design methods of ultimate resistance predictions

<table>
<thead>
<tr>
<th>Design Method</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williams and Pells (1981)</td>
<td>0.3</td>
<td>0.5</td>
<td>Mathematical Modeling</td>
</tr>
<tr>
<td>Hovarth et al. (1983)</td>
<td>0.2</td>
<td>0.5</td>
<td>Developed from 710 mm diameter drilled shaft into mudstone, smooth shaft interface</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.5</td>
<td>Developed from 710 mm diameter drilled shaft into mudstone, Rough shaft interface</td>
</tr>
<tr>
<td>Rowe and Armitage (1987)</td>
<td>0.45</td>
<td>0.5</td>
<td>Drilled shaft into weak rock, clean socket</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Hooley and Lefroy (1993)</td>
<td>0.15</td>
<td>0.5</td>
<td>disturbed rock and rocks of the slate, siltstone and argillite types</td>
</tr>
<tr>
<td>Zhang (1997)</td>
<td>0.2</td>
<td>0.5</td>
<td>Smooth Socket</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.5</td>
<td>Rough Socket</td>
</tr>
<tr>
<td>Rosenberg and Journeaux (1976)</td>
<td>0.34</td>
<td>0.52</td>
<td>Based on small diameter drilled shaft in shale</td>
</tr>
<tr>
<td>Hooley and Lefroy (1993)</td>
<td>0.3</td>
<td>1</td>
<td>( \sigma_c &lt; 0.25 ) MPa highly weathered rock</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.5</td>
<td>( 0.25 ) MPa &lt; ( \sigma_c ) &lt; 0.3 MPa</td>
</tr>
</tbody>
</table>

* Grooves of depth 0-10 mm, ** Grooves or undulations of depth greater than 10 mm, see Pells et al. (1980)

It is evident from the studies of O’Neill et al. (1995) and Seol and Jeong (2007) that for any given rock strength, very large variations in shaft resistance are possible. Design based entirely on empirical correlations with rock strength should therefore be very conservative. This variation suggests that there are other factors, which may significantly influence the shaft resistance.

Johnston and Lam (1989) made detailed investigations of the pile-rock interface with the goal of better understanding the shear load transfer based on constant normal stiffness (CNS) shear test. They observed that for a pile-rock interface, shearing results in dilation.
as one asperity overrides another. If the surrounding rock mass is unable to deform sufficiently, an inevitable increase in the normal stress, $\Delta \sigma_n$, occurs during shearing. This so-called normal stiffness $K_n$ can be determined conventionally by theoretical analysis of an expanding infinite cylindrical cavity in an elastic half-space (Boresi, 1965) as follows:

$$K_n = \frac{\Delta \sigma}{\Delta r} = \frac{E_m}{r(1 + \nu_m)}$$  \hspace{1cm} (2-65)

where $\Delta \sigma$ is the increased normal stress, $\Delta r$ is the dilation, $r$ is the radius of a pile, and $E_m$ and $\nu_m$ are the deformation modulus and the Poisson’s ratio of the rock mass, respectively.

Clearly, greater socket roughness will result in larger dilation for any given pile settlement once sliding at the pile rock interface has commenced. The – CNS boundary condition produces an increase in stress normal to the interface and a corresponding increase in the frictional resistance between pile and rock. In this method, many parameters of rock mass were required which makes the application of Johnston and Lam (1989a) method difficult.

2.5 DISCUSSION OF ANALYTICAL MODELS FOR LATERALLY LOADED ROCK SOCKETS

Each of the analytical methods described above has advantages and disadvantages for use in the design of rock-socketed shafts supporting structures.

Although the elastic continuum methods can predict lateral displacements under the working load conditions very well and can take into account the media continuity, they
cannot account for multiple rock mass layers, and cannot account directly for nonlinear M–EI behavior of a reinforced-concrete shaft. Moreover, the method of Zhang et al. (2000) requires numerical (computer) solution, not currently available commercially, and requires a larger number of rock mass material parameters. However, since the method of Carter and Kulhawy (1992) requires a single material parameter (rock mass modulus), it can be best used for preliminary design; for example, establishing the initial trial depth and diameter of rock-socketed shafts under the lateral and moment loading.

Although there is a lack of a strong theoretical basis for p-y curves, subgrade reaction methods, as implemented through the p-y curve method of analysis, offer some practical advantages for design. These include: (i) it can predict the full, nonlinear lateral load-deformation response, (ii) can incorporate multiple layers of soil and/or rock, (iii) computer programs using p-y accounts for nonlinear (M–EI) behavior of the reinforced concrete shaft section, and can provide structural analysis (shear, moment, rotation, and displacement) of the drilled shaft.

Reese (1997) interim rock p-y criterion was not well calibrated due to inadequate test data. The failure modes of rock mass were not well defined, and an estimation of ultimate lateral resistance did not include the effect of friction between rock mass and shaft. Determination of such parameters as constant $k_{rm}$ appears to be empirical. Cho et al. (2001) conducted a lateral load test on rock-socketed drilled shafts. Two drilled shafts, 30 inch in diameter and 10 feet to 13 feet of socket length were laterally loaded. Reese (1997) interim rock p-y criterion for weak rock was used to construct p-y curves. It was found
that the use of interim p-y curves underestimated the deflections of shafts when comparing with the measured values.

Gabr et al.’s (2002) p-y criterion is another p-y criterion for weak rock. However, it has not been further validated with other load tests. The equation for estimating modulus of subgrade reaction was based on Vesic (1961) equation for beam on elastic foundation.

Although it has been shown by Vu (2006) that the interim p-y criterion proposed in SJN 134137 in 2006 tends to be the most accurate one in predicting the behavior of laterally loaded drilled shaft socketed into weak rock, none of the aforementioned p-y criteria could predict the behavior of laterally loaded drilled shaft socketed into IGM. Also, the effect of discontinuities is not taken into account thoroughly in these criteria. Moreover, all these p-y criteria were derived either semi-empirically or based on the assumption that the rock is an elastic isotropic continuum.

In summary, a range of analytical tools are available to foundation engineers to design rock sockets under the lateral and moment loading. These include simple, closed-form equations requiring a small number of material properties (Carter and Kulhawy 1992). A more rigorous model that predicts the complete nonlinear response but requires more material properties is also available (Zhang et al. 2000). Highly sophisticated numerical models requiring extensive material properties and appropriate expertise (FEM analysis) exist and may be appropriate for larger projects. The p-y method of analysis is attractive to designers, as evidenced by its wide use; however, considerable judgment is required in the selection of the p-y curve parameters. All of the currently available methods suffer
from a lack of field data for verification and are best applied in conjunction with local ex-
perience.

2.6 BEDROCKS

Six of the groups or systems in the historical classification of rocks are present in the
outcrops of Ohio. The geological map, shown in Figure 2-6, provides the distribution of
those rock systems across the state. The cross section through the rocks of central Ohio
from the Indiana-Ohio border to the Ohio River is shown in Figure 2-7.

2.6.1 Rocks in Ohio

The bedrocks exposed at the surface in Ohio are all of sedimentary types formed from
unconsolidated sediments deposited in marine, brackish, or fresh waters (Lamborn et al.
1938). All the common varieties of the sedimentary series can be found, such as
limestone, shale, sandstone, dolomite, and conglomerate, as well as many thin beds of
coal, clay, and iron ore. In the western half of the State, the bedrock is predominantly
limestone and dolomite with minor amounts of calcareous shale. The calcareous shales
are most abundant in the southwestern part of the state, where thin limestone
interstratified with shale is the usual mode of occurrence. In the northwestern corner of
the state, non-calcareous shale is the bedrock lying immediately below the glacial drift.

Shale, highly distributed in eastern, southwestern, and northwestern of Ohio, is a
variety of sedimentary rock resulted from the consolidation of more or less thinly
laminated or bedded silts and clays (Lamborn et al., 1938). In general, shale is less
weather resistant than most other varieties of sedimentary rocks.
Shale beds are associated with sandstone, limestone, coal, and claystone. Shale may gradually become shaley sandstone and finally sandstone as quartz increases in percentage and size of grain. With an increase in the percentage of calcium carbonate, shale transits to limestone. As carbonaceous material becomes greater in amount, shale is transited to bone coal and coal; and as the fissility or shaley structure disappears, siltstone is produced from shale.

2.6.2 Rock Characterization

The difficulties in making predictions of the engineering responses of rocks and rock mass derive largely from their discontinuous and variable nature. In fact, rock is distinguished from other engineering materials by the presence of inherent discontinuities, which may control its mechanical behavior. The response of intact rock material itself may be complex and difficult to describe theoretically because the rock consists of an aggregation of grains of material having quite different physical properties. It may contain inter- and intra-granular micro-cracks and may have anisotropic and/or nonlinear mechanical properties (Brown, 1993).

Qualification and quantification of in-situ rock masses are some of the most important aspects of site characterization for design of foundations. These characteristics generally indicate the strength, deformability, and stability of the rock masses. For economical reason, it is not feasible to fully measure the characteristics of a complex rock masses.

Discontinuities, one of the major effects, may influence the engineering response of rock masses in a variety of ways such as: (i) the provision of planes of low shear strength along which slip might occur, (ii) reducing the overall shear and tensile strengths of the
rock masses, and (iii) influencing the stress distribution within the rock masses mainly because of their low stiffness and strengths.

2.6.3 Rock Categories

The behavior of many rocks is anisotropic because of some orientation of the fabric, or the presence of bedding, stratification, layering, and jointing. Hawk and Ko (1980) examined the orthotropic nature of two shales and concluded that the properties of both shales are represented well by a transversely isotropic model, although an isotropic model is also acceptable. Additionally, Sargand and Hazen (1987) conducted a series of triaxial tests on Ohio grey shales and concluded that the transverse isotropy is an appropriate model to simulate the stress-strain relations of these shales. However, since most of these layers are parallel to each other, transverse isotropy is generally assumed to describe the deformability and strength of these types of rocks.

Another type of geomaterial that is not investigated thoroughly as a media where the drilled shaft is socketed into is what referred to in the literature as IGM. IGM can have a wide array of properties with characteristics ranging from stiff or hard soil to soft weathered rock, including shale, siltstone, claystone, and some sandstones.
Figure 2-6 Geological map of Ohio, showing the pattern of surface rocks across the state.

Figure 2-7 Cross section through the rocks of central Ohio from the Indiana-Ohio border to the Ohio River (taken from Feldmann et al., 1996)
2.7 ROCK ANISOTROPY

Rock behavior, generally, can vary depending on the geological history and the depositional surroundings. The most important influence of these two factors is the formation of rock anisotropy. The one-dimensional nature of sediment compaction in large depositional basins induces fabric anisotropy resulting in long axes of the constituting geo-material particles stacked parallel to the horizontal axis. With time, these deposits show anisotropy in stiffness properties, (Leroueil and Vaughan (1990)). Shale, which is commonly encountered in North America, is one of the common geomaterials displaying this transversely isotropic property.

Worotnicki (1993), in a recent work, classified anisotropic rocks into four groups as follows.

1. Quartzofeldspathic rocks (e.g. granites; quartz; and, granulites and gneisses).
2. Basic/lithic rocks (e.g. basic igneous rocks such as basalt; greywacke sandstones and amphibolites).
3. Pelitic (clay) and pelitic (micas) rocks (e.g. mud-stones, slates, phyllites and schists).
4. Carbonate rocks (e.g. limestones, marbles and dolomites).

Through his study, Worotnicki (1993) concluded that quartzofeldspathic and basic/lithic rocks show low to moderate degrees of anisotropy with Young’s modulus ratio $E_{\text{max}}/E_{\text{min}}$ not more than 1.3 for approximately 70% of the rocks analyzed and not more than 1.5 in about 80%. This ratio was found not to exceed 3.5 (Figure 2-8(a)). Pelitic clay and pelitic mica rocks show the highest degree of anisotropy with $E_{\text{max}}/E_{\text{min}}$
less than 1.5 for about 33% of the rocks analyzed and less than two in about 50%. The modulus ratio was found not to exceed six, with most cases below four (Figure 2-8(b)). Finally, carbonate rocks were found to show an intermediate degree of rock anisotropy with $E_{\text{max}}/E_{\text{min}}$ not exceeding 1.7 (Figure 2-8(c)). However, rock mass often exhibits transversely isotropic stress strain behavior due to the inherent mineral grain orientation and the presence of bedding planes of parallel sets of joints. The transverse isotropy behavior of rock will be discussed in the following section.

To better understand the anisotropy of rock, Amadei (1996) presented a thorough discussion on the topic of rock anisotropy.

According to Amadei (1996), many rocks exposed near the Earth’s surface show well-defined fabric in the form of bedding, stratification, layering, foliation, fissuring or jointing. These rocks can then have properties (physical, dynamic, thermal, mechanical, and hydraulic) that differ with direction and are thus inherently anisotropic. This anisotropy can be found at diverse scales in a rock mass sorted from the anisotropic microstructure of intact laboratory-sized specimens to the anisotropic faulting and fracturing geometry of a complete rock mass.

When referring to the un-fractured rock, anisotropy is a characteristic of intact foliated metamorphic rocks (slates, gneisses, phyllites, schists). In these rocks, the fabric can be articulated in different ways. For instance, cleavages for the closely spaced fractured rocks are found in slates and phyllites as well. These rocks tend to come apart into planes due to the parallel orientation of microscopic grains of mica, chlorite or other platy minerals. The fabric in schists is produced by the parallel to sub-parallel arrangement of
large platy minerals such as mica, chlorite and talc. Foliation can also be uttered in the form of alternating layers of dissimilar mineral composition such as in gneisses, Milnes et al. (2006).

Non-foliated metamorphic rocks, such as marble, can also be an evidence for some anisotropy due to the favored orientation of calcite grains. Anisotropy is also a characteristic of intact laminated, stratified or bedded sedimentary rocks, such as shales, limestones, sandstones, coal, siltstones, etc. Anisotropy, in general, results from multifaceted physical and chemical processes associated with the transportation, deposition, compaction, cementation, etc. It is remarkable that rocks, which have experienced numerous formation processes may hold more than one direction of planar anisotropy such as the foliation and bedding planes in slates; in addition, these directions are not essentially parallel to each other. Rock mass anisotropy can also be found in volcanic formations like basalt and tuff and sedimentary formations consisting of alternating layers or beds of different isotropic and/or anisotropic rock nature.

In addition to being discontinuous, rock masses cut by one or a number of frequently spaced joint sets are anisotropic with the rock between the joints being either isotropic or anisotropic. It is normal to have several kinds of planar anisotropy in a rock mass such as joints and bedding planes or joints and foliation planes. When the joints expanded parallel to the foliation or bedding planes, they are called foliation joints or bedding joints, correspondingly.

Directional nature of the deformability properties of anisotropic rocks and rock masses is generally evaluated by field and laboratory testing, probably supported by numerical
modeling. Deformability test results on anisotropic rocks are usually explored in terms of
the theory of elasticity for anisotropic media by using the generalized form of Hooke’s
law. Rock has 36 elastic constants of which 21 are independent, as articulated by means
of the compliance matrix.

Still, for most practical cases, anisotropic rocks are modeled as transversely isotropic
or orthotropic media within a coordinate system attached to their arrangement or
directions of symmetry. In transversely isotropic rock media only five independent elastic
constants are needed to describe the rock deformation; while in orthotropic rock media
nine independent elastic constants are needed to describe rock deformation. Transverse
isotropy means that at each point in the rock there is an axis of rotational symmetry and
that the rock has isotropic properties in the plane normal to this axis. This plane can be
termed as plane of transverse isotropy. Orthotropy means that three orthogonal planes of
elastic symmetry be present at each point in the rock and that these planes have the same
orientation all through the rock.
Figure 2-8 Histograms of Emax/Emin ratios for: (a) quartzofeldspathic and basic/lithic rock, (b) pelitic clay and pelitic mica rocks and (c) carbonate rocks.
2.8 TRANSVERSELY ISOTROPIC ROCK

The term “transverse isotropy” refers to a class of material that exhibits isotropic properties in one plane, called the plane of transverse isotropy, and different properties in the direction normal to this plane. For a material that is transversely isotropic, only five independent elastic constants are needed to describe its elastic deformation behavior. Throughout this work, these elastic constants, illustrated in Figure 2-9, are denoted as \( E \), \( E' \), \( \nu \), \( \nu' \), and \( G' \), with the definitions given as follows: \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio respectively in the plane of transversely isotropy, while \( E' \), \( G' \), and \( \nu' \) are Young’s modulus, shear modulus, and Poisson’s ratio in the perpendicular plane.

The generalized Hooke’s law for the stress-strain relationship for elastic transversely isotropic rock mass, (Fig. 2-9), can be expressed as follows:

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E} & -\nu & -\nu' & 0 & 0 & 0 \\
-\nu & \frac{1}{E} & \frac{1}{E'} & 0 & 0 & 0 \\
-\nu' & \frac{1}{E'} & \frac{1}{E} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G'} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G}
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx} \\
\sigma_{xy}
\end{bmatrix}
\]

(2-66)

Theoretical work for the determination of the elastic constants of the transversely isotropic rock has been discussed in the literature and are briefly summarized herein. On the theoretical side of research, Cauwelaeter (1977) showed in a theoretical work that the five elastic constants could be reduced to only three. Other researchers, such as Barden (1963), Graham and Houlsby (1983), Pickering (1970), and Yu and Dakoulas (1993)
have also enriched the literature with their theoretical work and simplified representation of the elastic constants of a transversely isotropic rock.

Figure 2-9 Definition of elastic constants for the case of transverse isotropy.

The five elastic properties of transversely isotropic rocks cannot be any set of values; in fact, a number of inequalities linked with the thermodynamic constraints that the rock strain energy be positive definite must be fulfilled. For example, for transverse isotropy,
the five elastic properties $E$, $E'$, $\nu$, $\nu'$, and $G'$ must satisfy the following thermodynamic constraints:

$$-\sqrt{\frac{E'(1-\nu)}{2E}} < \nu' < \sqrt{\frac{E'(1-\nu)}{2E}}$$  \hspace{1cm} (2-67.a)

$$-1 < \nu < 1$$  \hspace{1cm} (2-67.b)

$$E, E', G' > 0$$  \hspace{1cm} (2-67.c)

Amadei et al. (1987) analyzed 98 measurements of elastic properties and found that for the majority of intact transversely isotropic rocks, the ratio $E/E'$ fluctuates between 1 and 4. Several cases of rocks with $E/E'$ less than unity were found, but this ratio did not fall below 0.7. The ratio $G/G'$ was found to vary between 1 and 3; the Poisson’s ratio, $\nu$, between 0.1 and 0.35; and $\nu'E/E'$ between 0.1 and 0.7.

2.8.1 Method for Estimating the Five Elastic Constants

In many geotechnical engineering applications, there is a need for determining the elastic constants of geo-medium, such as for settlement computation of shallow foundations, predicting deflections of drilled shafts under the lateral loads, computing deformations of underground excavation (tunneling or open excavation), among others. For sedimentary rocks, due to inherent grain orientations and preferred bedding planes, they can exhibit strong directional dependency in their mechanical properties, including elastic constants. Similarly, for rock mass possessing strong joint pattern or foliations, their elastic deformation response can generally be described as transversely isotropic. Due to their geological formation process, stiff clays with or without presence of fissures, also exhibit strong anisotropy properties. According to studies by Hawk and Ho (1980), Sargand and Hazen (1987), this research study, and Shatnawi (2008), the simplest form
of anisotropy, i.e., transversely isotropy, can be effectively used to represent the anisotropic behavior of these geo-materials.

Experimental methods in the laboratory for determining elastic constants of transversely isotropic rocks have been presented in the literature. Laboratory tests can be divided into two categories: dynamic and static methods. The dynamic laboratory methods include the resonant bar method (Goodman 1989) and ultrasonic pulse method (Youash 1970) by which the dynamic elastic constants $E_d$, $\nu_d$, and $G_d$ can be determined. Indeed, the ultrasonic pulse method can be used to determine all five independent elastic constants of transversely isotropic rocks, as illustrated by Liao et al. (1997). Nevertheless, Chou (2008) pointed out some shortcomings of Liao et al. (1997) in that the transversely isotropic plane has to be parallel to the longitudinal axis of the specimen. The available static methods were summarized in Chou (2008) to include the uni-axial compression test, conventional tri-axial compression test, true tri-axial compression test, hollow cylinder test, bending test, torsion test, and diametral compression test (Brazilian test).

The elastic constants of anisotropic rocks can be computed by substituting loading force and strain data recorded in testing into stress–strain equations. To evaluate the five independent elastic constants of transversely isotropic rocks, two cylindrical specimens with one loading direction are needed in a uniaxial compression test. Two cubic specimens with three loading directions are needed in a true triaxial compression test. Two hollow cylindrical specimens with two types of loading conditions are needed in a hollow cylinder test. And two discs with one loading direction are needed in a Brazilian
test. The numbers and geometry of specimens and types of loading condition needed in these methods are listed in Table 2-3 for comparison.

Table 2-3 Comparison of specimens needed in tests applied to evaluate elastic constants of transverse isotropic rocks.

<table>
<thead>
<tr>
<th>Test</th>
<th>Type of specimen</th>
<th>Number of specimen</th>
<th>Type of loading for each specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazilian Disc</td>
<td>Disc</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Hollow cylinder</td>
<td>Hollow cylinder</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>True triaxial compression</td>
<td>Cube</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Uniaxial compression</td>
<td>Cylinder</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

The in-situ test methods include a number of tests from which the five independent elastic constants could be evaluated, such as the Goodman jack tests, and the hydraulic chamber test (Amadei, 1996). These two tests are mainly used in tunnels and exert stresses on a limited section of the perimeter of the walls (as in the Goodman Jack) which makes it hard to interpret as the rigid surface in contact with the rock change with the pressure change. Kawamoto (1966) proposed using a pressurized borehole to estimate the elastic constants of anisotropic rock. Kawamoto’s method was only applicable to certain directions of rock anisotropy with respect to the holes.

In-situ tests are generally considered more appropriate for the determination of the geo-material’s deformation modulus due to the fact that they cause less sample disturbance compared to sampling and testing in the laboratory. Furthermore, in-situ tests are conducted in the geo-materials without altering significantly on-site state of stress. Kiehl (1983) showed that for a borehole sunk parallel to the schistosity, the mean values
for $E'$ and $G'$ for a transversely isotropic medium can be predicted by using the direct pressuremeter test. The description for Kiehl method was presented in Wittke (1990).

Figure 2-10 Dilatometer test in a transversely isotropic rock (taken from Wittke, 1990)

The orientation of a borehole in relation to the schistosity is defined by the angle $\Phi$ presented in Figure 2-10. This represents the angle between a perpendicular to the borehole’s axis and the line of dip measured in the plane including both the boreholes’ axis and the line of dip. The different directions in which the alterations in diameter are measured in dilatometer tests are distinguished by the angle $\theta$ between a perpendicular to the strike and the direction in question on the plane perpendicular to the boreholes’ axis. The direction $\theta=0$ is perpendicular to the schistosity’s strike and coincides with the direction of greatest rock mass deformability on the plane perpendicular to the borehole’s axis; whereas the direction $\theta=90$ is parallel to the schistosity’s strike and coincides with the direction of least rock mass deformability.
When the borehole is sunk perpendicular to the schistosity ($\Phi=0$), the modulus $E$ may be determined from the stress-deformation curve attained in a dilatometer test using the following equation:

$$E = (1 + \nu)d \frac{\Delta p}{\Delta d}$$  \hspace{1cm} (2-68)

The change in the diameter of a borehole sunk parallel to schistosity ($\Phi=90$) may be described as a function of the angle $\theta$ as follows:

$$\Delta d(\theta) = (U_0 \cos^2 \theta + V_0 \sin^2 \theta)d.\Delta p$$  \hspace{1cm} (2-69)

$U_0$ and $V_0$ are quantities which depend on the five elastic constants. To determine them from dilatometer test, the reciprocal values of the slopes of the stress-deformation curves for the different measurement directions A, B, C, and D divided by the borehole diameter $d$, ($\Delta d/\Delta p.d$), are plotted as a function of $\theta$. If possible, the measurement directions should be arranged with respect to the schistosity’s direction of strike in the manner shown in Figure 2-10. Figure 2-11 indicates how the quantities $U_0$ and $V_0$ are obtained as the extreme values in the plot of the test results.

Although $U_0$ and $V_0$ are virtually only a function of the moduli $E$, $E'$, and $G'$, the elastic constants cannot be explicitly determined from these quantities. Indeed, the value of one of the three moduli has to be already known.

It should be pointed out that $E$ can still be determined fairly accurately even when a borehole is not sunk perpendicular to the schisosity, as long as $\Phi<30$. The boreholes
intended to establish $U_0$ and $V_0$ need not necessarily be arranged parallel to schistosity.

In this case Figure 2-12 is used.

Figure 2-11 (a) dilatometer test in a borehole oriented parallel to the schistosity, (b) directions of measurement

Figure 2-12 (a) correlation of the measurement of $U_0$ and $V_0$, (b) diagram to determine the modulus ratios
Figure 2-13 Coefficient $k_1$ and $k_2$ determined from dilatometer test as a function of the modulus ratios for differing borehole orientations with respect to the schistosity.

Usually, it is convenient to reduce the number of unknown elastic constants by assuming shear modulus $G'$ to be dependent on other elastic moduli. On the other hand, it is $G'$ which is the most arduous to be experimentally determined due to the particular testing apparatus it needs and the special sample preparation it requires. Most of the literature studies on the examination of the dependency of the elastic constants of the transversely isotropic rocks were mainly concerned with the validity of the classic empirical Saint Venant’s (1863) expression:

$$G' = \frac{EE'}{E + E' + 2\nu E'}$$ \hspace{1cm} (2-70)

Because the five elastic constrains are theoretically independent, this equation is in theory incorrect. Even so, a sensible engineering estimate for the shear modulus $G'$ was found to be associated with this equation. In a survey of elastic constants of rocks exhibiting anisotropy, Amadei (1996) concluded that the majority of the available experimental data support, to some extent, the validity of Saint Venant’s (1863).
In a quite recent publication, Talesnick and Ringle (1999) suggested a modification for Saint Venant’s to comprise a multiplication factor that includes the relative difference between E and E’. The modified G’ takes the form of the following equation.

\[
G' = \frac{EE'}{E + E' + 2\nu E} \left[ \frac{2E - E'}{E} \right]
\]  

(2-71)

Wittke (1990) suggested that the estimate of shear modulus G’ might be obtained if the following Equation is adopted:

\[
G' = \frac{E'}{2(1 + \nu')}
\]  

(2-72)

By differentiating between the estimate of G’ assuming plane stress and G’ assuming plane strain, Exadakyllos (2001) proposed a relation for the estimation of G’. Also Cauwelaert (1977) and Kiehl (1980) provided an expression for the estimate of G’ depending on the other elastic constrains. During the work on this research study, an empirical relationship for the estimate of G’ was proposed and is also documented in more detail in Shatanawi (2008). A summary of these expressions is presented in Table 2-4.
Table 2-4: Empirical Equations for estimation of the shear modulus $G'$

<table>
<thead>
<tr>
<th>Author</th>
<th>Empirical Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saint Venant (1863)</td>
<td>$G' = \frac{EE'}{E + E'^2 + 2\nu'E'}$</td>
</tr>
<tr>
<td>Talesnick and Ringle (1999)</td>
<td>$G' = \frac{EE'}{E + E'^2 + 2\nu'E'} \left[ \frac{2E - E'}{E} \right]$</td>
</tr>
<tr>
<td>Wittke (1990)</td>
<td>$G' = \frac{E'}{2(1 + \nu')}$</td>
</tr>
<tr>
<td>Exadaktylos (2001)</td>
<td>$G' = \frac{\sqrt{2\nu'}}{2\nu' / E' + 1 / E' - 1 / E}$</td>
</tr>
<tr>
<td></td>
<td>$G' = \frac{\sqrt{2\nu'(1 + \nu)}}{2\nu'(1 + \nu)} \left[ \left( \frac{1 - \nu'^2}{E'} \right) \frac{E}{E'} - \frac{(1 - \nu^2)}{E} \right]$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{G'} = \left[ \frac{1}{E} + \frac{1}{E'} + 2k \right]$</td>
</tr>
<tr>
<td>Cauwelaert (1977)</td>
<td>$G' \leq \frac{E'}{2[\nu'(1 + \nu) + \sqrt{(E'/E - \nu'^2)(1 - \nu^2)}]}$</td>
</tr>
<tr>
<td>Kiehl (1980)</td>
<td>$G' = \frac{0.064}{(1 - e^{-0.064E'})}$</td>
</tr>
</tbody>
</table>

This Study
2.9 THE PRESSUREMETER

2.9.1 Definition

The term pressuremeter was first used by Menard to describe the testing equipment he developed in 1955. Baguelin et al. (1978) referred to the pressuremeter probe as a device that applies hydraulic pressure through a flexible membrane to the borehole walls. Mair and Wood (1987) further restricted the definition of a pressuremeter to a cylindrical device and this definition is recognized internationally by the ISSMFE (Amar et al., 1991). This widely recognized definition is as follows.

A pressuremeter is a cylindrical probe that has an expandable flexible membrane designed to apply a uniform pressure to the walls of a borehole.

2.9.2 The Basic Theory on Pressuremeter

2.9.2.1 Elasticity

Baguelin et al. (1977) produced a theoretical derivation of the full shear stress-shear strain relationship from pressuremeter test. The restrictions in the deviation include homogeneity, incompressibility and axisymmetric deformation.

\[ \tau = x(1 + x) \frac{dp}{dx} \quad (2-73) \]

where, \( x = \frac{\Delta v}{v_0} \); \( \Delta v \) = change in cavity volume from the initial volume \( v_0 \).

The stress and displacement fields around a laterally moving infinitely long cylinder, in plain strain, have been presented by Baguelin et al. (1977). The model used is a disk having a fixed outside radius of \( R \), representing the soil, with a rigid section fixed at its center, radius \( r_0 \), representing the cross section of the pile. Perfect pile to soil adhesion is
assumed and a load per unit depth, $T$, is applied to the pile causing uniform translation as shown in Figure 2-14.

![Figure 2-14: Mobilized radial and tangential reactions around a translating pile](image)

In polar co-ordinates, the stress distribution around the pile, circumference is expressed as:

\[
\sigma_r = \frac{T}{2\pi r_0} \cos \theta
\]  \hspace{1cm} (2-74)

\[
\sigma_\theta = -\frac{T}{2\pi r_0} \cos \theta
\]  \hspace{1cm} (2-75)

\[
\tau_{r\theta} = -\frac{T}{2\pi r_0} \sin \theta
\]  \hspace{1cm} (2-76)

The distribution of the components of the radial force and the tangential force can be expressed as:

\[
dQ = f_r = \sigma_r \cos \theta r_0 d\theta
\]  \hspace{1cm} (2-77)

\[
dF = f_\theta = \tau_{r\theta} \sin \theta r_0 d\theta
\]  \hspace{1cm} (2-78)
The pile displacement is given by:

\[
y = \frac{T}{16\pi G(1-\mu)} \left[ (3-4\mu) \ln \left( \frac{R}{r_0} \right)^2 - \frac{2}{3-4\mu} \right], \text{ when assumed}
\]

\[
\frac{R^2 - r_0^2}{R^2 + r_0^2} \approx 1
\]

(2-79)

Considering any element around the circumference of the pile a modulus of pile element resistance may be defined as:

\[
k_{pile} = \frac{\sigma_r \cos \theta + \tau_{r\theta} \sin \theta}{y^2/r_0}
\]

(2-80)

where \(y=\)horizontal translation of the pile

For the element where \(\theta = 0\), it can be obtained:

\[
k_{pile} = \frac{\sigma_r}{y/r_0} = \frac{8G(1-\mu)}{(3-4\mu) \ln \left( \frac{R}{r_0} \right)^2 - \frac{2}{3-4\mu}}
\]

(2-81)

Also, a modulus of pile resistance,

\[
k_{pile} = \frac{p}{y/r_0} = \frac{pr_0}{y}
\]

(2-82)

and since \(p = \frac{T}{2r_0}\), then

\[
k_{pile} = \frac{T}{2y} = \frac{8\pi G(1-\mu)}{(3-4\mu) \ln \left( \frac{R}{r_0} \right)^2 - \frac{2}{3-4\mu}}
\]

(2-83)
This shows, for equal model to pile ratios, \( \frac{R}{r_0} \), that the modulus of pile resistance is independent of change in the pile radius \( r_0 \). The ratio of the respective moduli for the pile and pressuremeter is given as:

\[
\frac{k_{\text{pile}}}{k_{\text{pmt}}} = \frac{\pi k_{\text{pile}}}{k_{\text{pmt}}} = \frac{4\pi (1 - \mu)}{(3 - 4\mu) \ln \left( \frac{R}{r_0} \right)^2 - \frac{2}{3 - 4\mu}}
\]  

(2-84)

2.9.2.2 Plasticity:

Considering pressuremeter expansion, the elements of material adjacent to the wall will yield initially and become plastic when the deviatoric stress reaches a certain limit. In purely cohesive material without volume change, a Tresca yield criterion is appropriate:

\[
\sigma_r - \sigma_0 = 2C_u
\]  

(2-85)

If the material has both cohesion and frictional properties and obeys the Mohr-Coulomb yield criterion without volume change, then the pressure in the cavity to initiate yield is given by:

\[
p_c = p_0 (1 + \sin \phi) + e \cos \phi
\]  

(2-86)

where \( p_0 \) = initial cavity pressure.

For both the purely cohesive material and combined cohesion and friction material, the plastic zone propagates into the material as the pressure increases and cavity expands. The limiting cavity pressure, \( p_L \), at which indefinite cavity expansion occurs is:
\[ p_L = p_0 + c_u \left[ 1 + \ln \left( \frac{G}{c_u} \right) \right] \]  

(2-87)

2.9.3 Methods for Design Pile under Horizontal Load using PMT Results

One of the most direct and obvious applications of pressuremeter test results is the design of piles subjected to horizontal loads. Indeed, some analogy exists between the cylindrical expansion of the PMT probe and the horizontal movement of a pile segment. This analogy is not a direct analogy and steps are necessary to go from pressuremeter expansion curve to the horizontal soil resistance curve (p-y curve) for the pile. It is in these steps that the various existing methods differ.

A total of nine (9) methods for design pile under horizontal load using pressuremeter test results are summarized by Briaud (1986).

2.9.3.1 Menard 1969 (method 1)

This method was developed originally by Menard (1969), Bourdon and Gambin (1969) and then expanded by Gambin (1979). The method uses the results of preboring pressuremeter tests. It considers that the p-y curve is bilinear elastic-plastic. The slope of the first linear portion of the curve is obtained from Menard’s equation for the settlement of a strip footing (Menard, 1975) and the second slope is half the first slope. The soil ultimate resistance, \( p_{\text{ult}} \), is given by the pressuremeter limit pressure. The critical depth varies from 3B to 8B (B=diameter or width of pile) depending on the soil type. A reduction factor, which varies from 1 at the critical depth to 0.5 at the surface, is applied to \( p_{\text{ult}} \) within the critical depth.
2.9.3.2 CAI Consultants 1980 (method 2)

This method was developed by CAI Consultants in 1980 for the rigid drilled shafts used for power line tower foundations. Both $\sigma_r$ and $\tau_{r0}$ resistances are combined into one lateral resistance model, which is a parabola cut off at $p_{ult}$, obtained by Hansen’s theory (Hensen, 1961). CAI’s experience with this method is that when the measured deflection is 2.5 cm the predicted deflection is between 1.2 cm and 3.7 cm.

2.9.3.3 Dunand 1981 (method 3)

This method was developed by Dunand (1981). The method is based on the results of preboring pressuremeter tests. An elastic-plastic model is used for the frontal reaction. The slope of the elastic portion is obtained from the pressuremeter modulus and elasticity theory, while the ultimate value is considered to be the limit pressure from the pressuremeter. A friction model is also proposed and the critical depth approach is the same as in Menard’s method (method 1). This method was developed specifically for pile foundations for electric power lines in France.

2.9.3.4 OYO Corporation (method 4)

This method was developed by the OYO Corporation; it uses the result of preboring pressuremeter tests. It is based on the closed form solution to the governing differential equation for the problem of an infinitely long laterally loaded pile embedded in a uniform soil with a constant modulus of subgrade reaction $k$. The solution, which gives the relationship between the horizontal load and the horizontal displacement at the ground surface, is then modified with empirical factors to match experimental data including non-linearity of the soil and scale effects. The value of $k$ is obtained from the
pressuremeter modulus value. This method has the advantage of being a hand calculation method.

2.9.3.5 Baguelin et al. 1978 (method 5)

This method was first proposed by Baguelin, Jeaequel and Shields in 1978; it was presented with more details in 1982 and was incorporated in a yet more complete form into a design manual by the French Petroleum Institute in 1983. It uses the results of self-boring pressuremeter tests. The p-y curve at a depth z for the pile is obtained from the pressuremeter expansion curve \( p^* - \frac{1}{2} \frac{\Delta V}{V_0} \) at the same depth z as follow:

\[
p = \eta p^* B
\]
\[
y = \frac{1}{2} \frac{\Delta V}{V_0} r
\]

where, \( \eta \) is the lateral resistance factor varying from 0.33 to 3, \( p^* \) is the net pressure \( (p-p_0) \) from the pressuremeter curve, \( r \) is the pile radius, \( V_0 \) is the initial volume, and \( \Delta V \) is the volume injected into the probe.

2.9.3.6 Briaud et al. (method 6)

This method was developed by Briaud, et al. (1981). It uses the results of preboring pressuremeter tests and considers that the p-y curve is made of a front resistance Q-y curve and a friction resistance F-y curve.

According to the theoretical distribution of the radial stress and tangential stress around a laterally moving pile, the distribution of the radial force, \( f_r \), and tangential force, \( f_0 \), is given by the following:
\[ f_r = dQ = \sigma_r \cos \theta \, r \, d\theta \]  
\[ f_\theta = dF = -\tau_{r\theta} \sin \theta \, r \, d\theta \]  

where

\( r = \) pile radius

The total load per unit depth of the pile for p-y curve is a combination of the two components, i.e. shear reaction, \( F \), and pressure reaction, \( Q \). Thus at any deflection, \( y \):

\[ P = Q + F = D[\sigma_r \times SQ + \tau_{r\theta} \times SF] \]

As shown in Figure 2-14, the two components \( Q \) and \( F \), can be calculated as follows:

\[ Q = 2 \int_0^{\pi/2} \sigma_{r,\text{max}} \cos \theta \cos \theta \, r \, d\theta = 2r \sigma_{r,\text{max}} \left[ \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi/2} = \frac{\pi}{4} D \sigma_{r,\text{max}} \]  
\[ F = 2 \int_0^{\pi/2} \tau_{r\theta,\text{max}} \sin \theta \sin \theta \, r \, d\theta = 2r \tau_{r\theta,\text{max}} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} = \frac{\pi}{4} D \tau_{r\theta,\text{max}} \]

(for circular pile)
Figure 2-15 The two components Q and F comprising the lateral force P

where

\[ D = \text{pile diameter or width} \]
\[ \sigma_r = \text{mobilized front pressure at } y \]
\[ \tau_{r0} = \text{mobilized shear stress at } y \]

\[ SQ = \text{shape factor for pressure reaction} = \frac{\pi}{4} \text{ for circular piles}=1.13 \text{ for square piles} \]
\[ SF = \text{shape factor for shear reaction} = 0.79 \text{ for circular piles} \]

\[ = 1.0 \text{ considering the additional contribution from shear behind the pile} \]
\[ = 1.76 \text{ for square piles} \]
\[ = 2.0 \text{ considering the additional contribution from shear behind the pile} \]

2.9.3.7 Fugro B. V. 1983 (method 7)

This method was developed by Fugro B. V. in 1983. It uses the results of push-in or full-displacement pressuremeter test results. The pressuremeter test derived parameters, namely shear modulus \( G \), undrained shear strength \( S_u \), friction angle \( \phi' \), and at rest
pressure coefficient $K_0$, are input in a pseudo three-dimensional finite element code. The material is modeled as a linear-elastic, perfectly-plastic material. This method has the advantage of modeling the soil as a continuum.

2.9.3.8 Robertson 1986 (method 8)

This method uses the results of driven pressuremeter tests for driven piles, and preboring or self-boring pressuremeter tests for bored piles (Hughes, 1979; Robertson, 1982, 1986). The p-y curve at a depth $Z$ for the pile is obtained point by point from the pressuremeter expansion curve at the same depth $z$ by using equations used in Baguelin et al. 1978 (method 5) with $\eta = 2$ for cohesive materials and $\eta = 1.5$ for cohesionless materials. For the pile, a unique value of critical depth equal to 4 pile diameters is proposed and the appropriate reduction factors are recommended within the critical depth.

2.9.3.9 Matlock and Reese 1974 (method 9)

This method was used by Woodward-Clyde Consultants for piles in clay (Davidson, 1986). It is a version of the Matlock and Reese method (Matlock, 1970; Reese, 1974). The Matlock and Reese method requires the undrained shear strength, $S_u$, and the strain at 50% of peak stress $\varepsilon_{50}$ in order to construct the various p-y curves. Davison and Bodine (1986) use $S_u$ obtained from preboring pressuremeter tests by the Gibson/Anderson method (Gibson, 1961), $\varepsilon_{50} = \frac{S_u}{E_R}$ for deflection less than 1.2 cm. where $E_R$ is the reload pressuremeter modulus, and $\varepsilon_{50} = \frac{S_u}{E}$ for deflections of 2.5 cm or larger, where $E$ is first load pressure modulus.
3.1 INTRODUCTION

It is common in practice to design laterally loaded rock-socketed drilled shafts using the p-y curve methods. However, the initial part (small deflection) of the existing p-y curve criteria for rock were derived either semi-empirically or based on the assumption that the rock is an elastic isotropic continuum. The assumption of isotropy may not be appropriate for rocks with intrinsic anisotropy due to its aligned grain orientation, or for rocks with joints and bedding planes. Therefore, there is a need to develop a p-y curve criterion that would be capable of taking into considerations of the effects of rock anisotropy on the small-deflection response of the laterally loaded shafts. This chapter presents a method for estimating the initial tangent $K_i$ to the p-y curve for rock that can be characterized as a transversely isotropic elastic continuum. It is important to be able to accurately determine the initial tangent to p-y curve because it governs to a large extent the accuracy of the predicted deflection of the laterally loaded drilled shaft in rock under the working load condition. The 3-D FE parametric study results of a laterally loaded drilled shaft socketed in a transversely isotropic continuum are used to develop a series of design charts for estimating $K_i$ as a function of the five elastic constants of a transversely isotropic medium.

From the review presented in section 2.3, it is apparent that the derivations of these cited equations are based on either limited field data or the assumption that the rock is an
isotropic continuum. Furthermore, it appears that the recommended empirical equations for determining the initial modulus of p-y curve criteria are almost universally relied upon the use of the rock mass modulus. Determination of representative rock mass modulus is not straightforward and as it will be shown in the next section that the accuracy of the rock mass modulus can exert significant influences on the accuracy of the derived p-y curve and the predicted shaft deflections.

3.2 SENSITIVITY ANALYSIS

A source of uncertainty in the existing p-y criteria that were based on the use of rock mass modulus comes from the choice of method for selecting rock mass modulus when there are more than one option available. In the literature, several empirical equations are available for determining the modulus of rock mass using empirical correlations with rock properties. For example, rock properties such as RMR, $q_{us}$, $E_i$, GSI, and RQD have been used to correlate with the modulus of rock masses. Justo et al. (2005) showed that the rock mass modulus estimated using these empirical equations can have a large variance. In their study, $E_m$ is shown to be in a wide range between 0.53 GPa to 15.2 GPa (i.e., three order of magnitude of difference). This issue of selecting representative and accurate rock mass modulus is most acute in the interim model proposed by Reese (1997).

The determination of GSI can be difficult and different person interpreting the geological and rock setting can come up with different values of GSI. The sensitivity of the response of the laterally loaded drilled shaft in rock to the interpreted GSI values is demonstrated herein using a real case documented by Gabr et al. (2002) and later studied
in the SJN 134137 report by Nusairat et al (2006). The drilled shaft is 2.5 feet in diameter with 13.3 feet rock socket in siltstone. The parameters used for generating the p-y curves for rock are summarized in Table 3-1. With the two GSI values (GSI = 60 and GSI = 70) that are deemed to be within the range of judgment difference by engineers, the corresponding rock mass modulus would be estimated to be (79.4 ksi, and 125.9 ksi) and the resulting p-y curves based on the p-y criterion suggested in SJN 134137 report are shown in Figure 3-1. Using the LPILE computer program with input of these p-y curves, the predicted deflection versus depth as well as the moment versus depth curves are shown in Figure 3-2(a) and Figure 3-2(b), respectively. It can be seen that even with a small variation of GSI values that is within an interpretation error by an engineer, the resulting difference of the predicted rock mass modulus, p-y curves, and the shaft response can be significantly different based on empirical equations (Equation (2-16)) and p-y curve criterion develop in SJN 134137. The high sensitivity observed in this case study has provided impetus for us to develop an alternative approach to characterize the rock mass properties; namely to use the five elastic constants of the transverse isotropic elastic theory, rather than a single value of rock mass modulus estimated from GSI values.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$\gamma'$ (kN/m$^3$)</th>
<th>$\sigma_{ci}$ (MPa)</th>
<th>GSI</th>
<th>mi</th>
<th>$E_m$ (MPa)</th>
</tr>
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<tr>
<td>0.61</td>
<td>25</td>
<td>12</td>
<td>55</td>
<td>9</td>
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</tr>
<tr>
<td>1.91</td>
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<tr>
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</tr>
<tr>
<td>3.81</td>
<td>13</td>
<td>24</td>
<td>75</td>
<td>14</td>
<td>349</td>
</tr>
</tbody>
</table>

Table 3-1 Input Rock Mass Parameters of I-40 Load Test
Figure 3-1 Effect of GSI on p-y criterion develop in SJN 134137 (2006)

Figure 3-2 Deflection and moment profile
3.3 FINITE ELEMENT ANALYSIS

In order to develop an elastic subgrade reaction solution for drilled shafts socketed into a transversely isotropic elastic rock, a methodology for deriving an empirical relationship for estimating the initial modulus of subgrade reaction of transversely isotropic rock is developed herein by employing the results of a parametric study for a 3-D FE simulations of a laterally loaded drilled shaft socketed into transversely isotropic continuum.

3.3.1 Description of FE Model

The nonlinear FE program ANSYS (ANSYS, 2007) is used for the parametric study. Due to the symmetric nature of the problem, only one-half of the problem domain is modeled as demonstrated in Figure 3-3. Also shown in Figure 3-3 is the 3D mesh generated by ANSYS program in which both the rock and the drilled shaft are modeled using 8-node brick elements (Solid 185). A mesh sensitivity and a solution convergence study was carried out, as shown in Figure 3-4, from which an optimum mesh density was selected for the subsequent parametric study. The previous related study was also reviewed to confirm the appropriateness of the dimension of the selected problem domain. In 2002, Wallace et al. (2002) found that 11D (D is the diameter of the drilled shaft) could be used to sufficiently define the dimension of the needed problem domain size. Wallace (2002) also demonstrated that the depth of rock mass beneath the drilled shaft tip needs to be at least 0.7 L, where L is the embedment length of the drilled shaft. These recommendations by Wallace (2002) were fully adopted in the selection of the problem domain dimensions.
3.3.2 Constitutive Models

The cylindrical drilled shaft is modeled as a linear elastic material, while the rock is modeled as a transversely isotropic elastic material. The interface between the drilled shaft and the rock mass is modeled using a surface based contact, where the shaft surface is treated as a contact surface while the rock surface is taken as target surface. In the tangential direction of the shaft-rock interface, the frictional interaction is simulated using a linear Coulomb friction theory, with the required input of a coefficient of friction. In the normal direction, the contact surface would transmit no contact pressure unless the nodes of the contact surface are in contact with the target surface.

3.3.3 FE Analysis

Since the determination of initial subgrade reaction modulus is the primary objective of this analysis, only the elastic response of rock and shaft is the concern of this parametric study. In the numerical simulations, incremental lateral loads are applied at the drilled shaft head. From the FE results, the rock resistance per unit shaft length \( p \) is calculated by double differentiating the moment versus depth profile, where the moment profile is obtained from the strain data around the shaft and then fitted by using the piecewise polynomial curve fitting technique. By extracting the deflections of the shaft under various load levels at the corresponding depth of the rock layer, \( p-y \) curves of the rock layer can be obtained. The initial subgrade reaction modulus \( K_i \) is determined as the initial tangent to the \( p-y \) curves.
Figure 3-3 FE mesh of a drilled shaft-rock system.

Figure 3-4 Mesh convergence.
3.3.4 FE Parametric Study Results

An extensive parametric study using ANSYS FE method program was carried out to examine systematically the effects of several influencing factors, such as shaft radius (R), Young’s modulus of the drilled shaft (E_p), Poisson’s ratio of the drilled shaft (ν_p), and the five elastic constants of a transversely isotropic rock mass (E, E’, G’, ν, ν’, and β), together with shaft and rock unit weight and the rock-shaft interface properties. The specific influencing parameters and the range of each parameter studied are summarized in Table 3-2. The parametric study is carried out systematically in two stages: (a) Stage 1: the effects of each parameter individually was studied by varying the values of one parameter only while keeping the values of the other parameters constant to ascertain the relative importance of each parameters that would affect K_i, (b) Stage 2: the values of the influencing parameters are randomly generated within the bounds selected in Table 3-2 to further enhance the data points for regression analysis.

Table 3-2 Parameters variation for the FEA

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<td>β</td>
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<td>-90°-90°</td>
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<td>10-25</td>
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<td>γ_rock (kN/m³)</td>
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<td>E_p (GPa)</td>
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<td>0.1-0.45</td>
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<tr>
<td>μ</td>
<td>0.5</td>
<td>0.1-0.9</td>
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*Baseline value is the reference value in the parametric study, when only one parameter is varied while keeping other parameters at the baseline value.*
The FE analysis results for varying the values of single input parameters are plotted in Figure 3-5 for the effects of five elastic constants and the angle between the shaft direction and the bedding plane direction. Similarly, the FE analysis results for varying the values of input parameters that reflect the properties of the drilled shaft and the interface are plotted in Figure 3-6. The relative importance (or sensitivity) of varying each single input parameter is calculated and summarized in Table 3-3 and plotted as a pie chart in Figure 3-7. The order of importance of the investigated parameters affecting $K_i$ can be ranked from high to low as follows: $(R, E, \beta, E_p, E', \nu, v, \gamma_{\text{rock}}, \gamma_{\text{shaft}}, \text{and } \mu)$. It can be observed that $\mu$, $\gamma_{\text{rock}}$, and $\gamma_{\text{shaft}}$ have exerted relatively no effects on $K_i$. These parameters can therefore be held constant throughout the FE simulations in the stage 2 (random parameter generation) parametric study. The number of runs for stage 1 is 150, while the number of successful runs in stage 2 is 100.

The results of FE simulations from both stage 1 and stage 2 are analyzed using a statistical analysis software SPSS (Statistical Package for the Social Sciences, 2003) program. It was assumed for the purpose of nonlinear regression, that $K_i$ could be computed as in Equation (3-1) by a product of various $\eta$ functions.

$$K_i = \eta(E)\eta(E')\eta(G')\eta(\nu)\eta(\nu')\eta(\beta)\eta(E_p)\eta(v_p)\eta(R)$$  \hspace{1cm} (3-1)

The built-in Levenberg-Marquardt algorithm (LMA) in the SPSS is used to perform the curve fitting to find various $\eta$ functions in Equation (3-1). In essence, the LMA performs an iterative numerical solution to the mathematical problem of minimizing a sum of squares of nonlinear functions that depend on a common set of parameters (i.e.,
curve fitting). The solution obtained from the SPSS is presented for convenience in design charts in Figure 3-8. These charts can be used together to find $K_i$ in accordance with the following equation:

$$\frac{K_i}{P_a} = 1 \times 10^6 \eta(E, \beta) \eta(E', G') \eta(\nu, \nu') \eta(E_p, \nu_p) \eta(R)$$ (3-2)

Goodness of the nonlinear regression analysis presented in the design charts was statistically examined using R-squared (i.e., statistical measure of how well a regression line approximates real data points, in which an R-squared of 1.0 indicates a perfect fit). R-squared was found to be 0.99 for the regression line between the empirically predicted $K_i$ using Equation 3-2 and the FEM calculated $K_i$ as shown in Figure 3-9. This R-squared value is considered excellent.
Figure 3-5 Effects of transversely isotropic parameters on $K_i$. 

- (a) Hoerl Model: $y = a(x^b + c)$
  - $a = 1.23$, $b = 0.98$, $c = 0.77$

- (b) Hoerl Model: $y = ax^c$
  - $a = 8.3$, $c = -0.08$

- (c) $y = 1.5023x^2 - 0.9541x + 7.4689$

- (d) $y = 5.08x + 7.0728$

- (e) For $\beta > -45$:
  - $(7.832 \times (0.7575 + 0.298 \times \cos(0.047 \times \theta - 0.62)))$
  - Otherwise: $(8/(5.5 + 0.127 \times \theta + 0.001 \times \theta^2))$
Figure 3-6 Effects of drilled shaft properties and rock density on $K_i$
### Table 3-3 Sensitivity analysis of $K_i$ (Stage I)

| Parameter | $K_{i_{\text{min}}}$ | $K_{i_{\text{max}}}$ | $|K_{i_{\text{max}}} - K_{i_{\text{min}}}|$ | $\frac{|K_{i_{\text{max}}} - K_{i_{\text{min}}}|}{\sum |K_{i_{\text{max}}} - K_{i_{\text{min}}}|} \times 100\%$ |
|-----------|---------------------|---------------------|------------------|----------------------------------|
| $E$       | 4.5                 | 11.8                | 7.3              | 18.09                            |
| $E'$      | 6.6                 | 8.7                 | 2.1              | 5.20                             |
| $G'$      | 6.25                | 7.5                 | 1.25             | 3.10                             |
| $\nu$     | 7                   | 7.5                 | 0.5              | 1.24                             |
| $\nu'$    | 7.1                 | 8.6                 | 1.5              | 3.72                             |
| $\beta$   | 3.5                 | 8.5                 | 5                | 12.39                            |
| $\gamma_{\text{shaft}}$ | 7.3                | 7.45                | 0.15             | 0.37                             |
| $\gamma_{\text{rock}}$ | 7.3                | 7.45                | 0.15             | 0.37                             |
| $R$       | 7                   | 28                  | 21               | 52.04                            |
| $Ep$      | 4.4                 | 9.2                 | 4.8              | 11.90                            |
| $\nu_p$   | 6.4                 | 7.8                 | 1.4              | 3.47                             |
| $\mu$     | 7.3                 | 7.3                 | 0                | 0.00                             |
| $\Rightarrow \sum$ | 40.35 | 100 | | |

Figure 3-7 Sensitivity analysis of $K_i$ (Stage I)
Figure 3-8  Design charts for estimating $K_i$
3.4 ESTIMATING THE TRANSVERSELY ISOTROPIC PARAMETERS

To determine the transversely isotropic elastic constants, different types of laboratory tests have been used such as uniaxial compression tests on samples with different orientation between the plane of anisotropy and the axial loading direction, Brazilian tensile strength test, uniaxial tension test, and ultrasonic wave velocity measurement. Moreover, Wittke (1990) described a method of using the direct dilatometer in-situ test to determine the five elastic constants. Using the laboratory tests, four of the constants (E, E', ν, and ν') can be measured directly; however, measuring the shear modulus (G') proves to be the most difficult task with the current in-situ or lab test methods (Wittke 1990). It is desirable to develop an appropriate empirical relationship to determine the shear modulus (G').

In order to develop the desired empirical relationship for G' so that the characterization procedures for transversely isotropic rock can be simplified, statistical analyses were
carried out on the experimental data of Gerrad (1975), Lo et al. (1986), Amadei et al. (1987), Johnston et al. (1993), Homand et al. (1993), Liu (1994), Amadei (1996), Liao et al. (1997), and Chen et al (1998). A large amount of experimental data of elastic modulus ratio and shear modulus ratio \((E/E', G/G')\) was collected as shown in Table 3-4. By plotting these ratios against each other and using some fitting techniques, an empirical formula for the shear modulus \((G')\) is derived as presented in Equation (3-3). The prediction of this equation is highly correlated with the measured values as shown Figure 3-10.

\[
G' = \frac{0.032E}{(1 + \nu)(1 - e^{-0.06 \frac{E}{E'}})} \tag{3-3}
\]

![Figure 3-10 Predicted vs. measured shear modulus (G')]
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Table 3-4 Summary of the Variation of Measured Elastic Constants
Table 3-4 Summary of the Variation of Measured Elastic Constants „continued“

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Note: The data represents variations in elastic constants at different depths. The values are given in a format that may include different units or numerical values indicative of the elastic properties of materials at these depths.
3.5 SENSITIVITY OF THE TRANSVERSELY ISOTROPIC PARAMETERS

To show the important effects of rock anisotropy behavior on the response of the laterally loaded drilled shaft, an actual lateral load test at Dayton documented in SJN 134137 by Nusairat et al. (2006) was studied. For this test, the Young modulus of the tested drilled shaft is estimated to be 4000 ksi with Poisson’s ratio of 0.25. The test shaft is 6 ft in diameter with 18 ft rock socket length. The rock is classified as Ohio shale with unconfined compressive strength for intact core shale as 5668 psi, GSI is 40, the unit weight of the rock mass is estimated as 0.038 pci. The elastic modulus of the gray shale was determined to be 590 ksi from unconfined compression test. One direct shear test was performed on a cored gray shale sample and resulted in the residual angle of internal friction of 24º along a bedding plane. By using the software RocLab (Rocscience, 2006) in which Hoek-Brown (Hoek, et al., 2002) rock strength criterion was used, the Mohr-Coulomb cohesion, $C_r$, and friction angle, $\phi'$, of rock mass are estimated as 200 psi and 23º, respectively. The elastic modulus of rock mass $E_m$ is also found to be 95 ksi, using the RocLab, which employs the Hoek et al. (2002) empirical correlation equation based on $q_o$ of intact rock core and rock type. The shale is assumed transversely isotropic, with parameters estimated from the rock mass properties. Three different sets of parameters are used.

The first analysis is carried out by assuming the modulus of elasticity in the plane of transversely isotropy to be equal to the rock mass modulus ($E=E_m=95$ ksi), and the modulus of elasticity in the direction perpendicular to the plane of transverse isotropy is equal to one-tenth of the rock mass modulus ($E'=9.5$ ksi). A typical value of Poisson’s
ratio is used ($\nu = 0.3$, $\nu' = 0.15$). The empirical Equation (3-3) is used to estimate the shear modulus ($G'=5.2$ ksi). Based on these parameters, the analysis is then carried out for different orientation of plane of transverse isotropy ($\beta=-45^\circ$, $0^\circ$, $45^\circ$, and $90^\circ$). The estimated $K_i$ for these parameters are summarized in Table 3-5. It can be seen that the orientation of the plane of transverse isotropy in reference to the vertical axis of the drilled shaft can exert a sinusoidal effect on $K_i$.

The second analysis is carried out for different $E/E'$ ratios (15, 10, 5, and 1), while keeping other parameters constant as follows: $E'=9.5$ ksi, $\nu = 0.3$, $\nu' = 0.15$, and $\beta=0$. The estimated $K_i$ for these parameters are summarized in Table 3-6. It is apparent that the degree of anisotropy, as represented by the ratio $E/E'$, can exert significant effects on the value of $K_i$, as the degree of anisotropy increases, the $K_i$ value increases.

For the $K_i$ values in Table 3-5 and Table 3-6, the load deflection curve and the maximum moment vs. load curve are generated using the elastic numerical solution given in SJN 134137, which is based on subgrade reaction theory. Figure 3-11 and Figure 3-12 show the effects of the orientation of plane of transverse isotropy on the deflections and moments of the drilled shaft in the elastic range. Figure 3-13 and Figure 3-14 show the significant effects of degree of anisotropy ($E/E'$) on the top deflection and the maximum moment of the laterally loaded drilled shaft. These analysis results clearly demonstrate the importance of characterizing the weathered rock using the transversely isotropic elastic parameters.

The last analysis was conducted for comparison purposes. For this analysis, $E/E'$ ratio
is assumed to be five, while the other parameters are kept as follows: $E' = 9.5$ ksi, $v = 0.3$, $v' = 0.15$, and $\beta = 0$. The elastic numerical solution is used again to predict the load deflection curve and to calculate the maximum moment. The predicted load-deflection curve at the shaft head and the load-moment curve are compared with the measured in Figure 3-15 and Figure 3-16. It can be seen by focusing on the elastic response of the drilled shaft (small load levels), that the predicted deflections and maximum moments are larger than the measured values. In general, a good agreement between the measured and the predicted can be observed.

Table 3-5  Effect of $\beta$ on $K_i$

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Table 3-6  Effect of rock anisotropy on $K_i$

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Figure 3-11 Effects of $\beta$ on the shaft head deflections

Figure 3-12 Effects of $\beta$ on the maximum moment
Figure 3-13 Effects of $E/E'$ on the shaft head deflections

Figure 3-14 Effects of $E/E'$ on the maximum moment
Figure 3-15 Comparison of load-deflection of test shaft at Dayton load test

Figure 3-16 Comparison of the maximum moment of test shaft at Dayton load test
3.6 SUMMARY AND CONCLUSIONS

A series of design charts for estimating the initial tangent to the p-y curve for transversely isotropic elastic rock were developed by utilizing the results of 3D FE analysis of a laterally loaded drilled shaft socketed into a transversely isotropic rock.

The importance of the initial tangent to p-y curve \( K_i \) in predicting the response of the laterally loaded drilled shaft, and the sensitivity of \( K_i \) to the transversely isotropic rock properties is investigated using a data set from a full-scale lateral load test. The significant effects of rock anisotropy on \( K_i \) are clearly demonstrated from this investigation.

Evaluations based on comparisons between the predicted and measured responses of a full-scale lateral load test on fully instrumented drilled shafts have shown the importance of having capability to reflect the anisotropy in predicting both deflection and moment in the drilled shaft under lateral load. Specific observations regarding the effect of anisotropy can be summarized below.

- It can be concluded that as the positive orientation of the plane of transverse isotropy in reference to the vertical axis of the drilled shaft increases, the drilled shaft head deflection increases. In addition, it can be concluded that the drilled shaft head deflection, when the orientation value is negative, is greater than the deflection when it is positive.

- It can be concluded that the larger the degree of anisotropy, as represented by the ratio \( E/E' \), the smaller the drilled shaft head deflection and the smaller the maximum moment.
CHAPTER IV: EQUIVALENT TRANSVERSELY ISOTROPIC MODEL FOR JOINTED ROCK

4.1 INTRODUCTION

In many engineering problems (e.g., foundations, dams, tunneling, or underground excavation), the ability to accurately determine the deformation behavior of jointed rock mass is very important, especially when the allowable displacements dominate the design criterion. The deformation behavior of a rock mass is greatly influenced by the presence of the discontinuities due to either the preferred orientation of the mineral grains, or the presence of bedding, stratification, layering, and jointing. Thus, the deformation behavior of most rocks is usually considered anisotropic.

Transversely isotropy is generally assumed to describe the deformation behavior of most of the jointed rock, considering that most of the joints and discontinuities are parallel to each other. Most foliated metamorphic rocks, such as schist, slates, gneisses, and phyllites, contain fabric with preferentially parallel arrangements of flat or long minerals (Tien, 2001). To achieve a realistic description of the stress-strain behavior of a jointed rock mass, it is desirable and of practical interests to develop an equivalent transversely isotropic homogeneous model for the rock mass with parallel joints.

Although many studies have been undertaken to take into account of joints effect on rock mass deformation behavior (e.g., Singh, 1973; Amadei, 1981; Gerrard, 1982; Wei, 1986; Wittke, 1990; and Sitharam, 2001), some of these models are complicated to apply due to the large number of required input parameters. Some of these models are empirical
in nature and derived on the basis of statistical and regression analysis of measurement data. Moreover, in some models, the discontinuities were assumed to have a negligible thickness, such that the Poisson’s effect of the joint filling material on the deformation behavior of a rock mass during loading was neglected.

In this chapter, the FE modeling of jointed rock masses was carried out to derive a set of transversely isotropic elastic constants as a function of both intact rock properties and joint material properties. An equivalent transversely isotropic homogeneous model was developed to represent a rock mass containing distinct joints. Specifically, the derived equivalent transversely isotropic elastic properties include the effects of joint spacing, joint thickness, and the Poisson’s effect of the joint filling, together with intact rock and joint filling material properties.

It should be mentioned that the developed equivalent transversely isotropic elastic constants were based on the following simplifying assumptions: the orientation of the joints are assumed to be known and that these joints are parallel to each other. These assumptions may pose limitations on the applicability of the model; nevertheless, they are reasonable since orientation and spacing can be determined and joints are often parallel.

4.2 LITERATURE REVIEW

Due to the complex geometry of the jointed rock, a rigorous estimation of its deformability is often difficult and challenging. One method to deal with this problem is to treat the jointed rock as an equivalent continuum. Various researchers have adopted this method using different techniques. For example, Sitharam (2001) developed an
equivalent continuum for a jointed rock mass, in which the effect of joints was taken into account by a joint factor, accounting for the joint orientation, joint frequency, and joint strength. The equivalent modulus was given as follow.

\[
E_j = E_i \left( 0.035 + 0.879 \exp \left( -\frac{J_f}{92.69} \right) \right) \quad (4-1)
\]

\[
J_f = \frac{J_n}{nr} \quad (4-2)
\]

where \(E_j\) is the tangent modulus of jointed rock, \(E_i\) is the tangent modulus of intact rock, \(J_f\) is the joint factor based on experimental data and statistical fitting for \(E_j\), \(J_n\) is the number of joints per meter depth, \(n\) is the inclination parameter depending on the orientation of the joint, \(r\) is the roughness or joint strength parameter dependent upon the joint condition.

Gerrard (1982) also used an equivalent continuum approach to describe the elastic deformation properties of a jointed rock mass. Equivalent homogeneous orthorhombic material properties were derived by expressing the compliance of an element as the sum of the compliances of the intact rock and those of the individual joint sets. Amadei and Goodman (1981) developed an equivalent continuum model to describe the jointed rock as a function of the intact rock properties and the normal and shear stiffness of the joints as follows. However, their derivations were based on the assumption that the joints have a negligible thickness,
\[ \frac{1}{E_i^*} = \frac{1}{E_i} + \frac{1}{k_n S_i} \]  

(4-3)

\[ \frac{1}{G_{ij}^*} = \frac{1}{G_{ij}} + \frac{1}{k_s S_i} + \frac{1}{k_s S_j} \]  

(4-4)

where \( E_i^* \) is the equivalent Young’s modulus, \( G_{ij}^* \) is the equivalent shear modulus, \( k_n \) and \( k_s \) are the normal and shear stiffness, respectively, and \( S \) is the joint spacing.

Wittke (1990) derived the equivalent five elastic constants for the jointed rock mass with the aid of volumetric content of the intact rock (\( \alpha \)) and the filling (\( \beta \)). He assumed that the intact rock and the filling material would deform completely independent of each other. Furthermore, the discontinuities in the rock mass were assumed to be very thin. The five transversely isotropic elastic constants are given as follows.

\[ E = \alpha E_{\text{rock}} \]  

(4-5)

\[ E' = \frac{1}{\left( \frac{\alpha E_{\text{rock}}}{E_{\text{rock}}} + \frac{\beta}{E_{\text{filling}}} \right)} \]  

(4-6)

\[ \nu = \nu_{\text{rock}} \]  

(4-7)

\[ \nu' = \frac{E_{\text{filling}} \nu_{\text{rock}}}{\alpha E_{\text{filling}} + \beta E_{\text{rock}}} \]  

(4-8)

\[ G = \alpha G_{\text{rock}} \]  

(4-9)

where \( E_{\text{rock}} \), \( G_{\text{rock}} \), and \( \nu_{\text{rock}} \) are the young’s modulus, shear modulus, and the Poisson’s ratio of the intact rock, respectively. \( E_{\text{filling}} \) is the young’s modulus for the filling.
materials. As will be shown later in this chapter, the proposed model is different from Wittke model as the thickness and Poisson’s effect of joints are included in the proposed new model.

4.3 EQUIVALENT HOMOGENEOUS MODEL

The basic concept of the proposed model is to model the jointed rock mass as an equivalent transversely isotropic linear elastic continuum. This means that the elastic constants must be chosen so that the shear, lateral, and normal deformations of the equivalent continuum are equal to those of the jointed rock under the applied loads.

For a jointed rock where both the intact rock and the discontinuity filling material are assumed to behave purely isotropic elastic, the equivalent values of the elastic constants for such a rock mass may be determined with the aid of volumetric content of the intact rock \( \alpha \) and the filling material \( \beta \). For the joint spacing \( d_1, d_2, \ldots d_n \), and joint thickness \( t_{F1}, t_{F2}, \ldots t_{Fn} \) shown in Figure 4-1, the average joint spacing \( \bar{d} \) and the average thickness \( \bar{t}_F \) is calculated as follows.

\[
\bar{t}_F = \frac{1}{n} \sum_{i=1}^{n} t_{Fi} \quad (4-10)
\]

\[
\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i \quad (4-11)
\]

From which \( \alpha \) and \( \beta \) can be calculated as
\[ \alpha = \frac{d}{t_F + d} \]  

(4-12)

\[ \beta = \frac{t_F}{t_F + d} = (1 - \alpha) \]  

(4-13)

Figure 4-1 Rock block with three discontinuities

In the next section, the behavior of a rock block with different discontinuities properties under three different types of loading is investigated.

- The behaviour of a rock block under uniaxial compressive stress perpendicular to the bedding.

For this type of loading shown in Figure 4-2(a), the mean uniaxial strain could be calculated as:

\[ \varepsilon_x = \alpha \varepsilon_{z(\text{rock})} + \beta \varepsilon_{z(\text{filling})} = \alpha \frac{\sigma_x}{E_{\text{rock}}} + \beta \frac{\sigma_x}{E_{\text{filling}}} \]  

(4-14)

However, this equation is valid only if the intact rock and the filling material deform completely independent of each other. This may occur if the discontinuities are very thin.
or the Poisson’s ratio of the discontinuity filling is very small. Since elastic behavior is pre-supposed, a lateral strain with the form depicted in Figure 4-3 will arise, if Poisson’s effect of filling material is significant which in turn will affect the uniaxial strain. To study the effect of this bulging phenomenon on the mean uniaxial strain, 3-D FE parametric analysis was performed. Factors such as filling Poisson’s ratio ($\nu_{\text{filling}}$), discontinuity filling volumetric content ($\beta$), and the discontinuity spacing were varied in the study.

For the spacing effect, a one meter length cubic rock block with a fixed discontinuity filling volumetric content ($\beta=0.03$) was modeled using ANSYS FE program for three different spacing as shown in Figure 4-4. A uniaxial compressive stress of $1\times10^3$ kPa was applied at the top of the block. The computational results of the mean uniaxial strain shown in Figure 4-5 clearly indicate that for a constant $\beta$, the discontinuity spacing exerts no effect on the mean uniaxial strain. The mean strains were found to be 4.94x$10^{-4}$, 4.91x$10^{-4}$, and 4.90x$10^{-4}$ for the model with one, two, and three discontinuities, respectively.

The jointed model shown in Figure 4-4(c) (i.e. 3 joints in the cubical) was analyzed to study the effect of the filling volumetric content ($\beta$) and Poisson’s ratio ($\nu_{\text{filling}}$) on the mean axial strain. $\nu_{\text{filling}}$ is varied from 0.001 to 0.49 while five different values of $\beta$ (0.003, 0.015, 0.01, 0.05, 0.1, and 0.2) were used. The ratios ($\eta_u$) between the uniaxial strains calculated using Equation (4-14) and the mean uniaxial strains determined from the FE analysis are plotted in Figure 4-6.
The mean uniaxial strain could be expressed as:

\[ \varepsilon_z = \frac{\alpha \sigma_z}{E_{\text{Rock}}} + \beta \frac{\sigma_z}{E_{\text{filling}}} \]

But

\[ \varepsilon_z = \frac{\sigma_z}{E_{\text{equ}}} \]

from Equations (4-15) and (4-16), the equivalent Young’s modulus could be derived as:

\[ \frac{1}{E_{\text{equ}}} = \frac{1}{\eta_u} \left( \frac{\alpha}{E_{\text{Rock}}} + \frac{\beta}{E_{\text{filling}}} \right) \]

Because the filling is so thin, and the Young’s modulus of the intact rock is generally considered larger than the Young’s modulus of the filling, it is expected that the lateral strain of the jointed rock block to be approximately equal to the lateral strain in the intact rock.

\[ \varepsilon_x = V_{\text{Rock}} \varepsilon_z(\text{Rock}) = V_{\text{Rock}} \frac{\sigma_z}{E_{\text{Rock}}} \]

To validate equation (4-18), the same 3-D FE model shown in Figure 4-4(c) was used for the FE analysis of the lateral strain. Figure 4-7 shows the lateral strain occurred in the rock block due to the uniaxial stress. The effect of the Poisson’s ratio of the filling material is clearly seen; therefore, a parametric study was performed to quantify this effect.
The Poisson’s ratio of the filling material ($\nu_{\text{filling}}$) is varied from 0.05 to 0.45 and four different values of $\beta$ (0.006, 0.03, 0.09, and 0.2) are used. The ratios ($\eta$) between the integration of the lateral strain along the block height determined from the FE analysis and the lateral strains calculated using Equation (4-18) are shown in Figure 4-8.

The mean lateral strain could be expressed as:

$$
\varepsilon_x = \eta \left( \nu_{\text{Rock}} \frac{\sigma_z}{E_{\text{Rock}}} \right)
$$

(4-19)

but

$$
\varepsilon_x = \nu_{\text{equ}} \varepsilon_z = \nu_{\text{equ}} \frac{\alpha \frac{\sigma_z}{E_{\text{Rock}}} + \beta \frac{\sigma_z}{E_{\text{filling}}}}{\eta_a}
$$

(4-20)

from Equations (21) and (22) the equivalent Poisson’s ratio could be derived as:

$$
\nu_{\text{equ}} = \eta_a \eta_i \frac{\nu_{\text{rock}}}{\alpha + \beta \frac{E_{\text{rock}}}{E_{\text{filling}}}}
$$

(4-21)

Thus elastic stress-strain behavior of the rock block with parallel joints under uniaxial compression may be simulated by the behavior of a homogeneous rock block with the equivalent modulus $E_{\text{equ}}$ and Poisson’s ratio $\nu_{\text{equ}}$. 

IV-9
Figure 4-2 Jointed rock block under different types of loading; (a) Uniaxial Loading perpendicular to the bedding, (b) Uniaxial Loading parallel to the bedding, and (c) shear loading parallel to the bedding

Figure 4-3 Effect of discontinuities properties on the axial strain
Figure 4-4 FE model for cubic rock block with three different spacing.

\[ E_{\text{rock}} = 5 \times 10^6 \text{kPa}, \nu_{\text{rock}} = 0.25, E_{\text{filling}} = 0.1 \times 10^6 \text{kPa}, \text{and} \nu_{\text{filling}} = 0.05 \]

Figure 4-5 Uniaxial strain for a rock block under uniaxial stress.
Figure 4-6 Effect of $\nu_{\text{filling}}$ and $\beta$ on the uniaxial strain.

Figure 4-7 Lateral strain in cubic rock block with $\beta=0.09$. 
The behaviour of a rock block under uniaxial compressive stress parallel to the bedding as shown in Figure 4-2(b).

Since the Young’s modulus of the intact rock is larger than the Young’s modulus of the filling, the filling material does not play any role in transmitting the stress; i.e., \( \sigma_{\text{filling}} = 0.0 \). Therefore, the stress will be redistributed resulting in an increase in the stress in the layer of intact rock by a factor of \( 1/\alpha \). The strain \( \varepsilon_x \) can be determined as:

\[
\varepsilon_x = \frac{\sigma_x}{\alpha E_{\text{Rock}}} \tag{4-22}
\]

The FE model shown in Figure 4-9 was used to validate the Equation (4-22) under a uniaxial stress, \( \sigma_x = 1 \times 10^3 \) kPa. The lateral strain was determined to be \( 0.210 \times 10^{-3} \) as shown in Figure 4-10, which is very close to the lateral strain of \( 0.2128 \times 10^{-3} \) calculated.
using Equation (4-22), but

\[ \varepsilon_x = \frac{\sigma_x}{E_{equ}} \quad (4-23) \]

The Young’s modulus \( E_{equ} \) can now be derived as:

\[ E_{equ} = \alpha E_{rock} \quad (4-24) \]

A similar argument applies to the lateral strain in the y direction, so that

\[ \nu_{equ} = \frac{\varepsilon_y}{\varepsilon_x} = \frac{\nu_{rock}\varepsilon_x(rock)}{\varepsilon_x(rock)} = \nu_{rock} \quad (4-25) \]

Figure 4-9 Rock block with three discontinuities (\( \beta=0.06, E_{rock}=5000\times10^6 \) Pa, \( E_{filling}=100\times10^6 \) Pa, \( \nu_{rock}=0.25, \nu_{filling}=0.45 \))
Figure 4-10 Behaviour of jointed rock block under lateral stress

- The behaviour of a rock block under shear loading parallel to the bedding.

For the shear loading shown in Figure 4-2(c) and (d), the shear strain will take place as follows.

\[
\gamma = \alpha \gamma_{\text{Rock}} + \beta \gamma_{\text{filling}} = \alpha \frac{\tau}{G_{\text{rock}}} + \beta \frac{\tau}{G_{\text{filling}}} \tag{4-26}
\]

Again, to validate Equation (4-26), the FE model shown in Figure 4-9 was analyzed under a shear stress, \(\tau = 1\times10^3\) kPa. The shear strain was determined to be 0.00221 as shown in Figure 4-11, which is exactly equal to the mean shear strain (0.002211) calculated using Equation (4-26).

but

\[
\gamma = \frac{\tau}{G_{\text{equiv}}} \tag{4-27}
\]
from (3-28) and (3-29), the equivalent shear modulus could be derived as:

\[
\frac{1}{G_{\text{equ}}} = \frac{\alpha}{G_{\text{rock}}} + \frac{\beta}{G_{\text{filling}}} \quad (4-28)
\]

Figure 4-11 Rock block behaviour under shear stress.

4.3.1 Summary of Derivations

Using the definitions of the elastic constants for the case of transverse isotropy given in Figure 2-9, along with the previous derivations, one can derive the five transversely isotropic elastic constants.

From step 1, it can be concluded that the derived elastic constants $E_{\text{equ}}$ and $\nu_{\text{equ}}$ are equivalent to $E'$ and $\nu'$ defined in Figure 2-9, while $E_{\text{equ}}$, $\nu_{\text{equ}}$ derived in step 2 are equivalent to $E$ and $\nu$ defined in Figure 2-9. The fifth elastic constant, $G'$, is equivalent to $G_{\text{equ}}$ derived in step 3.
The elastic constitutive equation for the equivalent transversely isotropic model of
jointed rock block is given by

$$\{ \varepsilon \} = [\mathbf{r}]^T [\mathbf{D}']^{-1} [\mathbf{r}] \sigma \} \quad (4-29)$$

where

$$[\mathbf{D}']^{-1} = \begin{bmatrix}
\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu'}{E'} & 0 & 0 & 0 \\
-\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu'}{E'} & 0 & 0 & 0 \\
-\frac{\nu'}{E'} & -\frac{\nu'}{E'} & \frac{1}{E'} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G'} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G'}
\end{bmatrix} \quad (4-30)$$

$$[\mathbf{T}] = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
\cos^2 \theta & 0 & \sin^2 \theta & 0 & 0 & -2 \sin \theta \cos \theta \\
\sin^2 \theta & 0 & \cos^2 \theta & 0 & 0 & 2 \sin \theta \cos \theta \\
0 & 0 & 0 & \cos \theta & -\sin \theta & 0 \\
-\sin \theta \cos \theta & 0 & \sin \theta \cos \theta & 0 & 0 & \sin^2 \theta - \cos^2 \theta \\
0 & 0 & 0 & -\sin \theta & -\cos \theta & 0
\end{bmatrix} \quad (4-31)$$

$$\theta = \text{orientation of plane of transversely isotropic}$$

$$E = \alpha E_{\text{rock}} \quad (4-32)$$

$$\frac{1}{E'} = \frac{1}{\eta_u} \left( \frac{\alpha}{E_{\text{rock}}} + \frac{\beta}{E_{\text{filling}}} \right) \quad (4-33)$$

$$\nu = \nu_{\text{rock}} \quad (4-34)$$
\[ ν' = \eta_u \eta_l \left( \frac{ν_{\text{rock}}}{α + β \frac{E_{\text{rock}}}{E_{\text{filling}}}} \right) \]  
(4-35)

\[ \frac{1}{G'} = \frac{α}{G_{\text{rock}}} + \frac{β}{G_{\text{filling}}} \]  
(4-36)

A nonlinear regression analysis on the data points of FE analysis results shown in Figure 4-6 and Figure 4-8 was carried out using SPSS statistical program. Two equations for predicting \( \eta_u \) and \( \eta_l \) are fitted to match \( \eta_u \) and \( \eta_l \) values obtained from FE parametric study. These empirical equations are given below.

\[ \eta_u = \begin{cases} 
\frac{1}{0.97 - 3.8ν_{\text{filling}}^{2.7}} & \frac{0.36}{1 - 72.1e^{-8ν_{\text{filling}}}} \ln β \quad \text{For } β > 0.003 \text{ or } ν_{\text{filling}} > 0.001 \\ 
1 & \text{Otherwise} 
\end{cases} \]  
(4-37)

\[ \eta_l = \begin{cases} 
e^{7.11ν_{\text{filling}}} & \text{For } β > 0.003 \text{ or } ν_{\text{filling}} > 0.001 \\ 
1 & \text{Otherwise} 
\end{cases} \]  
(4-38)

4.3.2 Model Verification

In order to validate the derived equivalent elastic constants, the model with distinct joints shown in Figure 4-9 and the homogeneous continuum model shown in Figure 4-12 were analyzed using ANSYS under three different types of loading. The model in Figure 4-9 is a rock block with three discontinuities (\( β=0.06, E_{\text{rock}}= 5000×10^6 \text{ Pa}, E_{\text{filling}}=100×10^6 \text{ Pa}, ν_{\text{rock}}=0.25, ν_{\text{filling}}=0.45 \)), while the model in Figure 4-12 is a rock block with the equivalent transversely isotropic properties derived from the proposed method (\( E=4700×10^6 \text{ Pa}, E'=2696.7×10^6 \text{ Pa}, ν=0.25, ν'=0.169, G'=452.7×10^6 \text{ Pa} \)). The
deformations of the two models under the different loadings are in good agreement as it can be clearly seen in Figure 4-13, thus providing validation of the proposed model.

Figure 4-12 Equivalent transversely isotropic model (E=4700×10^6 Pa, E’=2696.7×10^6 Pa, ν=0.25, ν’=0.169, G’=452.7×10^6 Pa)

4.4 SUMMARY AND CONCLUSIONS

In this chapter, the elastic deformation behavior of a jointed rock mass with parallel joints has been modeled successfully as an equivalent transversely isotropic elastic homogeneous continuum. The properties of this equivalent continuum model are derived and expressed as a function of the intact rock and joint filling material properties, as well as volumetric content of the filling materials. This proposed model is valid for a jointed rock with multi parallel discontinuities with any joint thicknesses. Furthermore, the effect of Poisson’s ratio of joint filling material is taken into account as well. The validation study has shown that the deformation behavior of a rock block with the derived equivalent elastic properties under different types of loading is very close to the deformation behavior of the jointed rock block.
Figure 4-13 Comparison of deformation behaviour of (a) equivalent transversely isotropic continuum, and (b) rock with three distinct joints

Under Axial Stress ($\sigma_z = 1 \times 10^3$ kPa)

Under Lateral Stress ($\sigma_x = 1 \times 10^3$ kPa)

Under Shear Stress ($\tau_x = 1 \times 10^3$ kPa)
CHAPTER V: ULTIMATE SIDE SHEAR RESISTANCE OF ROCK-SOCKETED DRILLED SHAFT

5.1 INTRODUCTION

Rock-socketed drilled shafts are widely used as a foundation to support large structural loads. For the laterally loaded drilled shaft socketed into rock, the lateral rock resistance is derived from two components: side shear resistance and front resistance as shown in Figure 5-1. Although the side shear resistance is relatively small compared to the front resistance, it will nevertheless affect the response of the drilled shaft under lateral load, particularly at a small shaft deflection.

![Figure 5-1](image-url)  
Figure 5-1 The mobilized normal force and shear resistance at peak lateral load (from Zhang and Einstein, 2000)

Kong et al. (2006) has shown that the development of the side shear resistance of a laterally loaded rock-socketed drilled shaft is significantly influenced by the roughness of the interface between the drilled shaft and the surrounding rock mass, in addition to the properties of rock mass and drilled shaft. The construction of drilled shaft in rock
generally involves drilling of a socket in the rock and placing concrete in the socket to form a rock socketed drilled shaft. The sides of the socket as drilled generally exhibit some degree of roughness, and consequently when the concrete is placed, the interface between the concrete and the rock mass takes the form of a rough interface. When the drilled shaft is loaded, the side shear component of the total resistance develops because of the reaction of this rough interface to the applied shear forces.

Empirical correlations, proposed by many researchers, between the ultimate side shear resistance and the unconfined compressive strength of an intact rock core have been widely used for estimating the ultimate side shear resistance of a drilled shaft socketed in rock. In general, these empirical correlations are expressed as.

\[ \tau_{ul} = \alpha \sigma_c^\beta \]  

(5-1)

\( \tau_{ul} \) = ultimate side shear resistance (MPa)

\( \sigma_c \) = unconfined compression strength of an intact rock (MPa)

\( \alpha \) and \( \beta \) = regression constants determined empirically from the load tests

The empirical constants \( \alpha \) and \( \beta \), proposed by a number of researchers were summarized in Table 2-2. Most of these empirical relationships were developed from specific and limited data sets, which may not be readily extended for application to other rock formations different from that of the original data.

It is evident from the studies of O’Neill et al. (1995) and Seol and Jeong (2007)
summarized in section 2.4 that for a given intact rock core strength, very large variations in side shear resistance can be obtained from these empirical equations. Estimation of ultimate side resistance based on the empirical correlation equations that only involve one single variable (e.g., the intact rock core strength or the rock mass modulus) could be very conservative. To improve the accuracy of the empirical equations for predicting the ultimate side shear resistance, it appears that the empirical equations need to expand to include other important variables as well.

As can be seen in Table 2-2, the empirical constants $\alpha$ and $\beta$ vary greatly, depending upon the original data from which these constants were derived. It seems that there is an inherent deficiency in adopting a single variable type of correlation equation such as Equation (5-1), in which $\sigma_c$ is the sole correlation parameter. A more versatile and accurate predictive equation could be developed by incorporating multiple influencing factors in the mathematical equation.

In this chapter, a series of FE simulations was undertaken to quantify the effects of various influencing factors on the ultimate side shear resistance between the rock and drilled shaft, including the interface strength parameters, the modulus of the drilled shaft and rock mass, and the drilled shaft geometry. The FE simulation involves applying a torque at the top of the drilled shaft to mobilize the lateral side shear resistance along the drilled shaft length in pure shear. Based on the results of the FE parametric study, an empirical equation was developed that can be used to estimate the ultimate side shear resistance as a function of multiple influencing factors. As an evaluation of the proposed empirical equation, comparisons between the predictions of the developed solution and
the actual measured field data are presented.

5.2 THEORETICAL MODEL FOR INTERFACE BEHAVIOR

The shear strength of the interface between the drilled shaft and rock mass is assumed to follow Mohr-Coulomb strength criterion.

\[ \tau = C_{ad} + \sigma_n \mu \]  \hspace{1cm} (5-2)

where \( \sigma_n \) is the normal stress at the interface, \( C_{ad} \) is the interface “adhesion”, and \( \mu \) is the coefficient of friction of the interface.

It has been shown by Williams et al. (1980) and Johnston et al. (1987) that the stress normal to the interface is directly proportional to the stiffness of the surrounding rock. Furthermore, the stress normal to the interface is dependent on the overburden pressure. Malmgren et al. (2004) pointed out that the interface adhesion is mainly dependent on the roughness of the rock surface and the mineral composition of the rock. Therefore, the ultimate side shear resistance could be related to the interface strength through an empirical coefficient \( k \).

\[ \tau_{ult} \propto (C_{ad} + \sigma_v' \mu) \]  \hspace{1cm} (5-3)

\[ \tau_{ult} = k(C_{ad} + \sigma_v' \mu) \]  \hspace{1cm} (5-4)

where \( \sigma_v' \) is the overburden pressure at the depth of the drilled shaft that is of interest,
and \( k \) is a non-dimensional empirical coefficient that takes into account the effect of drilled shaft and rock properties on the ultimate side shear resistance.

5.3 FINITE ELEMENT STUDY

A methodology for deriving an empirical relationship for estimating the ultimate side shear resistance is developed herein by employing the results of FE parametric study of a 3-D model of a drilled shaft socketed into rock. Figure 5-2 shows a schematic diagram of the model, together with the model parameters to be varied in the parametric study. In this diagram, \( L \) is the drilled shaft length embedded in rock, \( R \) is the drilled shaft radius, \( E_m \) is the rock mass modulus, \( G_s \) is the drilled shaft shear modulus, \( C_{ad} \) is the interface adhesion, and \( \mu \) is the coefficient of friction for the interface between the drilled shaft and the rock.

5.3.1 3-D FE Modelling

Figure 5-3 shows a FE mesh of a single drilled shaft-rock system generated using the computer program ANSYS. The cylindrical drilled shaft and the rock are modeled using the 8-node brick elements (Solid 65) in the ANSYS element library. The bottom and the outer boundary of rock mass is fixed. The interaction between drilled shaft and rock mass is modeled using a surface based contact. The shaft surface is treated as the contact surface, while the rock surface is taken as the target surface. The frictional interaction is simulated using linear Coulomb friction theory, which requires an input of a coefficient of friction and interface adhesion.
5.3.2 Constitutive Models

The drilled shaft is modeled as an elastic material characterized by Young’s modulus and Poisson’s ratio. The rock mass is modeled as a Drucker-Prager Model. According to ANSYS User’s Manual (2007), the Drucker-Prager model is an elastic-perfectly plastic model, in which the yield surface does not change with progressive yielding of the rock mass. The main input parameters for the Drucker-Prager model include $E_m$ (elastic modulus of rock mass), the cohesion value, $C_r$ (must be greater than zero), the angle of internal friction, $\varphi'$, and the dilatancy angle. The dilatancy angle is used to indicate the relationship between the volume change and yielding.

Figure 5-2 3D Model for drilled shaft socketed into rock under applied torque
5.3.3 FE Analysis Simulation

A torque is applied monotonically at the top of the drilled shaft until the interface failure along the entire shaft length occurs. The angular rotation of the drilled shaft top is calculated from the linear movement of the drilled shaft’s circumference at the head of the drilled shaft. A typical plot of the applied torque vs. drilled shaft head rotation is shown in Figure 5-4. Using the notation previously defined in Figure 5-2, Equation (5-5) could be used to calculate the ultimate side shear resistance. In this equation, $T_f$ is the maximum torque applied at which failure at the drilled shaft interface occurs.

$$
\tau_{ult} = \frac{T_f}{2\pi LR^2}
$$

(5-5)
By equating Equations (5-4) and (5-5), the non-dimensional value, \( k \), could be determined as follows.

\[
k = \frac{T_f}{2\pi L^2 (C_{ad} + \sigma_v' \mu)}
\]

(5-6)

In the parametric study, the effect of the selected model parameters on the \( k \) value is analyzed. The selected model parameters include drilled shaft geometry represented by the ratio of the drilled shaft embedment length over the drilled shaft diameter (L/D), shear modulus of the drilled shaft (\( G_s \)), rock strength properties (\( C_r, \Phi \)), rock mass modulus (\( E_m \)), and the rock-shaft interface strength properties (\( C_{ad}, \mu \)). Table 5-1 provides a summary of the values of these selected model parameters in the parametric study. The goal here is to select a wide range so that the trend between these parameters and the non-dimensional factor, \( k \), could be adequately captured.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Line Value*</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>0.3</td>
<td>0.1-1</td>
</tr>
<tr>
<td>( C_{ad} ) (Pa)</td>
<td>0</td>
<td>0-2.5 \times 10^6</td>
</tr>
<tr>
<td>( \gamma ) (N/m^3)</td>
<td>20000</td>
<td>15000-20000</td>
</tr>
<tr>
<td>( C_r ) (Pa)</td>
<td>1.5 \times 10^6</td>
<td>1 \times 10^4 - 5 \times 10^7</td>
</tr>
<tr>
<td>( \Phi' )</td>
<td>0</td>
<td>0-45</td>
</tr>
<tr>
<td>( E_m ) (Pa)</td>
<td>1.0 \times 10^{10}</td>
<td>1 \times 10^8 - 5 \times 10^{10}</td>
</tr>
<tr>
<td>( E_s ) (Pa)</td>
<td>1.0 \times 10^{10}</td>
<td>1 \times 10^7 - 1 \times 10^{11}</td>
</tr>
<tr>
<td>L (m)</td>
<td>2.25</td>
<td>1.5-40</td>
</tr>
<tr>
<td>D (m)</td>
<td>1.5</td>
<td>1-3</td>
</tr>
</tbody>
</table>

*Baseline value is the reference value in the parametric study, when only one parameter is varied while keeping other parameters at the baseline value.
5.4 FACTORS AFFECTING THE ULTIMATE SIDE SHEAR RESISTANCE

It has been shown by researchers (e.g. Kong et al., 2006) that the development of the side shear resistance of a rock-socketed drilled shaft is significantly influenced by the roughness of the interface between the drilled shaft and the surrounding rock mass. According to Dykeman et al. (1996), the interface roughness depends on the type of drilling technique used and the hardness of the rock.

In this FE parametric study, the variation of roughness was considered by varying the range of interface properties through the coefficient of friction and the adhesion of the interface. Figure 5-5 present the results of the multiple FE parametric study runs in which \( \mu \) is varied, while other parameters are kept at the baseline values. From the best-fit regression linear data fit, the slope (i.e., the k value) is found to be 9.7. Similar plots for varying the adhesion, or the overburden pressure, while keeping other parameters at the
reference baseline values are shown in Figure 5-6 and Figure 5-7, respectively. The corresponding best linear fit gives k value of 9.564 and 9.342, respectively. Combining all these sets of FE parametric study results in Figure 5-5 through Figure 5-7, one can obtain the best fit k value of 9.564 as shown in Figure 5-8.

5.4.1 Effect of Rock Mass Properties (Em, Cr, \( \Phi_r \))

The stiffness (or modulus \( E_m \)) of rock mass can exert significant effect on side shear resistance, as the stiffer the rock mass, the larger will be the shearing-induced normal stress at the interface. Thus, the effect of roughness is considerably greater for the rock with higher modulus than the rock with lower modulus. Figure 5-9 provides the parametric study results of the effect of \( E_m \) on the k value, while keeping the other FE model parameters at the baseline values. The plot in Figure 5-9 confirms that the k value can be linearly related to \( E_m \).

The effect of rock mass strength (\( C_r \) and \( \Phi' \)) on k values, as obtained from the FE parametric study, is plotted in Figure 5-10 and Figure 5-11. As it can be seen, an exponential curve could fit the relation between \( C_r \) and k. The FE parametric study results shown in Figure 5-11 suggest that variation of the friction angle \( \phi' \) of rock mass does not seem to affect the k value.

As reviewed earlier, the existing empirical equations for predicting the side shear resistance rely on one single variable (i.e., the uniaxial compressive strength). However, it is foreseeable that for a given uniaxial compressive strength of a typical rock core sample, the rock mass modulus may vary greatly depending on the degree of jointing.
Therefore, it is clearly evident that the empirical predictive equations should at least expand to include the effect of rock mass modulus.

5.4.2 Effect of Drilled Shaft Geometry and Shear Modulus

The effect of drilled shaft geometry has been investigated experimentally by Smith (1985). In his torque-testing model study, the ultimate side shear resistance of model shafts was observed to decrease with an increase in the L/D ratio, where L is the drilled shaft embedment length and D is the drilled shaft diameter. In the current FE parametric study, the effects of the (L/D) ratio on the side shear resistance is also investigated. As shown in Figure 5-12, the k value decreases with an increase in L/D when L/D is greater than 2. For shaft with L/D less than 2, the k value seems to be constant with the value of approximately 10. It is interesting to note that according to Carter and Kulhawy (1992), when L/D is greater than 2, the drilled shaft is considered as flexible based on the following proposed condition. Figure 5-13 shows the effect of the drilled shaft modulus on the k value.

\[
\frac{L}{D} \geq \left( \frac{E_s}{G^*} \right)^{2/7} \tag{5-7}
\]

\[
G^* = G_m \left( 1 + \frac{3}{4} \nu \right) \tag{5-8}
\]
Figure 5-5 Effect of coefficient of friction on $\tau_{ult}$

Figure 5-6 Effect of the interface adhesion on $\tau_{ult}$
Figure 5-7 Effect of the $\sigma'_v$ on $\tau_{ult}$

Figure 5-8 Relationship between the ultimate side shear resistance and the stress calculated using Equation (5-2)
Figure 5-9 Effect of rock mass modulus on $k$

Figure 5-10 Effect of rock cohesion, $C_r$ on $k$
Figure 5-11 Friction angle $\phi$ vs. $k$

$$\Phi = 32.612x^{-1.5492}$$
$$R^2 = 0.9809$$

Figure 5-12 Effect of drilled shaft geometry on $\tau_{ult}$
5.5 SUGGESTED EMPIRICAL RELATIONSHIPS

Based on the parametric study presented in the previous sections, the following observations may be drawn.

- The ultimate side shear resistance exhibits a linear relationship with $C_{ad} + \sigma_v \cdot \mu$ with the coefficient $k$;

- the coefficient $k$ is affected by the rock properties ($E_m, C_t$), drilled shaft geometry (L/D), and the shear modulus of the drilled shaft ($G_s$);

- $k$ value is linearly related to the modulus of rock modulus $E_m$;

- for a flexible shaft, the $k$ value is affected by L/D through a power relationship;
• one of the strength parameters for rock, that is the friction angle, does not seem to exert any significant effect on k value;

• an exponential curve could be used to fit the relationship between the shear modulus of the drilled shaft ($G_s$) and the cohesion of rock mass ($C_r$).

A nonlinear regression analysis on the results from the FE parametric study is carried out, using the SPSS program, to develop the empirical relationship in Equation (5-9) for estimating the $k$ value. The coefficient of determination is more than 0.97 for the goodness of fit shown in Figure 5-14. Using the developed Equation (5-9) for $k$, the ultimate side shear resistance could be calculated using Equation (5-4).

$$k = 2.39 \times 10^{-6} \eta_{L/D} \left( \frac{E_m}{p_a} \right) \left( \ln \left( \frac{C_r}{p_a} \right) + 2.61 \left( \ln \left( \frac{G_s}{p_a} \right) - 3.6 \right) \right) \quad \text{(5-9)}$$

where, $p_a$ is the atmospheric pressure in the same unit as other parameters

$$\eta_{L/D} = \begin{cases} 1 & \text{Rigid} \\ 3.43 \left( \frac{L}{D} \right)^{-1.55} & \text{Flexible} \end{cases} \quad \text{(5-10)}$$
Figure 5-14 Comparison of FEM predictions and empirical predictions for k value

5.6 VALIDATION OF THE EMPIRICAL EQUATION

To verify the accuracy of the developed empirical equations for predicting the ultimate side shear resistance, an effort was made to collect and compile test results from both laboratory and field load tests. The pertinent information of test results, including the properties of the test drilled shafts and the surrounded rock mass for eight cases, is provided in Table 5-2.

It should be noted that most parameters in Table 5-2 are extracted directly from the original literature. However, some parameters are interpreted by the authors based on the unconfined compression test results of intact rock cores and the interpreted Mohr-Coulomb cohesion and friction angle of the rock mass. The effects of joints, fillings, cracks, and other secondary structures of the rock were taken into account by using the Hoek-Brown (Hoek, et al., 2002) rock strength criterion, where $q_u$, $E_i$, and the Geological Strength Index (GSI) were the required input. Using the free software RocLab
(Rocscience, 2006), the Mohr-Coulomb cohesion, $C_r$, and the friction angle, $\phi$, of rock mass are estimated and summarized in Table 5-3. The elastic modulus of rock mass $E_m$ is also interpreted through the use of the RocLab software, which employs the Hoek et al. (2002) empirical correlation equation based on $q_u$ and rock type. Since the rock type in Patros et al. (1989) and Yang et al. (2004) are the same, the value of adhesion $C_{ad}$ is purposely selected to be the same. The unit weight is determined based on the rock type according to Goodman (1990).

For the interface roughness illustrated in Figure 5-15, Brady (2004) recommended the use of Equation (5-11) to calculate the sliding force, $S$, where $\Phi$ is the interface friction angle. By assuming the interface friction angle equal to $2/3\phi$, Equation (5-12) is recommended to calculate the coefficient of friction.

\[
\frac{S}{N} = \tan(\Phi + \psi) \tag{5-11}
\]

\[
\mu = \tan\left(\frac{2}{3}\phi + \psi\right) \tag{5-12}
\]

The input of the parameters for predicting $\tau_{ult}$ and the predicted values of $\tau_{ult}$ using the proposed equation are presented in Table 5-3. In order to assess the accuracy of the proposed empirical equation for predicting $\tau_{ult}$ in rock, a comparison between the predicted and measured values of $\tau_{ult}$ is given in Table 5-4, in which the relative error of each prediction calculated using Equation (5-13) is computed as follows.

\[
\text{Relative Error} \% = 100 \times \frac{\text{Abs}(\text{Measured - Predicted})}{\text{Measured}} \tag{5-13}
\]
Also presented in Table 5-4 are the assessments of the other predictive equations. It can be seen that the proposed empirical equation can provide conservative estimate for the ultimate side shear resistance. Furthermore, it can be seen from the mean values of the relative error presented in Table 5-4 that the proposed equation could predict the ultimate side shear resistance more accurate than the other empirical equations.

![Idealized surface roughness model](image)

**Figure 5-15 Idealized surface roughness model (After Brady, 2004)**

### 5.7 SUMMARY AND CONCLUSIONS

This chapter presents the results of a series of FE parametric analysis of rock socketed drilled shafts subjected to a torque at the head of the shaft. From a systematic analysis of the FE simulation results, together with the aid of a statistical regression analysis, an empirical equation was developed for predicting the mobilization of the ultimate side shear resistance in a rock socketed drilled shaft. A number of important parameters affecting the ultimate side shear resistance have been systematically examined by the FE parametric study. It was shown that the magnitude of the ultimate side shear resistance depends not only upon the strength parameters of rock mass, but also upon the strength properties of the interface between the shaft and the rock, the L/D ratio of the drilled shaft, and the modulus values of the drilled shaft and the rock mass. These FE analysis results suggest that the apparent scattering of predictions by the current existing empirical
equations, which are based solely on correlations with a single variable (i.e., intact rock core uniaxial unconfined compressive strength) is largely due to the lack of inclusion of other equally important parameters in the equation, such as the strength and modulus of rock mass, interface strength, L/D ratio, and shear modulus of the drilled shaft.

The developed empirical equation is validated with available load test data from both laboratory and field. The prediction by the proposed empirical equation is shown to be conservative but with improved accuracy compared with other existing empirical equations. From the statistical analysis shown in this chapter, the proposed empirical equation can improve our prediction capability for the ultimate side shear of a rock socket.
Table 5-2  Field and laboratory side shear resistance tests information

<table>
<thead>
<tr>
<th>Test</th>
<th>Rock Type</th>
<th>Interface Properties</th>
<th>Rock Properties</th>
<th>Soil Properties</th>
</tr>
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<tr>
<td>Load on soil</td>
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<td></td>
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</tbody>
</table>

Field Test Materials

- Field Test (2004)
- Field Test (1996)
- Field Test (1999)
- Field Test (1999)
- Field Test (2004)
- Field Test (1999)

Notes:
- V-22
- Table 5-2: Field and laboratory side shear resistance tests information

Where, PS=Perfectly Smooth Socket (ψ=0), S=Smooth Socket (ψ=5 degree), R=Rough Socket (ψ=30 degree)

<table>
<thead>
<tr>
<th>Test</th>
<th>Rock Type</th>
<th>Interface Properties</th>
<th>Rock Properties</th>
<th>Soil Properties</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>1st Test</td>
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<td>Field Test</td>
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<tr>
<td>Load on soil</td>
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</tr>
</tbody>
</table>

Field Test Materials

- Field Test (2004)
- Field Test (1996)
- Field Test (1999)
- Field Test (1999)
- Field Test (2004)
- Field Test (1999)

Notes:
- V-22
- Table 5-2: Field and laboratory side shear resistance tests information

Where, PS=Perfectly Smooth Socket (ψ=0), S=Smooth Socket (ψ=5 degree), R=Rough Socket (ψ=30 degree)
Table 5-3 Input data for the proposed equation estimated using the information given in Table 5-2

<table>
<thead>
<tr>
<th>Test</th>
<th>μ</th>
<th>( C_{ad} ) (Pa)</th>
<th>( C_r ) (Pa)</th>
<th>( E_m ) (Pa)</th>
<th>( G_s ) (Pa)</th>
<th>( \sigma' ) (at the middle depth) (Pa)</th>
<th>L/D</th>
<th>( \tau_{ult} ) (Pa)</th>
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<td>Partos (1989)</td>
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<td>1.90E+06</td>
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Table 5-4 Side shear resistance of drilled shafts, a comparison of experimental values and those predicted by empirical equations

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<th>Relative Error %</th>
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Mean % 58 113 89 139 186
CHAPTER VI: ULTIMATE LATERAL RESISTANCE OF TRANSVERSELY ISOTROPIC ROCK

6.1 INTRODUCTION

The accuracy of the p-y curve method in predicting the behavior of the drilled shaft under lateral load depends to a large extent on the accuracy and representativeness of the p-y curves used in the analysis. Regardless of the specific mathematical equations used for generating the pertinent p-y curves for the lateral shaft-rock interaction, there are two important parameters that regulate the shape of the p-y curve: the subgrade modulus or the initial tangent to the p-y curve ($K_i$) and the ultimate lateral resistance ($p_u$). The value of $K_i$ tends to affect the initial portion of the p-y curve; whereas, the $p_u$ represents the asymptotic value of $p$ at a large shaft deflection in the soil or rock that exhibits ductile behavior.

In previous attempts to formulate p-y curves for rock (e.g. Reese 1997; Gabr 1993; and SJN 134137 2006), the rock mass in general was treated as an isotropic medium. However, most rocks exhibit direction-dependent stress-strain and strength properties, it is therefore very important to identify the lateral load direction in relation to the dominant planes of anisotropy in rock mass. For example, the lateral capacity of a foundation loaded obliquely to the bedding of a rock mass may be less than one half of that of the foundation when the load is applied perpendicular or parallel to the bedding (Goodman, 1989). Thus, a method that takes into account the effect of the bedding orientation on the p-y curves for the laterally loaded drilled shafts would provide more reliable analysis.
results than those p-y curves that ignore the influence of anisotropy.

As described in chapter II, an empirical equation is presented in which $K_i$ can be expressed as a function of various influencing parameters, such as shaft radius ($R$), Young’s modulus of shaft ($E_p$), Poisson’s ratio of shaft ($v_p$), and five elastic constants of transversely isotropic rock ($E, E', G, v, v'$, and $\beta$). However, there is a lack of a detailed study on the effect of transversely isotropic behavior on the ultimate lateral resistance ($p_u$) for transversely isotropic rock. This chapter presents a pertinent method for determining the ultimate lateral resistance of drilled shafts in transversely isotropic rock mass. There are numerous researches (e.g. Reese 1974; To et al.2003; and Yang 2006) in the past supporting the formation of different types of ultimate failure modes in the soil medium where the drilled shaft is pushed laterally to the failure state in the soil. For example, a wedge type of failure occurs in soils at shallow depth. The effects of bedding plane orientation on the failure modes at both shallow and great depths remain to be investigated.

The objective of this chapter is to present a method for computing the ultimate lateral resistance, $p_u$ of a transversely isotropic rock at both shallow and great depths due to the laterally loaded drilled shaft. To achieve this objective, a series of 3-D FE model simulations of a laterally loaded drilled shaft socketed into jointed rock using ANSYS FE program were performed. The results of these FE simulations, in conjunction with theoretical mechanics derivations, lead to the development of semi-analytical equations to estimate the ultimate lateral rock resistance per unit shaft length, $p_u$ at shallow and
great depths for the laterally loaded drilled shafts in a transversely isotropic rock mass.

6.2 ROCK FAILURE AT SHALLOW DEPTH

To further understand the failure mode of the jointed rock near the ground surface (at shallow depth), a series of FE simulations was conducted for a model consisting of a drilled shaft embedded in rock mass with joints. The nonlinear FE program ANSYS was used for the FE simulations. Due to the symmetric nature of the problem, only one-half of the problem domain was modeled as shown in Figure 6-1. The developed 3D mesh generated by ANSYS program is shown in Figure 6-2, in which both the rock and the drilled shaft are modeled using 8 node brick elements (Solid 65). The FEM mesh was very dense at and near the region of the drilled shaft and near the joint location to allow for capture of high stress gradient in the region. A mesh sensitivity study was carried out to determine the optimum mesh size for the production of a full-range parametric analysis.

The focus of the FE analysis was on the rock response; therefore, the drilled shaft was modeled as an elastic material. On the other hand, the intact rock itself was using the Modified Drucker-Prager model. The joint between the intact rock and the interface between the drilled shaft and the rock were modeled using a surface based contact element in the ANSYS program. The drilled shaft surface was treated as contact surface, while the rock surface was taken as the target surface.

In carrying out the numerical simulation, incremental lateral loads were applied at the drilled shaft head until a failure along the predefined joint occurred. The lateral capacity was defined as the maximum applied lateral load before the numerical convergence
problem occurred. Again, the strength of the shaft was artificially assigned to high values, thereby ensuring that failure would have occurred in the joint only. As can be seen in Figure 6-3, at failure, the rock in front of the shaft showed a forward and upward movement. The maximum shear stress distribution in the rock is shown in Figure 6-4, from which one can observe that the locus of maximum shear stresses was a curved surface. The upward and forward movements shown in Figure 6-3, together with the locus of the maximum shear, lead us to develop a more refined failure mode as the one shown in Figure 6-5.

![3D Model for drilled shaft socketed into rock](image)

Figure 6-1 3D Model for drilled shaft socketed into rock
Figure 6-2 FE mesh of a drilled shaft-rock system.

Figure 6-3 The forward movement of rock mass along the joint
6.2.1 Derivation of \( p_u \) at shallow depth

In order to derive \( p_u \) for the failure mode depicted in Figure 6-5, a two stage approach was used. The first stage was to derive the theoretical \( p_u^* \) for a straight edge failure wedge shown in Figure 6-6. The second stage was to carry out a detailed FEM parametric study to determine the actual total lateral resistance \( p_u \) for the curved failure block. By comparing the results calculated from the two stages, one can determine a correction factor (CF), used in Equation (6-1)

\[
p_u = CF \times p_u^*
\]  

(6-1)

The derivation of \( p_u^* \) for the straight wedge in Figure 6-6 is based on force equilibrium principles. The notations used in Figure 6-6 are defined herein as follows. \( F_{\text{net}} \) = the total net rock resistance, \( D \) = the diameter of the drilled shaft, \( H \) = the height of the wedge, \( F_a \)
= the active lateral force exerted on the drilled shaft, \( F_s \) = the friction force on the sides of the wedge, \( F_n \) = the normal force applied to the side faces, \( F_{sb} \) = the friction force on the bottom surface, \( F_{nb} \) = the normal force on the bottom surface determined through force equilibrium in the vertical direction, \( W \) = the weight of the wedge, \( \sigma'_{v0} \) = effective overburden earth pressure on the top of rock, \( \beta \) = the wedge angle. The ultimate lateral resistance per unit shaft length, \( p^*_u \), for the triangular rock block can be derived using force equilibrium in both vertical and the lateral loading directions as follows.

\[
p_u^* = \frac{dF_{net}}{dH}
\]  

(6-2)

where

\[
F_{net} = 2F_s \sin \beta + F_{ab} \sin \beta + F_{nb} \cos \beta - F_a
\]  

(6-3)

\[
F_a = \frac{1}{2} K_0 \gamma D (H - z_0)^2
\]  

(6-4)

\[
F_s = c'A_x + F_n \tan \phi'
\]  

(6-5)

\[
F_n = K_0 \sigma'_{v0} A_x + \frac{1}{6} K_0 \gamma H^3 \tan \beta
\]  

(6-6)

\[
F_{nb} = \frac{\sigma'_{v0} A_x + W + 2F_s \cos \beta + (c')_{jo int} A_h \cos \beta}{\sin \beta - \tan \phi'_{jo int} \cos \beta}
\]  

(6-7)

\[
F_{sh} = (c')_{jo int} A_h + F_{nb} \tan \phi'_{jo int}
\]  

(6-8)

\[
K_a = \tan^2 (45 - \phi' / 2)
\]  

(6-9)
\[ K_0 = 1 - \sin \phi' \]  
(6-10)

\[ z'_0 = \frac{2c'}{\gamma' |K_a|} \]  
(6-11)

\[ A_s = \frac{1}{2} H^2 \tan \beta \]  
(6-12)

\[ A_h = DH \sec \beta \]  
(6-13)

\[ A_t = DH \tan \beta \]  
(6-14)

\[ W = \frac{1}{2} \gamma' H^2 D \tan \beta \]  
(6-15)

From which

\[ p_a^* = \frac{dF_n}{dH} = 2 \sin \beta \frac{dF_s}{dH} + \sin \beta \frac{dF_{sb}}{dH} + \cos \beta \frac{dF_{nb}}{dH} - \frac{dF_a}{dH} \]  
(6-16)

\[ \frac{dF_a}{dH} = \gamma' K_a (H - z'_0)D \]  
(dFa/dH \geq 0)  
(6-17)

\[ \frac{dF_n}{dH} = K_0 H \tan \beta (\sigma'_{v0} + \frac{1}{2} \gamma' H) \]  
(6-18)

\[ \frac{dF_s}{dH} = c'H \tan \beta + \tan \phi' \frac{dF_n}{dH} \]  
(6-19)

\[ \frac{dF_{sb}}{dH} = \tan \phi'_{jo} \left( \frac{dF_{nb}}{dH} + (c')_{jo} D \sec \beta \right) \]  
(6-20)

\[ \frac{dF_{nb}}{dH} = \frac{D \tan \beta (\sigma'_{v0} + H\gamma')}{-\sin \beta - \tan \phi'_{jo} \cos \beta} \]  
(6-21)
In order to determine the correction factor (CF) defined in Equation (6-1), a series of parametric analysis using 3-D FE modeling techniques was performed in this study. Factors that may have affected CF were investigated in this FE parametric study.

It should be noted that $p_u$ was determined from the FE output by double differentiating the moment versus depth profile along the wedge height, where the moment versus depth profile was obtained from the FE computed strains around the
drilled shaft and then fitted using the piecewise polynomial curve fitting technique.

6.2.2 FE Parametric Study Results

An extensive parametric study using ANSYS FE program was carried out to examine systematically the effects of several influencing factors on the correction factor (CF), including overburden pressure \( (\sigma_{v0}') \), strength properties of the intact rock \( (C_r, \Phi') \), rock unit weight \( (\gamma') \), the depth of the joint \( (H/D) \), the joint interface strength properties \( (C_{\text{joint}}, \mu) \), and the joint orientation \( (\beta) \). The range of all the parameters studied is listed in Table 6-1. The parametric study is carried out systematically in two stages: (a) Stage 1: studying the effects of each parameter individually by changing the numerical values of that particular parameter while keeping the values of the other parameters to be constant; (b) Stage 2: randomly selecting different combinations of the values of the parameters so that the effects of these random variations of the parameters on CF can be captured.

Table 6-2 summarizes the parametric sensitivity analysis results for stage 1, in which the calculated values of CF for the range of each parameter are given. The same results shown in Table 6-2 are plotted as a pie chart in Figure 6-7. It can be seen that the order of importance of these parameters affecting CF can be ranked from high to low as follows: \( (C_r/C_{\text{joint}}, H/D, \sigma_{v0}', \beta, \Phi, \mu, \text{and } \gamma') \). It can also be observed that \( \mu \) and \( \gamma' \) have exerted relatively small effects on CF. These two parameters are therefore held constant throughout the additional FE simulations in the stage 2 of the parametric study.
Table 6-1 Parameters variation for the FE study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Line Value*</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{v0}/p_a$**</td>
<td>0</td>
<td>0-50</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>20-75</td>
</tr>
<tr>
<td>$\Phi'$</td>
<td>30</td>
<td>20-45</td>
</tr>
<tr>
<td>$C_r$ (psi)</td>
<td>500</td>
<td>50-10000</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5</td>
<td>0.2-0.9</td>
</tr>
<tr>
<td>$C_{joint}$ (psi)</td>
<td>20</td>
<td>10-5000</td>
</tr>
<tr>
<td>$\gamma'$ (pci)</td>
<td>0.05</td>
<td>0.03-0.1</td>
</tr>
<tr>
<td>$H/D$</td>
<td>1</td>
<td>0.1-2.5</td>
</tr>
</tbody>
</table>

**$p_a$=atmospheric pressure

The results of FE simulations from both Stage 1 and Stage 2 are compiled and further analyzed by using SPSS (2003) program. It was assumed for the purposes of nonlinear regression analysis that $CF$ can be computed as in Equation (6-22) by a product of various $\eta$ functions.

$$CF = \eta \left( \frac{C_r}{C_{joint}} \right) \eta \left( \frac{H}{D} \right) \eta(\beta) \eta(\phi) \eta(\sigma_{v0}')$$ \hspace{1cm} (6-22)

The curve fitting is performed using the SPSS program to find various $\eta$ functions in Equation (6-22). The solutions obtained from the SPSS are presented for convenience in three design charts in Figure 6-8 through Figure 6-10. These charts can be used together to find $CF$ in accordance with the following equation:

$$CF = \eta(\beta, \phi) \eta(C_r, C_{joint}, H / D) \eta(\sigma_{v0}')$$ \hspace{1cm} (6-23)

Goodness of fit of the nonlinear regression analysis presented in the design charts was
statistically examined using R-squared. R-squared was found to be 0.872 for the regression line between $CF_{\text{Predicted}}$ and $CF_{\text{FEM}}$ as shown in Figure 6-11. This R-squared value is considered to be fairly good, especially for such a highly nonlinear and complex combination of parameters.

In summary, to estimate the ultimate lateral resistance of jointed rock at shallow depth where the failure mode is governed by a block failure, the ultimate lateral resistance ($pu^*$) of the straight wedge shown in Figure 6-6 is calculated first using Equation (6-2). The design charts from Figure 6-8 to Figure 6-10 are then used to estimate CF by way of Equation (6-22). Finally, the ultimate lateral resistance can be obtained as the multiplication of ($pu^*$) and CF.

It should be cautioned that, when using this method for transversely isotropic rock, the values of $C_{\text{joint}}$ (cohesion) for the bottom face of the failure wedge should be obtained from laboratory tests by shearing the samples along the plane of transverse isotropy. Moreover, for rock mass without the distinctive joints, then the previous derivations could still be used to calculate the ultimate resistance if the rock mass strength ($Cr$) is used instead of ($C_{\text{joint}}$), and the value of $\beta$ is taken as $45^\circ + \phi'/2$ according to Hoek (1983), who pointed out that the failure plane of a rock mass is $45^\circ + \phi'/2$ because the rock mass also follows the Mohr-Coulomb failure criterion.
Table 6-2 Sensitivity analysis of CF (Stage I)

| Parameter     | CF<sub>min</sub> | CF<sub>max</sub> | | CF<sub>max</sub> - CF<sub>min</sub> | \( \frac{\sum |CF_{max} - CF_{min}|}{\sum |CF_{max} - CF_{min}|} \times 100\% \) |
|---------------|-----------------|-----------------|-----------------------------|---------------------------------|-----------------------------------|
| \( \sigma_{v0'} \) | 1.372           | 1.512           | 0.140006                    | 13.82                           |
| \( \beta \)    | 1.368           | 1.40            | 0.040019                    | 3.95                            |
| \( \Phi' \)    | 1.365           | 1.38            | 0.022105                    | 2.18                            |
| \( C_r/C_{joint} \) | 1.310          | 1.710           | 0.41                        | 40.46                           |
| \( H/D \)      | 1.101           | 1.50            | 0.4                         | 39.47                           |
| \( \gamma' \)  | 1.373           | 1.373           | 0.000321                    | 0.03                            |
| \( \mu \)      | 1.3722          | 1.37            | 0.000856                    | 0.08                            |

\[ \Rightarrow \sum \]

1.013307

100

Figure 6-7 Sensitivity analysis of CF (Stage I)
Figure 6-8 Design charts to estimate CF for the effects of $\beta$ and $\Phi$

Figure 6-9 Design charts to estimate CF for the effects of $C_r$, $C_{joint}$, and $H/D$
Figure 6-10 Design charts to estimate CF for the effects of $\sigma_{v0}'$

Figure 6-11 Comparison of the predicted CF with CF obtained from the FEM

$y = x$

$R^2 = 0.8719$
6.3 ROCK FAILURE AT GREAT DEPTH

At great depth, the wedge type of failure is unlikely to occur due to high overburden pressure that would have prevented the wedge from sliding upward. At this depth, the failure occurs when both the side shear and the normal pressure in the rock have reached their limiting values due to the laterally loaded drilled shaft.

To understand the stress field (i.e., both normal and shear stresses) at failure around a laterally loaded drilled shaft socketed into a transversely isotropic rock, a representative FE model of a drilled shaft has been developed using the ANSYS. A 3.3 feet long drilled shaft segment 5 feet in diameter embedded in a transversely isotropic rock as shown in Figure 6-12 was used in the analysis.

Based on the FE analysis on the shaft-rock system shown in Figure 6-12, it can be concluded that the maximum friction between the shaft and the rock would be reached first. Then, the maximum compressive stress would have reached the compressive strength of the rock. Therefore, the ultimate lateral resistance of rock at the great depth can be determined from the compressive strength of the rock and the friction strength (interface shear strength) between the shaft and rock. This is similar to the assumption made by Carter and Kulhawy (1992). Based on the mobilization of the compressive stress shown in Figure 6-13 and the mobilization of friction shown in Figure 6-14, the ultimate normal and shear stress distribution at failure is proposed to be the one shown in Figure 6-15, where $\tau_{\text{ult}}$ is the ultimate side shear resistance and $p_L$ is the rock compressive strength. Based on the assumed failure stress distribution in Figure 6-18, the lateral
ultimate rock resistance per unit length of shaft, \( p_u \), for rock at great depth can be computed by integrating the normal and shear stress as follows:

\[
p_u = 2\int_0^{\pi/2} p_l D / 2 \sin^2 \alpha d\alpha + 2\int_0^{\pi/2} \tau_{ult} D / 2 \sin(2\alpha) \cos \alpha d\alpha
\]

\[
p_u = \frac{\pi}{4} D p_L + \frac{2}{3} D \tau_{ult}
\]  

(6-24)  

(6-25)

Figure 6-12 FE mesh of a drilled shaft-rock system.
Figure 6-13 Normal stress distribution at the load direction.

Figure 6-14 Shear distribution on the shaft-rock interface
6.4 STRENGTH OF TRANSVERSELY ISOTROPIC ROCK

The strength criterion of a transversely isotropic rock was proposed by Tien and Kuo (2001), who had observed the failure modes in the laboratory when conducting triaxial tests on many rock specimens with different inclination ($\theta$) of the plane of transverse isotropy and under different confining pressures. The underlining principle of Tien and Kuo (T-K) failure criterion is based on the Jaeger’s criterion (1960) and the maximum axial strain theory, in which the axial strain is calculated from the constitutive law of the transversely isotropic rocks.

The T-K failure criterion is based on two distinct failure modes: one is the sliding mode where the failure is caused by sliding along the discontinuity, and the other one is the non-sliding mode where the failure is controlled by the rock strength. For the non-sliding mode, Tien and Kuo assumed that the failure occurs when the axial strain exceeds
its maximum limiting value under a specific confining pressure. They adopted Hooke’s law to calculate the axial strains and the strength ratio of specimens with various $\theta$ under a specified confining pressure.

Since the rock failure at great depth due to the lateral deflection of the shaft is the main objective of this section, the non-sliding mode of the T-K failure criterion is considered herein. Equation (6-26) represents the failure condition of the transversely isotropic rocks for the non-sliding mode as a function of transversely isotropic parameters as proposed by Tien and Kuo.

$$\frac{S_{1(\theta)}}{S_{1(90)}} = \frac{\sigma_{1(\theta)} - \sigma_3}{\sigma_{1(90)} - \sigma_3} = \frac{k}{\cos^4 \theta + k \sin^4 \theta + 2n \sin^2 \theta \cos^2 \theta}$$  (6-26)

where

$$k = \frac{E}{E'}$$  (6-27)

$$n = \frac{E'}{2G'} - v'$$  (6-28)

$\theta$ = orientation of the plane of transversely isotropy in the sample as shown in Figure 6-16

$S_{1(\theta)}$ = the major deviatoric stress at failure for the specimens with $\theta$.

$\sigma_{1(\theta)}$ = major principal stress at failure for the specimens with $\theta$.

$\sigma_3$ = minor principal stress at failure for the specimens with $\theta$. 

VI-20
A special case of Equation (6-26) is obtained by considering a case where n=1 and k=1. The model when using these parameters is reduced to the original Jaeger’s criterion.

As shown in the schematic diagram in Figure 6-17, if the minor principal effective stress, $\sigma_3'$, is assumed to be equal to the effective overburden pressure, $\gamma'z$, and the major principal effective stress, $\sigma_1'$, is assumed to be equal to the limit normal stress, $P_L$, then the following expression for the normal limit pressure of a transversely isotropic rock, $p_L$, can be derived from Equation (6-26).

$$P_L = \gamma'Z + \frac{k(\sigma_{c(90')} - \gamma'Z)}{\cos^4(90 - \beta) + k \sin^4(90 - \beta) + 2n \sin^2(90 - \beta) \cos^2(90 - \beta)} \quad (6-29)$$

The parameters of Tien–Kuo’s failure criterion can be obtained by conducting triaxial tests at four angles of orientation, say 0, 30 , 60, 90 degrees. The shear strength parameters of the discontinuity are determined by conducting the triaxial compressive tests on the specimens with $\theta=60$ degree, and where the failure of specimens is in the sliding mode. More details on the method to determine these parameters in the T-K strength criterion can be found in Tien and Kuo (2001).
As an illustrative example, a full scale lateral load test at Dayton is used herein to demonstrate the use of the developed method for estimating the ultimate lateral resistance. Three different sets of parameters are used, one set is for verification, and the other two
sets are for studying the effect of anisotropy on $p_u$.

The first set of analysis is carried out using the proposed method assuming the rock is isotropic (i.e. $E=E'=E_m$, $\nu=\nu'=0.3$, $G'=E_m/(2+2\nu)$, and $\beta=45+\phi/2$). Equation (6-30) is used to estimate $p_u$ at different depths. Figure 6-18 shows the estimated $p_u$ in comparison with $p_u$ estimated using SJN 134137 (2006) method. As it can be seen, for isotropic rock the results of the proposed method are very close to the results based on SJN 134137 (2006) method.

$$p_u = \min(CF \times p_u^*, \frac{\pi}{4} Dp_L + \frac{2}{3} D\tau_{uh}) \quad (6-30)$$

The second set of analysis is carried out assuming the modulus of elasticity in the plane of transverse isotropy to be equal to the rock mass modulus ($E=E_m=95$ ksi), and the modulus of elasticity in the direction perpendicular to the plane of transversely isotropy is equal to one-tenth of the rock mass modulus ($E'=9.5$ ksi). A typical value of Poisson’s ratio is used ($\nu = 0.3$, $\nu' = 0.15$). The empirical equation (3-3) is used to estimate the shear modules, $G'$. Based on these parameters, the calculated anisotropy coefficients $k$ and $n$ as defined in Equation (6-27 and 6-28) are 10, and 0.76, respectively. The analysis is then carried out for two different orientations of the plane of transverse isotropy ($\beta=30$ degree, and $\beta=45$ degree). The estimated ultimate lateral resistance versus depths for the aforementioned parameters are shown in Figure 6-19. The effects of the orientations of the plane of transverse isotropy on the ultimate lateral resistance at both shallow and great depth are clearly seen in this Figure 6-19.
The last set of analysis is carried out for two different k values, \( k = E_i/E_m = 6.2 \) and \( k = 2.0 \). The other parameters in this last set of analysis are kept the same as follows: \( E = E_i = 9.5 \) ksi, \( \nu = 0.3 \), \( \nu' = 0.15 \) and \( \beta = 30 \) degree. The ultimate lateral resistances based on these assumed parameters are calculated at different depths and plotted with depth in Figure 6-20. As it can be seen that the degree of anisotropy, defined by the k value, can exert significant effects on the ultimate resistance at the great depths.

![Graph showing Pu vs Z/D](image)

Figure 6-18 Comparison of the proposed method and SJN 134137 (2006) method for estimating \( P_u \)
Figure 6-19 Ultimate lateral resistance calculated for two different $\beta$

Figure 6-20 Ultimate lateral resistance calculated for two different $E/E'$ ratios
6.6 SUMMARY AND CONCLUSIONS

The main objective of this chapter was to present a method for computing the ultimate lateral resistance of the drilled shaft embedded in a rock mass that exhibits transversely isotropic behavior due to the presence of bedding planes or the inherent strength anisotropy. Toward this objective, the 3-D FEM model for simulating a laterally loaded drilled shaft in jointed rock was utilized to develop a better understanding of the failure modes for the rock at shallow and great depth. For the rock at shallow depth with distinctive discontinuities, the failure mode due to the lateral deflection of the shaft was found to be sliding on a block that possessed curved side surfaces. Using the force balance principles, the equations for estimating the ultimate lateral resistance per unit shaft length was derived. For the rock at great depth, where the high overburden earth pressure prevented the movement of the block, the failure mode of the rock was found to be controlled by the shear failure on the rock-shaft interface and the compressive strength failure in the front side of the shaft. Based on the FE simulation results, the distribution of the stress fields at failure was developed, from which the ultimate lateral resistance per unit shaft length was obtained. The Tien and kou (2001) strength criterion for transversely isotropic media and the improved empirical equation for the ultimate side shear resistance presented in chapter V were incorporated in the derived equations. A case study using a full-scale test was presented as an example to illustrate the use of the developed equations for computing the ultimate lateral resistance per unit shaft length in the drilled shaft that is embedded in a transversely isotropic rock. The case study showed that for isotropic rock, the proposed method would calculate the lateral resistance to be
very close to that calculated by an earlier model proposed in SJN 134137 (2006). However, if the rock exhibits significant cross anisotropy as reflected by the two Young’s moduli ratio, then the computed ultimate lateral resistance would be significantly different from that computed by the isotropic model of proposed in SJN 134137 (2006). The specific observations based on the case study presented in this chapter can be summarized below.

- It can be concluded that as the orientation of the plane of transverse isotropy in reference to the vertical axis of the drilled shaft increases, the ultimate lateral resistance increases at great depth. While at shallow depths, it can be concluded that the smaller the orientation angle, the larger the ultimate lateral resistance.

- Although it was found that the degree of anisotropy, as represented by the ratio E/E’ exerts negligible effects on the ultimate lateral resistance at shallow depths, it was found that the larger the degree of anisotropy at great depth, the larger the ultimate lateral resistance.
CHAPTER VII: HYPERBOLIC P-Y CRITERION FOR TRANSVERSELY ISOTROPIC ROCK

7.1 INTRODUCTION

Based on theoretical derivations and numerical (finite element) parametric analysis results described in chapters III through VI, a methodology for estimating the main ingredient of a p-y criterion (i.e. $K_i$ and $p_u$) has been developed for the rock mass that exhibits transverse isotropy due to intrinsic mineral grain orientation and/or the influences of parallel sets of joints. In this chapter, the p-y curve will be succinctly described, together with the methods for determining the pertinent rock parameters to enable the construction of the p-y curve. More interestingly, this chapter will present the evaluation of the proposed p-y criterion by performing a parametric study on hypothetical cases of a rock socketed drilled shaft under the lateral load. In these cases, a series of parameters differentiating the isotropic vs. transversely isotropic p-y curves are selected in a systematic numerical study using the LPILE computer program with the p-y curves generated from the p-y criterion described in this chapter. Based on this parametric study, the insights on the influence of rock anisotropy on the predicted response of the rock socketed drilled shaft under lateral load are discussed. Both the orientation of the plane of transverse isotropy and the degree of anisotropy ($E/E'$) have shown to exert significant influences on the main two parameters used to characterize the p-y curve: the subgrade modulus ($K_i$) and the ultimate lateral resistance ($p_u$). An analysis example is also presented to illustrate the application of the developed p-y curve criterion for the analysis.
of laterally loaded drilled shafts in complex rock formations. Finally, the developed p-y criterion is shown to predict the behavior of a laterally loaded shaft.

7.2 GENERAL SHAPE OF P-Y CURVE IN ROCK

Based on a study conducted in SJN 134137 (by Nusairat et al. 2006) who investigated different mathematical representation techniques for field load test derived p-y curves, it was concluded that hyperbolic mathematical modeling can provide the best fit to the data set. Therefore, a hyperbolic equation is deemed appropriate for mathematical modeling of the p-y curves for transversely isotropic rock as well. Two parameters are key controlling factors in characterizing the hyperbolic curve; namely, the initial tangent slope and the asymptote. For the proposed hyperbolic p-y model, these correspond to the subgrade modulus \( K_i \) and the ultimate resistance \( p_u \). The hyperbolic p-y relationship can be written as.

\[
p = \frac{y}{\frac{1}{K_i} + \frac{y}{p_u}} \tag{7-1}
\]

where

\( p = \) force per unit shaft length \( (F/L) \); \( y = \) lateral displacement of shaft \( (L) \); \( K_i = \) initial subgrade reaction modulus of the soil/rock \( (F/L^2) \); \( p_u = \) the ultimate lateral resistance force per unit length of shaft \( (F/L) \).
7.3 CONSTRUCTION OF P-Y CURVES FOR TRANSVERSELY ISOTROPIC ROCK

In the following, a brief recap is presented for the mathematical equations for estimating $p_u$ and $K_i$

1. Compute $K_i$

The $K_i$ is computed using Equation (7-2), in which the values of the $\eta$ functions can be interpreted from the design charts shown in Figure 3-8.

$$\frac{K_i}{p_a} = 1 \times 10^6 \eta(E, \beta)\eta(E', G')\eta(\nu, \nu')\eta(E_p^*, \nu_p^*)\eta(R)$$  \hspace{1cm} (7-2)

Note that the five elastic constants needed as part of input in Equation (7-2) could be determined by either in-situ direct pressuremeter test or one of the appropriate laboratory tests (e.g., Brazilian tensile strength, uniaxial tension, and ultrasonic wave velocity). Furthermore, an empirical Equation (3-3) was also developed to allow for empirically estimating $G'$.

For jointed rock, an equivalent transversely isotropic homogeneous model to describe the jointed rock’s stress-strain behavior to get the five transversely isotropic elastic constants was developed in this report. This model takes into account the joint spacing, joint thickness, and the Poisson’s effect of the joints filling with the aid of the volumetric content of the intact rock ($\alpha$) and the filling ($\beta^*$). The five elastic constants for this model are summarized in section 4.3.1.
2. Compute $p_u$

There are two separate equations for calculating $p_u$ depending on the mode of failure of the rock mass due to the lateral deflection of the shaft. Equations (7-3) and (7-4) are used for computing $p_u$ for a sliding of a block of rock. In Equation (7-3), $p_u^*$ is the ultimate resistance of a triangle wedge depicted in Figure 6-6 and could be computed by differentiating the net force acting on this wedge ($F_{net}$) over the wedge height as described in section 6.2.1. Equations (7-5) through (7-7) are used for calculating $p_u$ for a failure mode corresponding to strength failure of rock mass at a great depth below the ground surface. The value of $p_u$ is the smaller value calculated using these two equations. The design charts shown in Figure 6-8 to Figure 6-10 could be used to calculate the correction factor in Equation (7-4). The main parameters required for calculating $p_u$ at the great depth, other than the properties of a drilled shaft and the elastic constants include the following: the normal lateral resistance of a transversely isotropic rock, $P_L$ described in section 6.4, and the side shear resistance, $\tau_{ult}$ described in section 5.5. Equation (7-6) can be used to compute $P_L$, while Equations (7-7) can be used to estimate $\tau_{ult}$.

\[ p_u = CF \times p_u^* \quad (7-3) \]

\[ CF = \eta(\beta, \phi) \eta(C_r, C_{joint}, H/D) \eta(\sigma_{v0'}) \quad (7-4) \]

\[ p_u = \frac{\pi}{4} D p_L + \frac{2}{3} D \tau_{ult} \quad (7-5) \]

\[ P_L = \gamma'Z + \frac{k(\sigma_{c(90)} - \gamma'Z)}{\cos^4(90 - \beta) + k \sin^4(90 - \beta) + 2n \sin^2(90 - \beta) \cos^2(90 - \beta)} \quad (7-6) \]
7.4 SENSITIVITY ANALYSIS

A series of sensitivity analyses were carried out using the proposed p-y criterion to gain insight on the significant effects of rock anisotropy on the response of the laterally loaded drilled shaft. For this purpose, the pertinent data related to an actual full-scale lateral load test at MOT-CR-32 in Dayton Ohio reported in SJN 134137 (by Nusairat et al. 2006) is used. Some of the pertinent information related to this test site is summarized in section 3.5.

To represent the assumed transversely isotropic of the rock mass, some educated guess was exercised based on the rock mass properties. Furthermore, to shed more lights on the effects of anisotropy, five different case studies were performed as shown in Table 7-1. The first three cases are selected to study the effects of the orientation of the plane of transversely isotropy on the drilled shaft response, while the last two cases are related to the degree of anisotropy effects on the drilled shaft response. For these analyses, the modulus of elasticity in the plane of transversely isotropy is taken to be equal to the rock mass modulus ($E = E_m = 95$ ksi), while a typical value of Poisson’s ratio is used ($\nu = 0.3, \nu' = 0.15$). The empirical Equation (3-3) is used to estimate the shear modules ($G'$). Three different orientations of plane of transversely isotropy ($\beta = 30^\circ, 45^\circ, \text{and } 80^\circ$) are studied in the first three cases. The corresponding $K_i$ and $P_u$ computed from these values and the proposed criterion for these cases are also summarized in Table 7-1.

Using the initial modulus of subgrade reaction, and the ultimate lateral resistance

$$\tau_{ult} = k'(C_{ad} + \sigma_v'\mu)$$  \hspace{1cm} (7-7)
summarized in Table 7-1, three hyperbolic p-y curves at three different depths are generated as shown in Figure 7-1. Figure 7-2 shows three p-y curves generated at the shallow depth of 10 in, while Figure 7-3 shows the p-y curves generated at the great depth of 100 in for three different $\beta$. Figure 7-4 and Figure 7-5 show the effects of degree of anisotropy on the p-y curves generated at shallow and great depths, respectively. Of interest was the influence of $\beta$ on the p-y curve components. Specifically, at shallow depth this transversely isotropic rock is found to have its maximum effect at an orientation angle $\beta = 30^\circ$ or $80^\circ$, and its minimum effect at an orientation of $45^\circ$, while at great depth, the effect increases with the increase in $\beta$.

For all the five cases in Table 7-1, the relevant p-y curves are input into LPILE computer program (Reese, et al. 2004) to compute the response of the test shaft under the applied lateral loads. The predicted load deflection curves are shown in Figure 7-6 and Figure 7-7 for different $\beta$ angle and for different $E/E'$ ratio, respectively. The predicted moment versus depth profiles are shown in Figure 7-8 and Figure 7-9 for the similar effects. The effect of the orientation of plane of transverse isotropy and the degree of anisotropy on the response of the drilled shaft in terms of deflection and moment is clearly seen in these figures.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta$</th>
<th>E/E'</th>
<th>$K_i$ (psi)</th>
<th>Pu (lb/in) at (Depth, in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
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<td>375000</td>
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<tr>
<td>2</td>
<td>45</td>
<td>5</td>
<td>300000</td>
<td>6084</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>5</td>
<td>190000</td>
<td>11324</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>10</td>
<td>465000</td>
<td>16278</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>1</td>
<td>230000</td>
<td>16278</td>
</tr>
</tbody>
</table>

Table 7-1 Effect of anisotropy on the p-y curve parameters
Figure 7-1 p-y curves for case No. 1

Figure 7-2 Effect of $\beta$ on p-y curves at shallow depths
Figure 7-3 Effect of $\beta$ on p-y curves at great depths

Figure 7-4 Effect of $E/E'$ on p-y curves at shallow depths
Figure 7-5 Effect of $E/E'$ on $p$-$y$ curves at great depths

Figure 7-6 Load deflection curve for different $\beta$
Figure 7-7 Load deflection curve for different E/E' ratios

Figure 7-8 Predicted moment profile for different β
7.5 COMPARISON OF DAYTON TEST RESULTS WITH PROPOSED P-Y CRITERION

In this section, an effort was made to use the proposed p-y criterion to predict the measured response of the test shaft at Dayton site. Some judgments are needed to interpret the required input parameters and these best estimates of the input parameters are summarized herein. The E/E’ ratios is calculated as $E/E_m=6.2$, while the other parameters are kept as follows: $E'=9.5 \text{ ksi}$, $\nu = 0.3$, $\nu' = 0.15$, and $\beta=30$. Based on these parameters, hyperbolic p-y curves are generated from the hyperbolic p-y criterion and then submitted to the computer program LPILE to compute the response of the test shaft under the applied lateral loads. The predicted load-deflection curve at the loading point is compared with the actual measured in Figure 7-10. In general, a good agreement between the measured and the predicted can be observed. The comparison of the measured
moment versus depth curves from the measured strain gage readings and those predicted from the LPILE predictions are shown in Figure 7-11. In general, the predicted values match those measured quite well.

Figure 7-10 Comparison between the predicted and measured load deflection curve

Figure 7-11 Comparison between the predicted and measured a moment profile
7.6 ILLUSTRATIVE EXAMPLE

Due to the lack of actual lateral load tests in jointed rock, a hypothetical case is designed to demonstrate the use of the developed method for deriving the p-y curve for jointed rock. For this case, the Young modulus of the tested drilled shaft shown in Figure 7-11 is assumed to be 10 GPa with Poisson’s ratio of 0.25. The test shaft is 1 m in diameter with 5 m rock socket length.

For the joint thicknesses shown in Figure 7-11, the volumetric content of the filling material and the intact rock defined in Equation (4-12) and (4-13) are calculated as follows.

\[ \beta^* = \frac{1 + 2 + 1 + 2.5 + 1.5}{500} = 0.016. \]

\[ \alpha = 1 - \beta^* = 0.984 \]

For the intact rock and joint properties assumed for this hypothetical case, the calculated five elastic constants using the proposed equivalent model described in Equations (4-32) through (4-36) are (E=4.9 GPa, E’=4.6 GPa, ν=0.25, ν’=0.24, G’=1 GPa). Equation (7-2) is used to estimate the initial tangent to p-y curve, while the smaller value calculated using Equations (7-3) and (7-5) is used as the ultimate lateral resistance. The estimated K_i for these five elastic constants is 0.8 GPa. The estimated ultimate lateral resistance at each joint depth shown in Figure 7-12 from top to bottom assuming a wedge type of failure are (30 MPa, 100MPa, 400 MPa, 550 MPa, and 750 MPa), respectively. At the same depths, the estimated ultimate lateral resistances assuming that the stresses in
the rock reach its strength limit are (34.5 MPa, 34.7 MPa, 34.8 MPa, 34.9 MPa, and 35 MPa). It can be seen from these estimations that a wedge type of failure is likely to occur at the top depth (30 MPa < 34.7 MPa), while at the depth below that, the rock strength is controlling the lateral resistance. Using the estimated initial modulus of subgrade reaction and ultimate lateral resistance, three hyperbolic p-y curves constructed based on the proposed p-y criterion at different depths are shown in Figure 7-13. These curves are input into LPILE computer program to compute the response of the test shaft under the applied lateral loads. The predicted load deflection curve and moment versus depth profile are shown in Figure 7-14 and Figure 7-15, respectively.

Figure 7-12 Illustrative Example profile
Figure 7-13 p-y curves for the hypothetical case

Figure 7-14 Load deflection curve for the jointed rock socketed drilled shaft
SUMMARY AND CONCLUSIONS

A hyperbolic p-y criterion for transversely isotropic rock was developed in this chapter that can be used to predict the deflection, moment, and shear responses of the drilled shaft in rock under the applied lateral loads. The essential mathematical equations and the accompanied design charts necessary for constructing the hyperbolic p-y curve were succinctly described herein. However, the emphasis of the chapter was placed on presenting the results of a sensitivity analysis to shed lights on the effects of rock anisotropy (the orientation of bedding plane and the Young’s modulus ratio) on the p-y curves and the response of the laterally loaded drilled shafts. In addition, the load test data at Dayton test site was compared to the predicted shaft response based on the p-y curve criterion and the best estimate of the on-site rock properties. The comparison
results seem to support the applicability of the proposed p-y curve criterion. Finally, a practical hypothetical example was presented to demonstrate the step-by-step procedure for generating the pertinent p-y curves that reflect in-situ anisotropic rock properties.

Based on the sensitivity study results presented in this chapter, the following conclusions could be drawn concerning the effects of anisotropy on the p-y curves and the corresponding drilled shaft behavior.

- At a small lateral load, the drilled shaft is found to have its maximum deflection at an orientation angle $\beta = 45^\circ$ and its minimum deflection at an orientation of $30^\circ$, while at a large lateral load, the minimum deflection was found at an orientation angle of $80^\circ$. The maximum moment was found at an orientation angle $\beta = 45^\circ$ and the minimum at $\beta = 30^\circ$.

- It can be concluded for the same $\beta$ that the larger degree of anisotropy, as represented by the larger ratio of $E/E'$, the smaller the drilled shaft head deflection and the smaller the maximum moment would be at the same applied lateral load.
8.1 INTRODUCTION

The objective of this chapter is to present a unified p-y criterion for cohesive soils and IGM using the hyperbolic mathematical formulation. The 3-D FE parametric study results, of a laterally loaded drilled shaft in clay, are used to develop empirical equations for calculating two important parameters in the hyperbolic p-y formulation. These two parameters are the initial tangent to p-y curve, $K_i$, and ultimate resistance, $p_u$.

Two of the full scale lateral load tests conducted during the previous research project (SJN 134137) were dominantly in IGM. The results of the two tests are used to verify the proposed model. Another four full-scale fully instrumented lateral load test results are used to validate the proposed p-y criterion in predicting the load deflection and bending moment of the drilled shafts under the lateral loads.

8.2 BACKGROUND

A number of p-y curves for clays have been developed by Matlock (1970), Reese and Welch (1975), Evans and Duncan (1982), Gazioglu and O’Neill (1984), Dunnavant and O’Neill (1989), Hsiung and Chen (1997), Yang and Liang (2005). These existing p-y curves were developed based on a limited number of model or field lateral load tests on piles or drilled shafts. Furthermore, Yang and Liang (2005) have shown that the existing Reese & Welch (1975) and Gazioglu and O’Neill (1984) p-y criteria did not well predict
the response of the shafts for their cases. The accuracy and applicability of the existing p-y curves for clays can be further validated or improved by using additional lateral load test results that have become available since their original developments.

The advancement of computer technology has made it possible to study lateral soil-drilled shaft interaction problems with rigorous FE method. Brown and Shie (1990) have conducted a series of three-dimensional FE analyses on the behavior of single pile and pile group using an elastic soil model. They derived p-y curves from FE analysis results and provided some comparison with the empirical design procedures in use. Bransby and Springman (1999) utilized two-dimensional FE method to find load-transfer relationships for translation of an infinitely long pile through undrained soil for a variety of soil-constitutive models. Yang and Jeremić (2002) performed a FE study on pile behavior in layered elastic-plastic soils under lateral loads and generated p-y curves from the FE results. It was found that the FE results generally agree well with centrifuge data.

In this chapter, the hyperbolic function is suggested for constructing p-y curves for both soft and stiff clays under short-term static loading. The methods to compute $K_i$ and $p_u$ are based on FEM simulation results and are discussed in section 8.4.

8.3 LITERATURE REVIEW

A brief review is given below of the existing equations for calculating the two key parameters in the hyperbolic p-y curve for cohesive material.
8.3.1 Initial Tangent to p-y Curve

By fitting the subgrade reaction solution with the continuum elastic solution for beam on elastic foundation, Vesic (1961) proposed Equation (8-1) as an elastic solution for the modulus of subgrade reaction, $K_i$.

$$K_i = \frac{0.65E}{1-\nu^2} \left[ \frac{ED^4}{E_p I_p} \right]^{1/12} \quad (8-1)$$

where, $E$ = modulus of elastic materials; $\nu$ = Poisson’s ratio; $D$ = beam width; and $E_p I_p$ = flexural rigidity of beam.

Bowles (1988) suggested to double the value of $K_i$ in Equation (8-1) for piles under lateral loading since the pile would have contact with soils on two sides. However, in reality, soils do not fully contact with the pile when the lateral loads are applied. Based on field test data, Carter (1984) modified Vesic’s equation as follows to account for the effect of pile diameter.

$$K_i = \frac{1.0ED}{(1-\nu^2)D_{ref}} \left[ \frac{ED^4}{E_p I_p} \right]^{1/12} \quad (8-2)$$

where the reference pile diameter, $D_{ref} = 3.3$ ft, $E_p I_p$ = flexural rigidity of piles or drilled shafts. As noted in the FEM Modeling Section later in this chapter, the effect of Poisson ratio on $K_i$ by Carter (1984) seems to be different from the FEM parametric study results presented in this chapter.
8.3.2 Ultimate Resistance, $p_{ult}$

Based on a wedge failure mechanism, Reese (1958) provided an equation for estimating, $p_{ult}$, near the ground surface as follows

$$p_{ult} = \left( 2S_u + \gamma'z + \frac{2.83S_uz}{D} \right)D \quad (8-3)$$

Based on the flow-around failure theory for clay at a great depth, $P_{ult}$ for clay at the great depth is given as

$$p_{ult} = 11S_uD \quad (8-4)$$

Later, Matlock (1970) suggested computing the $p_{ult}$, using the smaller of the values given by the equations below.

$$p_{ult} = \left( 3 + \frac{\gamma'}{S_u}z + \frac{J}{D}z \right)S_uD \quad (8-5)$$

$$p_{ult} = 9S_uD \quad (8-6)$$

where $\gamma'$ = average effective unit weight from ground surface to the depth $z$ under consideration, $z$ = depth of the p-y curve, $S_u$ = undrained shear strength at depth $z$, $D$ = diameter of drilled shaft, $J$ = 0.5 for a soft clay, and $J$=0.25 for a medium clay. A value of 0.5 is frequently used for $J$.

8.4 FE MODELING

The commercial ABAQUS FE program is used for modeling the soil-drilled shaft interaction, as shown in Figure 8-1. The drilled shaft is modeled as a cylinder with elastic
material properties. The solid elements C3D15 and C3D8 available in ABAQUS are used to develop a mesh representation for shaft and clay, respectively. The surface interface technique is employed to simulate the soil-shaft interface.

![Figure 8-1 3-D FEM model](image)

8.4.1 Initial Tangent to p-y curve, $K_i$

Since the determination of initial tangent to p-y curve constitutes the primary objective of this part of FEM study, only the elastic response of clay is the main concern of this part of parametric study.

8.4.1.1 Effect of Elastic Modulus of Soils, $E_s$

The effect of elastic modulus of soils, $E_s$, is studied by varying it from 725 psi to 5075 psi. Other pertinent parameters are kept as constant as follows: $D=3.3$ ft, $E_p=2900$ ksi, $v_s=0.3$. The FEM computed results are plotted in Figure 8-2(a), from which one can see that a power function fits the relationship between $K_i$ and the elastic modulus of the soil.
8.4.1.2 Effect of shaft Diameter, D

The effect of shaft diameter is investigated by varying the diameter from 3.3 ft to 13.1 ft. Other pertinent parameters are kept as constant as follows: $E_p=2900$ ksi, $E_s=2900$ psi, $\nu_s=0.3$. The $K_i$ generated at a depth of 6.6 ft is shown in Figure 8-2(b). It can be seen that $K_i$ increases linearly with the shaft diameter.

8.4.1.3 Effect of Poisson’s Ratio of Soils, $\nu_s$

The Poisson’s ratio for cohesive soil is varied from 0.1 to 0.35, while other pertinent parameters are kept as constant as follows: $D=3.3$ ft, $E_p=2900$ ksi, $E_s=14,500$ psi. The $K_i$ generated at a depth of 3.3 ft is shown in Figure 8-2(c), where a power function can fit the data well. It is noted that in Carter’s empirical equation (Carter, 1984), an increase in Poisson ratio leads to an increase in $K_i$. The present FEM results show that $K_i$ decreases with an increase in Poisson ratio.

8.4.1.4 Effect of Elastic Modulus of Shafts, $E_p$

The effect of elastic modulus of the drilled Shaft, $E_p$, is studied by varying it from 2900 ksi to 58,000 ksi, while other parameters are kept as constants as follows: $D=3.3$ ft, $E_s=14,500$ psi, and $7250$ psi, $\nu_s=0.3$. Figure 8-2(d) shows the relationship between $K_i$ and $E_p$. It can be seen that a power function fits very well for the relationship between $K_i$ and $E_p$.

8.4.1.5 Effect of Depth, Z

The relationship between initial tangent of p-y curve and the depth is also investigated by estimating $K_i$ at different depths ranging from 1.0 m to 4.0 m. Other pertinent parameters are kept as constant as follows: $D=1$ m, $E_p=2.0 \times 10^7$ kPa, $\nu_s=0.3$. The FEM
results are computed for two values of $E_s$ (10,000 kPa, and 20,000 kPa) as shown in Figure 8-2(e), from which one can see that a power function can also fit the relationship between $K_i$ and $Z$.

8.4.1.6 Suggested Empirical Equation for $K_i$

A nonlinear regression analysis on the results from the FE parametric study is carried out using the SPSS program. Equation (8-7) for estimating the $K_i$ is developed, which appears to fit the $K_i$ values obtained from the FE parametric study as shown in Figure 8-2(f). The coefficient of determination is more than 0.97.

$$
K_i = 0.95 \frac{D}{D_{ref}} \left( \frac{Z}{Z_{ref}} + 0.1 \right)^{0.016} \left( \frac{E_s}{E_p} \right)^{1.033} \left( \frac{\nu_s}{\nu_p} \right)^{-0.078}
$$

(8-7)

where

$Z_{ref} = 1.0$ m

$D_{ref} = 1.0$ m

8.4.2 Ultimate Lateral Resistance, $p_{ult}$

The strength of saturated clay is usually characterized by undrained shear strength, $S_u$; therefore, clay is modeled as a simple Von Mises material in the FEM simulation. The FE model used to obtain the ultimate resistance is the same as the one used to obtain $K_i$. The drilled shaft with a diameter of 6.6 ft is modeled as an elastic material, while the undrained shear strength of cohesive soils is varied from 1.45 psi to 29 psi. The ultimate
resistances computed from FE analyses are shown in Figure 8-3(a) to (c) for three representative values of $S_u$. It can be seen that the results from FE analyses agree generally well with those from Matlock’s method. The ultimate resistance of in-depth clay from FEM is generally larger than $9S_uD$, but smaller than $11S_uD$. Therefore, $10S_uD$ is adopted in this chapter. The ultimate resistance of clays can be calculated by using the smaller of the values given by the following equations:

$$p_{ult} = \left( 3 + \frac{z'}{S_u} z + \frac{J}{D} z \right) S_u D$$

(8-8)

$$p_{ult} = 10S_u D$$

(8-9)
Figure 8-2 Effects of Shaft and Soils Parameters on $K_i$

- **(a)** shows the relationship between elasticity modulus of soils and $K_i$.
- **(b)** demonstrates the impact of diameter on $K_i$.
- **(c)** illustrates the correlation between Poisson ratio and $K_i$.
- **(d)** exhibits the effect of elastic modulus on $K_i$.
- **(e)** presents the dependency of $K_i$ on depth.
- **(f)** reflects the empirical prediction of $K_i$.

Mathematical expressions and best fit equations are provided for each graph, along with corresponding $R^2$ values.
Figure 8-3 Ultimate resistances vs. depth
8.5 DOCUMENTATION OF TWO LATERAL LOAD TEST RESULTS

This section presents two lateral load tests conducted as part of SJN 134137 research project. The subsurface investigation, Pressuremeter testing results, and lab test results on rock core samples are provided. The information pertinent to test drilled shafts and instrumentation are also presented. The lateral load test results, including deflections at shaft head at various loading levels, the shaft deflection profiles along depth of shaft, and measured strain with depth are presented.

8.5.1 Lateral Load Test at JEF-152-1.3

This project is a landslide repair job along and to the west of SR 152, approximately 800 feet north of the intersection of SR 152 and TR 125 in Jefferson County, Ohio.

8.5.1.1 Test Site

The area exhibiting instability is approximately 280 feet long. The Ohio Department of Transportation (ODOT) worked on design and construction of drilled shaft foundation to stabilize the slope. The final design was to install 42 inches diameter shafts, spaced 7 feet on center. Each shaft was socketed 20 feet into bedrock.

Considering the large lateral load acting on the drilled shafts due to the slide movement, the lateral loads are the controlling design load for these drilled shafts. Due to a lack of well established design methods for analyzing the laterally loaded drilled shafts in rock, a lateral load test was carried out on two production shafts.

Field rock pressuremeter tests were conducted at this site as can be seen in Figure 8-2. The limiting pressure and the modulus of elasticity, \( E_m \), measured from this test are
summarized in Table 8-1. The top of rock was found to be at 25 feet from the ground surface. All weathered rock core samples were inspected and specific samples were chosen for laboratory testing. Eight (8) samples were used for unconfined compression testing at the University of Akron laboratory. Table 8-2 summarizes the results of these tests.

Figure 8-4 Test Site in JEF-152-1.3

Figure 8-5 Pressuremeter Test at JEF-152-1.3
Table 8-1 Pressuremeter Test Results

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Depth (ft)</th>
<th>Limit Pressure (psi)</th>
<th>Qu (psi)</th>
<th>Em (psi)</th>
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</thead>
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<td>710</td>
<td>100</td>
<td>15140</td>
</tr>
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<td>2</td>
<td>31.5</td>
<td>905</td>
<td>125</td>
<td>13300</td>
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</tbody>
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Table 8-2 Laboratory Test results

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Top (ft)</th>
<th>Bottom (ft)</th>
<th>Qu (psi)</th>
<th>Ei (psi)</th>
<th>Poisson’s ratio</th>
</tr>
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<td>27.5</td>
<td>21</td>
<td>4200</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Rock: Red mudstone, water flowing, slickensided, decomposed

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Top (ft)</th>
<th>Bottom (ft)</th>
<th>Qu (psi)</th>
<th>Ei (psi)</th>
<th>Poisson’s ratio</th>
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<td>32</td>
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<td>0.38</td>
</tr>
</tbody>
</table>

Rock: Red mudstone, water flowing, slickensided, highly weathered, RQD=65%

8.5.1.2 Test Set-up and Test Procedure

Two drilled shafts, shaft #1 and shaft #2 were constructed to perform the lateral load test on. Test shaft# 1 was isolated from overburden using a 6’ diameter casing seated at the top of rock (25’ below ground). The shafts were reinforced with 24 #11 bars. Casing was used to build the shaft above rock. Test shaft# 2 was set as a reaction shaft. The top elevation of bedrock at the test drilled shafts was observed at 25 feet below ground surface. The total length of the test drilled shafts was 43 feet. Rock socket length was 15 feet. For both shafts, the socket was drilled 6 feet deeper to extend the inclinometer below the tip of the shaft to measure any movement at the tip. The test shafts extended 3 feet above ground and the distance between the top of the test drilled shafts and the jacking point was 2 feet.
The 28-day unconfined compressive strength of concrete was 5115 psi. The diameter of the drilled shaft socketed in bedrock was 3.5 feet. The reinforcement of the shaft was 24 #11 bars and the cover was 3 inch. To fully mobilize the rock-shaft interaction and isolate the effect of the overburden soils, a 6 feet diameter casing was used to form a gap between the test drilled shaft #1 and the soils above bedrock. This means that all the lateral forces were resisted by the bedrock during the lateral load test. The details of the instrumentation are provided in Figure 8-7.

The lateral load was applied by pushing the two drilled shafts apart via a jack and a reaction beam as shown in Figure 8-7. A load cell was installed between the reaction shaft #2 and the jack to measure the actual applied lateral loads. The loading sequence consisted of applying the lateral load in increments of 15 kips or 20 kip, followed by an unloading. The maximum load applied was 165 kips. Each load was held until the rate of deflection at the top of shaft was less than 0.04 inch/min and the inclinometer reading of each shaft was taken.

Figure 8-6 Lateral Load Test at JEF 152-1.3

Inclinometers
8.5.1.3 Lateral Load Test Results

The measured data during the lateral load test on the two drilled shafts include load-deflection curves at the loading point, the deflection versus depth profiles measured from inclinometers, and the strain along shaft length.
Figure 8-8 and Figure 8-9 present the load-deflection response measured at the loading point during the incremental lateral loading for shaft #1 and shaft #2, respectively. The deflections were taken as an average of the two dial gage readings for each shaft at each loading level. It can be seen that the measured deflection at the loading point at shaft #1 is much larger than that at shaft #2 due to the isolation of overburden soils using the 6’ diameter casing. The maximum deflections at the 165 kips lateral load in shaft #1 and shaft #2 are 8 inch and 0.75 inch, respectively.

The deflection profiles of test shaft #1 and shaft #2 along shaft length, deduced from inclinometer readings, are presented in Figure 8-10. From this Figure, it can be seen that the deflection at the bedrock elevation is very small. This means that most of the applied lateral load was resisted by the soils for shaft #2. However, for shaft #1, the lateral loads were resisted by the rock due to the use of a casing to isolate the soil reactions. The movement at the top of rock in shaft #1 was approximately 0.6 inches and movement was noticed in the upper 6 feet of the socket.

The strain readings were recorded by a data logger connected to a laptop during the test. The tension strain and the compression strain profiles of shaft #1 under various loading levels are presented in Figure 8-11. The depth shown in this figure starts from the top of the drilled shafts.
Figure 8-8 Measured load-deflection curves at loading point of JEF-152-1.3 test for shaft #1

Figure 8-9 Measured load-deflection curves at loading point of JEF-152-1.3 test for shaft #2
Figure 8-10 Deflection-depth profiles of drilled shaft #1 and shaft #2 at JEF-152-1.3 test

Figure 8-11 Tension and compression strain profiles of test shaft #1 of JEF-152-1.3 test
8.5.2 Lateral Load Test at WAR-48-21.02 (Warren County, Ohio)

The SR 48 over Clear Creek project is located in Warren County, Ohio. The lateral load testing was performed by the Foundation Test Group to proof the adequacy of the designed shaft length in rock.

8.5.2.1 Test Site

The drilled shafts were constructed using the “dry” methods, with temporary casings for excavation support. All shafts had a completed length of about 23.0 feet. The subsurface stratigraphy encountered during drilling of the 48-inch diameter shafts consisted of shale from the ground surface to the bottom of the shafts. At shafts #34 and #36, a limestone layer approximately 1-ft thick was encountered at a depth of 3-ft below the ground surface.

Field rock dilatometer tests were conducted at this site. The limiting pressure ultimate capacity, the modulus of elasticity, and Em, measured from this test in addition to the laboratory unconfined compressive strength (Qu) are summarized in Table 8-3.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Depth (ft)</th>
<th>Limit Pressure (psi)</th>
<th>Qu (psi)</th>
<th>Em (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.5</td>
<td>1,860</td>
<td>188</td>
<td>23,300</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>1,700</td>
<td>200</td>
<td>24,800</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>2,310</td>
<td>246</td>
<td>58,000</td>
</tr>
</tbody>
</table>
8.5.2.2 Test Set-up and Test Procedure

Figure 8-12 and Figure 8-13 show a four 48-inch diameter drilled shafts installed by Koker Drilling. The constructed shaft length was 20.0 ft. below the ground surface. The drilled shaft holes were drilled 6 feet deeper than the bottom elevation to extend the inclinometer deep enough to monitor the movement at the tip of the shaft. All drilled shafts were instrumented with inclinometers for measuring the deflection along the shaft length and the dial gages for measuring the deflection at the loading point. One dial gage was installed at each test drilled shaft. All the drilled shafts were instrumented with 7 levels of vibrating wire strain gages along the shaft length for monitoring the rebar strains. The details of the instrumentation are provided in Figure 8-14. The unconfined compressive strength of the concrete in the test shafts was 5000 psi. The diameter of the drilled shaft socketed in bedrock was 4 feet. The reinforcement of the shaft was 12 #11 and 6# 14 bars as shown in Figure 8-14.

A total load of 400 kips was applied to the shafts. Load was applied two feet below the top of the shaft in increments of 50 kips. Each increment was held for a total of 30+/− minutes. A total of 8 increments of load up to a maximum load of 400 kips was applied to the shafts. After the load of 400 kips was reached, the shaft was unloaded in 4 step to a load of zero kips. The lateral load was applied by pushing the two drilled shafts apart via a jack and a reaction beam placed between them.
Figure 8-12 Test Drilled shaft at WAR-48-21.02

Figure 8-13 Lateral Load Test at WAR-48-21.02
Figure 8-14 Test set-up of WAR-48-21.02 lateral load test
8.5.2.3 Lateral Load Test Results
The measured data during the lateral load test on the four drilled shafts include load-deflection curves at the loading point, the deflection versus depth profiles measured from the inclinometers, and the strain gage readings along the shaft length. Figure 8-15 and Figure 8-16 present the load-deflection response measured at the loading point during the incremental lateral loading for the four tested drilled shafts.

The deflection profiles along shaft lengths of test shafts #29 and #31, deduced from the inclinometer readings, are presented in Figure 8-17. The strain readings were recorded by a data logger connected to a laptop during test. The compression and tension strain profiles under various loading levels are presented in Figure 8-18 and Figure 8-19. The depths shown in the figures starts from the top of the drilled shafts.

Figure 8-15 Measured load-deflection curves at jacking point of WAR-48-21.02 test shaft #29
Figure 8-16 Measured load-deflection curves at jacking point WAR-48-21.02 test shaft #31

Figure 8-17 Deflection-depth profiles of drilled shafts #29 and #31 at WAR-48-21.02 test
Figure 8-18 Tension and compression strain profile of test shaft #29 at WAR-48-21.02 test

Figure 8-19 Tension and compression strain profile of test shaft #31 at WAR-48-21.02 test
8.6 CASE STUDIES

To validate the proposed p-y criterion for cohesive IGM, the results of the fully instrumented lateral load tests presented in Section 8.5.1 and Section 8.5.2 (e.g. load-deflection curves, deflection vs. depth curves, and the maximum moments) are compared with the results predicted using LPILE and the proposed hyperbolic p-y criterion for the same test data. Another lateral load test reported by (Liang, 1997) for the drilled shaft socketed into rock that could be characterized as cohesive IGM is used as an additional verification for the proposed model.

In Sections 8.6.1 through 8.6.6, a comparison is performed between the proposed p-y criterion and Reese and Welch p-y criterion for stiff clays (1975) in predicting the response of six laterally loaded drilled shaft socketed into cohesive material with undrained shear strength between (1 tsf to 9 tsf).

8.6.1 Ohio JEF-152-1.3 Test

The proposed hyperbolic p-y criterion was used to generate the p-y curves as input in the LPILE program to predict the load-deflection curve of the test shaft. Figure 8-20 shows the predicted and measured load-deflection curves. The predicted and measured deflection vs. depth curves are compared in Figure 8-21. The comparison of the predicted and measured maximum bending moment is given in Table 8-4. It can be seen that the proposed p-y criterion allows LPILE program to predict the measured data very well.

8.6.2 Ohio WAR-48-21.02 Test

The load-deflection curves and the deflection with depth profile predicted by the
LPILE program using the p-y curves constructed based on the proposed criterion are shown in Figure 8-22 and Figure 8-23, respectively. It can be seen that the proposed hyperbolic criterion provides the best match in comparison with Reese (1997) weak rock criterion.

8.6.3 Ohio LOR-6 Test (Liang, 1997)

The soils at the test site are composed primarily of silt clay, underlain by gray clay shale. The soil profile at the LOR-6 test site is presented in Figure 8-24. The undrained shear strength of clay is correlated to undrained elastic modulus as follows (U.S. Army Corps of Engineers, 1990)

\[ E_s = K_c S_u \]  

(8-10)

where \( K_c \) is a correlation factor. The value of \( K_c \) as a function of the overconsolidation ratio and plasticity index, PI, is estimated to be 500.

The geometry and dimension of test shaft as well as the instrumentation layout are shown in Figure 8-24. The diameter of the shaft in clay is 4ft, while the diameter of the shaft socket in the shale is 3 ft. Inclinometer casing and strain gages were installed inside the shaft as shown. The lateral deflections of the shaft top were measured by the dial gages.

Figure 8-25 shows the comparison of the predicted and measured load-deflection curves. It can be seen that the predicted load-deflection curve using the proposed hyperbolic p-y criterion agrees very well with the measured. A comparison of the predicted and measured deflections vs. depth is shown in Figure 8-26. The average
maximum bending moment prediction errors using the proposed p-y criterion is 0.33 as can be observed from Table 8-4, and is less than 0.45, the average prediction error using Reese and Welch p-y criterion (1975).

8.6.4 Salt Lake International Airport Test (Rollins et al, 1998)

The test pile was 1 ft I.D. closed-end steel pipe with a 0.37 in wall thickness driven to a depth of approximately 30 ft. The elastic modulus of the steel was 29,000 ksi and the minimum yield stress was 48,000 psi. Prior to conducting the lateral pile load testing, inclinometer casing and strain gauges were placed inside the pile. The pile was then filled with concrete. The soil profile at the test site as well as the pile instrumentation detail is presented in Figure 8-27. The soils near the ground surface are clays and silts with undrained shear strength typically between 3.5 psi to 7 psi. Some layers showed undrained shear strength of 15 psi. The underlying cohesionless soil layer consists of poorly graded medium-grained sands and silty sands. Based on EM 1110-1-1904, for PI=25, $E_s$ of cohesive soils can be estimated to be 1000$S_u$.

The proposed hyperbolic criterion and the Matlock (1970) criterion are used to generate two sets of p-y curves for the clay layers. Reese et al. (1975) p-y criterion is used to generate the p-y curves for the sand layers. The LPILE analysis results using these p-y curves are compared with the measured load-deflection curve in Figure 8-28. The proposed p-y curves provide good match with the measured load-deflection curve. Also, the bending moment prediction errors, of the proposed p-y criterion are less than Matlock (1970) p-y criterion prediction errors, as can be seen in Table 8-4.
8.6.5 Ohio CUY-90-15.24 Test (Liang, 2000)

The soils near the ground surface are mainly composed of silts, soft to stiff clays. Deposits in great depth are shale with a trace of gravels. The test shaft is 148 ft long with an embedded length of 141 ft. The lateral load was applied at the point 2 ft above the ground surface. The diameter of test shaft is 6 ft, reinforced by 24#18 rebar and a built-up beam. Figure 8-29 shows the soil profile at the site as well as the details of reinforcement inside the shaft.

Figure 8-30 and Figure show the predicted and measured load-deflection curves and the deflection-depth curves, respectively. It can be seen that the proposed p-y criterion can predict the load-deflection of the drilled shaft much better than Reese & Welch (1975) p-y criterion.

At five different loading levels out of six as shown in Table 8-4, the maximum bending moment prediction errors calculated based on the proposed p-y criterion results are less than those calculated based on Reese & Welch (1975) p-y criterion.


The soil profiles at the test site with the soil parameter interpreted from the pressuremeter test as well as the instrumentation of the test shaft are presented in Figure 8-32. The shaft is relatively short, with an embedment length of 15 ft. Therefore, the resistance of the shaft tip is a significant component of the overall lateral resistance.

The comparisons between the predicted and measured load-deflection curves are
shown in Figure 8-33. A discrepancy is observed, which may have been contributed by the shear resistance at the base of the shaft. This is another case proving that the maximum bending moment prediction using the proposed p-y criterion is better than the prediction using Reese & Welch (1975) p-y criterion as shown in Table 8-4.

8.7 SUMMARY AND CONCLUSIONS

A hyperbolic p-y criterion for IGM was developed in this study that can be used in conjunction with either COM624P or LI PLE computer programs to predict the deflection, moment, and shear responses of the shaft under the applied lateral loads. Based on 3-D FEM simulation results, a new empirical equation was developed for calculating the initial tangent to the p-y curve.

Evaluations based on comparisons between the predicted and measured responses of full-scale lateral load tests on fully instrumented drilled shafts socketed into IGM have shown the practical uses of the proposed p-y criterion

Using the results of three full-scale lateral load tests reported in the literature on fully instrumented drilled shafts socketed into cohesive material, a comparison study was performed between the proposed p-y criterion and Reese and Welch p-y criterion for stiff clay (1975). The proposed hyperbolic p-y criterion was shown to be capable of predicting the load-deflection and bending moments of these drilled shafts better than Reese and Welch p-y criterion.
Table 8-4. Maximum Bending Moment for the Six Cases.

<table>
<thead>
<tr>
<th>Site</th>
<th>Lateral Load</th>
<th>Max. Bending Moment (kN.m)</th>
<th>Prediction Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Proposed</td>
<td>Reese/Matlock</td>
</tr>
<tr>
<td>LOR-6</td>
<td>1601</td>
<td>3101</td>
<td>3470</td>
</tr>
<tr>
<td></td>
<td>1868</td>
<td>3812</td>
<td>4243</td>
</tr>
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<td>2580</td>
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<td>16699</td>
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Figure 8-20 Predicted and Measured Load Deflection Curves of Ohio JEF-152 Test

Figure 8-21 Predicted and measured deflection vs. depth curves of Ohio JEF-152 Test
Figure 8-22 Predicted and measured load deflection of Ohio WAR-48 Test

Figure 8-23 Predicted and measured deflection vs. depth curves of Ohio WAR-48 Test
Figure 8-24 Soil Profile and Shaft Dimension at Ohio LOR-6 Test Site
Figure 8-25 Comparison of Predicted and Measured Load-Deflections of Ohio LOR-6 Test.

Figure 8-26 Comparison of Predicted and Measured Deflections vs. Depth of Ohio LOR-6 Test.
Figure 8-27 Soil Profile and pile detail at Salt Lake International Airport Test (After Rollins et al.1998)
Figure 8-28 Predicted and Measured Load-Deflection Curves of Salt Lake International Airport
<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil Description</th>
<th>$S_3$ (kPa)</th>
<th>$e_0$ $\times 10^6$</th>
<th>$\gamma$ (kN/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 2</td>
<td>FILL: Dense black fine to coarse sand, little fine to coarse gravel, trace SILT</td>
<td>33</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>2 - 4</td>
<td>FILL: Dense black fine to coarse sand, little fine to coarse gravel, trace SILT</td>
<td>52</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>4 - 12</td>
<td>Stiff to very-stiff gray SILTY CLAY, trace fine to medium SAND, horizontal structure Medium-stiff to stiff gray SILTY CLAY, trace fine GRAVEL, few lenses of silt</td>
<td>65</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>12 - 16</td>
<td>Stiff to very-stiff gray SILTY CLAY, trace fine to coarse SAND, few lenses and lenses of SILT</td>
<td>83</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>16 - 30</td>
<td>Very-stiff to hard, stiff to very-stiff gray SILTY CLAY, trace fine to coarse sand, few lenses and lenses of SILT</td>
<td>110</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>30 - 36</td>
<td>Very-stiff to hard gray CLAY, little fine to coarse sand, fine fine GRAVEL</td>
<td>213</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>36 - 38</td>
<td>Dense gray fine to medium sand, little CLAYEY SILT, trace fine GRAVEL</td>
<td>310</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>38 - 43</td>
<td>Dense gray fine to coarse GRAVEL, some fine to coarse sand, trace CLAYEY SILT</td>
<td>255</td>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

$E_p$=500 $\text{S}_a$

$E_p$(avg.)=30GP$n$

Figure 8-29 Soil Profile and Shaft Dimension at CUY-90 Test Site
Figure 8-30 Predicted and Measured Load-Deflection Curve of Ohio CUY-90 Test

Figure 8-31 Predicted and Measured Deflections vs. Depth of Ohio CUY-90 Test.
Figure 8-32 Soil Profile and Shaft Dimension at Colorado I-225 clay site
Figure 8-33 Predicted and Measured Load-Deflection Curves of CDOT Clay Site
CHAPTER IX: PRESSUREMETER TO DETERMINE ELASTIC CONSTANTS OF TRANSVERSELY ISOTROPIC ROCK

9.1 INTRODUCTION:

In many geotechnical engineering applications, there is a need for determining the elastic constants of geo-medium, for computing elastic settlement of shallow foundations, predicting deflections of drilled shafts under lateral loads, computing deformation of underground excavation (tunneling or open excavation), among others. For sedimentary rocks, due to inherent grain orientations and preferred bedding planes, they can exhibit strong directional dependency in their mechanical properties, including elastic constants. Similarly, for rock mass possessing strong joint pattern or foliations, their elastic deformation response can generally be described as transversely isotropic. Stiff clays with or without presence of fissures, due to their geological formation process, also exhibit strong anisotropy properties. According to findings in this research and studies by Shatnawi (2008), Hawk and Ho (1980), and Sargand and Hazen (1987), the simplest form of anisotropy, i.e., transverse isotropy, can be effectively used to represent the anisotropic behavior of these geo-materials.

In-situ tests are generally considered more reliable for the determination of the geo-material’s deformation modulus due to the fact that they cause less sample disturbance compared to sampling and testing in the laboratory. Furthermore, in-situ tests are conducted in the geo-materials without altering significantly in-situ state of stress. The pressuremeter test (PMT) is widely used in weak rocks as an in-situ testing method to
determine the stress and deformation relationship of a geo-material. There are two types of pressuremeter devices which differ by the way of measuring deformations during testing. In the indirect pressuremeter type, the deformation of the surrounding geomaterial is measured based on the amount (volume) of fluid injected to dilate the membrane. In the direct pressuremeter test, the deformation of the geomaterial is measured by the variation of diameter of the membrane. It should be noted that most commercially available pressuremeters belong to the indirect pressuremeter type due to their cost advantage over the direct pressuremeters.

For a linearly elastic isotropic geomaterial, the linear slope of the pressure–expansion curve of the PMT has been theoretically derived for determining shear modulus. This method was based on the infinitely long cylindrical cavity expansion theory. Essentially, the equation for the radial expansion of a cylindrical cavity in an infinite isotropic elastic medium was first formulated by Lamé (1852); however, Ménard (1961) was the first to suggest the use of the theory for the estimation of the shear modulus. Until recently, in-situ pressuremeter test has not been well adopted in deducing the elastic constants of transversely isotropic rocks, primarily due to lack of associated research work. However, Kiehl and Wittke (1983) showed that for a borehole sunk parallel to the schistosity, the mean values of $E'$ and $G'$, (i.e., Young’s modulus and shear modulus, respectively, in a plane perpendicular to the plane of anisotropy), for a transversely isotropic medium can be predicted by using the direct pressuremeter test. The requirements of the borehole direction, together with the use of the direct pressuremeter, limit the wide application of Kiehl’s theory.
The main objective of this chapter is to present a new and practical approach for determining the five elastic constants of a transversely isotropic rock mass using the in-situ indirect pressuremeter test device. The in-situ indirect pressuremeter test involves the measurement of the membrane volume change when the membrane is subjected to an increase in the applied membrane pressure. Through conducting a series of finite element simulations of pressuremeter tests in a transversely isotropic elastic medium, the interrelationships were developed between the five elastic constants and the initial tangent, $K_i$, to the $(\Delta p, \Delta V/V_o)$ curves obtained from the simulated pressuremeter tests, where $\Delta p$ is the change in the applied membrane pressure, $\Delta V$ is the change in the membrane volume due to pressure applied, and $V_o$ is the initial volume of the membrane. A typical pressuremeter test result in pressure versus volume change plot is shown in Figure 9-1. The contributions of this chapter lie in the development of a series of regression charts for using $K_i$ to estimate the Young’s modulus $E$ and $E’$ for different Poisson’s ratio $\nu$ and inclination angle $\theta$ between the axis of the borehole and the plane of the symmetry of the transversely isotropic elastic medium.

9.2 THEORITICAL BACKGROUND

The term “transverse isotropy” refers to a class of material that exhibits isotropic elastic properties in one plane, called the plane of transversely isotropy, and different elastic properties in the direction normal to this plane. For a material that is transversely isotropic, only five independent elastic constants are needed to describe its elastic deformation behavior. Throughout this report, these elastic parameters, illustrated in
Figure 9-2, are denoted as $E$, $E'$, $\nu$, $\nu'$, and $G'$, with the definitions given as follows: $E$ and $\nu$ are Young’s modulus and Poisson’s ratio respectively in the plane of transversely isotropy, while $E'$, $G'$, and $\nu'$ are Young’s modulus, shear modulus, and Poisson’s ratio in the perpendicular plane.

The stress-strain relationship of an elastic transversely isotropic material follows the modified Hooke’s law as given in Equations (9-1), taken from Rajeeb (2006):

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} = 
\begin{bmatrix}
\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu'}{E'} & 0 & 0 & 0 \\
-\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu'}{E'} & 0 & 0 & 0 \\
-\frac{\nu'}{E'} & -\frac{\nu'}{E'} & \frac{1}{E'} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G'} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G'}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]  

(9-1)

9.2.1 Available Methods

9.2.1.1 Analytical Approach

Methods for the determination of the elastic constants of the transversely isotropic rock have been discussed in the literature review (Section 2.8.1) and are briefly summarized herein. On the theoretical side of research, Cauwelaeter (1977) showed used three fundamental constants: $E$, $\nu'$, and the degree of anisotropy ($n$) defined as $E/E'$ instead of five elastic constants. Graham and Houlsby (1983) presented a theoretical framework for describing the elastic transverse isotropy in a triaxial test. Other researchers, such as Barden (1963), Pickering (1970), and Yu and Dakoulas (1993) have also developed simplified representation of the elastic constants of transversely isotropic rock.
9.2.1.2 Experimental Approach

Experimental methods based on laboratory small scale testing for determining elastic constants of transversely isotropic rocks have also been presented in the literature review (Section 2.8.1). Laboratory tests can be divided into two categories: dynamic and static methods. The dynamic laboratory test methods include the resonant bar method (Goodman, 1989) and the ultrasonic pulse method (Youash, 1970) in which the dynamic elastic constants $E_d$, $\nu_d$, and $G_d$ can be determined. The ultrasonic pulse method could be used to determine all five independent elastic constants of transversely isotropic rocks, as illustrated by Liao et al. (1997). Chou and Chen (2008) pointed out some shortcomings of Liao et al. (1997) in that the transversely isotropic plane of the test specimen has to be parallel to the longitudinal axis of the specimen.

The available static methods for laboratory determination of the five elastic constants of transversely isotropic rocks were summarized in Chou and Chen (2008). They include the following tests:

(1) Uni-axial compression test where two cylindrical specimens with one loading direction are needed.

(2) True tri-axial compression test in which two cubic specimens with three loading directions are needed.

(3) Hollow cylinder test where two hollow cylindrical specimens with two types of loading condition are needed, and

(4) Diametral compression test, (Brazilian test), in which two discs with one loading
direction are needed.

The in-situ methods include a number of tests from which the five independent elastic constants could be evaluated, such as the Goodman jack tests, and the hydraulic chamber test (Amadei, 1996). These two tests are mainly used for tunnels and they exert stresses on a limited section of the perimeter of the walls (as in the Goodman Jack) which makes it hard to interpret, as the rigid surface in contact with the rock change with the pressure change. Kawamoto (1966) proposed using a pressurized borehole to estimate the elastic constants of anisotropic rock. His method was only applicable to certain directions of rock anisotropy with respect to the direction of the holes.

9.3 RESEARCH METHODOLOGY

This research utilizes the finite element (FE) method for formulation of the governing equations and experimental data for calibration of derived equations. FE method was used in this study to simulate the indirect pressuremeter tests in a transversely isotropic rock. Essentially, 3-D mesh was generated to allow for simulation of an expansion of a finite length pressuremeter membrane in a linearly elastic, homogenous, transversely isotropic rock media. The ABAQUS computer program was used to simulate the pressuremeter test and to conduct parametric study by varying the five elastic constants.

9.3.1 Numerical Simulations and Results

The 3-D mesh generated for simulating pressuremeter test is shown in Figure 9-3. The radius of the pressuremeter membrane was set to be one unit and the membrane length was set to be 10 units (i.e., length to diameter ratio of 5, in conformance with
The length of the mesh in the radial and axial (i.e., vertical) direction was equal to 25 units and 15 units measured from the center of the pressuremeter membrane, respectively. The finite element mesh used was made up of 8 node hexahedral brick elements. Due to symmetry, only half of the transversely isotropic rock mass was used in the FE model. A total of 37,120 elements with 41,123 nodes were used in generating the mesh. The size of the mesh in all directions was set to be sufficiently large so that the boundary effects would have an negligible influence on the numerical results. The pressuremeter was assumed to be always in contact with the surrounding transversely isotropic rock mass.

For validation purpose, the theoretical solutions developed by Hue-San (2000) and given in Equations (9-2) through (9-4) for the case of elastic expansion of a thick walled cylinder in a linear elastic and isotropic medium, as shown in Fig.9-4, are compared with FE simulation results. Hue-San (2000) equations are as follow:

\[
\sigma_r = - \frac{p_b b^2 (r^2 - a^2)}{r^2 (a^2 - b^2)} - \frac{p_o a^2 (b^2 - r^2)}{r^2 (a^2 - b^2)} \quad (9-2)
\]

\[
\sigma_\theta = - \frac{p_b b^2 (r^2 + a^2)}{r^2 (a^2 - b^2)} + \frac{p_o a^2 (b^2 + r^2)}{r^2 (a^2 - b^2)} \quad (9-3)
\]

\[
\delta_r = \frac{p - p_o}{2G\left(\frac{1}{a^2} - \frac{1}{b^2}\right)} \left[1 - \frac{2\nu}{b^2} r + \frac{1}{r}\right] \quad (9-4)
\]

where \(p\) and \(p_o\) are the internal and external pressures acting on the hollow cylinder surfaces respectively, \(a\) and \(b\) are the inner and outer radii of the cylinder, \(r\) is any radius within the thick walled cylinder, \(\nu\) is Poisson’s ratio, and \(G\) is the elastic shear modulus.
The comparisons are plotted in Figure 9-5(a) and Figure 9-5(b) for the normalized radial stresses with the maximum applied internal pressure $p_{\text{max}}$ vs. radial distance and the radial displacement vs. the radial distance, respectively. It is noted that tangential stresses are equal to the radial stresses in magnitude, but different in the direction only. Good match between the theoretical results and the numerical FE simulation results can be seen in Figure 9-5, thus providing a check on the FE modeling and simulation techniques used in this paper.

9.3.2 Finite Element Parametric Study Results

In order to examine systematically the effects of the five elastic constants of a transversely isotropic rock mass ($E$, $E'$, $G'$, $\nu$, and $\nu'$) on the pressuremeter test results, an extensive parametric study was carried out using ABAQUS program. In addition to five elastic constants, a parameter ($\theta$) characterizing the orientation of the plane of anisotropy of the rock in relation to the axis of pressuremeter membrane (or borehole’s axis of symmetry) was also added. The range of each parameter studied is summarized in Table 9-1.

The FE parametric study was carried out systematically in two stages: Stage I involved varying the values of one parameter only while keeping the values of other parameters as the baseline value shown in Table 9-1 to study the relative importance of each parameter that would affect the value of $K_i$. Stage II involved randomly varying the parameters listed in Table 9-1 to provide additional numerical cases for subsequent regression analysis.
The FE analysis results for the relationships between \(K_i\) and six variables selected in the parametric study are plotted in Figure 9-6(a through f) for the effects of five elastic constants and the dip angle (\(\theta\)) presented as \(\sin \theta\), respectively. It should be noted that the values of \(K_i\) plotted in Figure 9-6 are normalized values defined as follows.

\[
\overline{K}_i = \frac{K_i (kPa)}{1000 P_a (kPa)} \quad \text{(9-5-a)}
\]

\[
\overline{E} = \frac{E (kPa)}{1000 P_a (kPa)} \quad \text{(9-5-b)}
\]

\[
\overline{E'} = \frac{E' (kPa)}{1000 P_a (kPa)} \quad \text{(9-5-c)}
\]

\[
\overline{G'} = \frac{G' (kPa)}{1000 P_a (kPa)} \quad \text{(9-5-d)}
\]

Based on Figure 9-6(a) through Figure 9-6(c), it can be seen that \(K_i\) is exponentially related to \(E\), \(E'\), \(G'\). As shown in Figure 9-6(d) through Figure 9-6(e), that the value of \(K_i\) decreases slightly with increasing Poisson’s ratio \(\nu\) and \(\nu'\). Finally, Figure 9-6(f) shows that \(K_i\) increases with increasing \(\sin \theta\) up to 0.79 (\(\theta=52^\circ\)), after which value, \(K_i\) decreases with increasing \(\sin \theta\) for the range between 0.79 and 1 (\(\theta\) between 52\(^\circ\) and 90\(^\circ\)).

The relative importance (or the effect) of varying each parameter on the \(K_i\) is calculated by dividing the absolute difference, \(\left|K_{i_{\text{max}}} - K_{i_{\text{min}}}\right|\), as affected by each individual parameter, by the summation of the absolute of all values, \(\sum \left|K_{i_{\text{max}}} - K_{i_{\text{min}}}\right|\), as affected by all parameters.

The results are summarized in Table 9-2: Sensitivity analysis of \(K_i\). The order of
importance of the parameters affecting $K_i$ (i.e., the initial tangent to $\Delta p, \Delta V/V$, curve) can be ranked from high to low as follows: $G'$, $E$, $E'$, $\theta$, $\nu$, and $\nu'$. It can also be observed that $\nu$ and $\nu'$ have negligible effects on $\overline{K}_i$ with the importance percentage of 2.7% and 0.7%, respectively. Thus, it is reasonable to assume a constant value of $\nu$ and use typical values for different rock types in the subsequent discussions.

The computational results of FE simulations were analyzed by using a statistical analysis software SPSS [22] to form the empirical equation, Equation (9-6), for estimating $K_i$ from the six parameters in Table 9-1.

$$
\overline{K}_i = -0.002 \overline{G}^{0.54} \left[ \frac{-10000 + \sin(\theta)}{0.84 - 1.57 \sin(\theta) + \sin^2(\theta)} \right] \left[ e^{-(\frac{G}{E} + 0.2 \nu)} - e^{-0.008 (\frac{G}{E} + 25 \nu + E)} \right] 
$$

(9-6)

where $P_a$ is the absolute atmospheric pressure and equal to 101.3 kPa and $\overline{K}_i, \overline{E}, \overline{E}$, and $\overline{G}'$ are defined earlier in Equation (9-5). A comparison between the FEM computed $\overline{K}_i$ and the empirical prediction of $\overline{K}_i$ using Equation (9-6) was statistically examined from which R-squared and which was found to be 0.94 as shown in Figure 9-7.

### 9.4 PROPOSED PROCEDURE

In this Section, the initial tangent $K_i$ to the $(\Delta p, \Delta V/V,)$ curve of a typical pressuremeter test result is used as the key measured pressuremeter data for deducing the five elastic constants. The FE simulation is concerned only with initial, elastic, response of the pressuremeter test, before tensile cracking in the rock mass could be initiated due to membrane expansion. The work by Johnston (1990) supported this assumption of no
tensile cracking in the initial elastic deformation stage.

A step by step procedure for the determination of the five elastic constants based the methodology proposed in this study is presented in the next subsections.

9.4.1 Step1. Determination of Young Modulus E

A solution exists for interpreting indirect pressure test results to determine the modulus, E, for a transversely isotropic rock mass in soft rock testing, Handbook No. 362-81(Vicksburge, 1982), provided the test borehole is sunk perpendicular to the schistosity (or plane of anisotropy) as illustrated in Figure 9-8. The Young modulus E is calculated from the (P, ΔV) curve from Equation (9-7)

\[ E = 2(1 + \nu)(V_m + V_i) \frac{\Delta P}{\Delta V} \]  

(9-7)

Where, \( \nu \) = Poisson’s ratio

\( V_i \) = initial volume of the empty membrane

\( V_m \) = mean volume of the membrane due to applied pressure P

Kiehl (1980) reported that E can still be determined quite accurately using Equation (9-7) even when borehole was not sunk perpendicular to the schistosity as long as \( \hat{\theta} \) defined as the angle between a perpendicular to borehole’s axis and the line of dip, measured in the plane of both the borehole’s axis and the line of the dip, is within \( \hat{\theta} \leq 30^\circ \). The accuracy of Kiehl (1980) finding is examined herein using the FE simulations. Table 9-3 provides a summary of the FE simulations in terms of the standard error associated with the
inclination of the pressuremeter with respect to the borehole schistosity. It can be seen that within $\hat{\theta} = 30^\circ$, the standard error in the determination of $E$ using Equation (9-7) may reach up to 17% while it can be up to only 10% when $\hat{\theta}$ is within $25^\circ$. Thus, for the determination of $E$ using Equation (9-7), it is recommend to carry out the pressuremeter test with the angle $\hat{\theta}$ less than $25^\circ$.

9.4.2 Step 2 Determination of Shear Modulus $G'$

There are several empirical relationships for estimating shear modulus $G'$ from other elastic constants. For example, Amadei (1996) found that most of the published experimental data can fit the Saint-Venant approximation equation (Equation (9-8)) for anisotropic rock.

$$G' = \frac{EE'}{E + E' + 2\nu E'}$$  \hspace{1cm} (9-8)

Statistical analysis presented in Chapter X of this report and in Shatnawi (2007) was carried out on compiled experimental data that was collected extensively from the available literature, including Amadei (1996) and Amadei (1987), Chen et al (1998), Gerrad (1975), Homand et al. (1993), Johnston et al. (1994), Liao et al. (1997), and Lo et al. (1986) for transversely isotropic rock samples. The following empirical expression for estimating the shear modulus $G'$ was proposed.

$$G' = \frac{0.032E}{(1 + \nu)(1 - e^{-0.06\frac{E}{E'}})}$$  \hspace{1cm} (9-9)

Comparing with FE parametric results, it was found that Eq. (9-9) performed better
than Equation (9-8) in terms of the time needed to approach convergence in the iterative procedure required according to this study. Consequently, this study recommends the use of empirical correlation presented in Equation (9-9) for the determination of the shear modulus $G'$. However, step 2 and step 3 need to be solved iteratively.

Although Poisson’s ratio $\nu$ exerts negligible effect on $K_i$, the error associated with the assumed $\nu$ will directly affect the determination of Young’s modulus $E$ and the shear modulus $G'$ as in Equations (9-7) and (9-8), respectively. Therefore, this effect was further examined by choosing a combination set of data within that data range of the parametric study. The data used are that expected to show greater effect on estimated values of $E$ and $G'$. The estimate of the error associated with the estimation of $E$ due to mistaken Poisson’s ratio the data used were ($K_i=0.75$ GPa, and $\nu=0.4$); and for the same purpose in estimating $G'$, the data used were ($E=(1-50)$ GPa, $E'=(0.5-50)$ GPa, and $\nu=0.4$). Possible sets of data were studied by varying Poisson’s ratio within a range of up to 20%. The effect of Poisson’s ratio on Equation (9-7) and Equation (9-9) was further explored and summarized in Table 9-4. As expected, it can be seen that the error associated with mistaken Poisson’s ratio exerts negligible effect on the estimation of $E$ and $G'$ with a relative error less than 6% which is reasonable for practical use.

9.4.3 Step 3. Determination of Young Modulus $E'$

As a final step, plug Equation (9-9) into Equation (9-6) and solve Equation (9-6) for $E'$. Also, several charts based on FE simulation results are presented in Figure 9-9(a) through Figure 9-9(d) for different $\theta$ values to enable the determination of $E'$ from pressuremeter obtained $K_i$ by way of Equation (9-6). For a known $\theta$ angle, a calculated
value of $E$ from Equation (9-7) and the assumed typical values for Poison ratio $v$, the value of $E'$ can be determined from the appropriate chart.

9.5 VALIDATION OF THE PROPOSED METHOD -BASED ON FIELD RESULTS

In this section, an illustrative example is presented to demonstrate the suggested procedure and validate the obtained results. The elastic constants were obtained from Talesnick and Ringle (1999) for Loveland sandstone and Indiana Limestone. The values for these rocks are listed in Table 9-5 and labeled as the measured values. The pressuremeter results for $K_i$ were obtained from FE simulation using the known measured elastic constants as the input material parameters while the orientation of the dip angle with respect to the ground surface was assigned as $\theta=45^\circ$. Two angles, $\hat{\theta}=45^\circ$ and $\hat{\theta}=15^\circ$, were used for Indiana Limestone and Loveland Sandstone, respectively. The FE simulation results are graphically shown in Figure 9-10(a) and Figure 9-10(b) for Indiana Limestone and Loveland Sandstone, respectively. The results show good comparison between the measured and predicted elastic constants as summarized in Table 9-5.

9.6 SUMMARY AND CONCLUSIONS

This chapter presented a practical procedure for the estimation of the elastic constants for the transversely isotropic rock by using the indirect pressuremeter test. A semi-empirical equation was developed relating $K_i$ with five elastic constants and the dip angle $\theta$. A case study presented in this chapter to validate the results obtained from the proposed procedure. Based on the FE simulation and parametric study results, the
following conclusions can be drawn:

1. The elastic constants, $G'$, $E$, and $E'$ can exert significant influences on $K_i$. The larger the values of $G'$, $E$, or $E'$, the stiffer the initial slope to the $(\Delta p, \Delta V/V_0)$ curve. Also the more inclined schistosity (i.e., the larger $\theta$ value), the steeper the slope to the $(\Delta p, \Delta V/V_0)$ curve up to $\theta$ equal $52^\circ$. Thereafter, the reverse trend was observed.

2. The variation of Poisson’s ratios $\nu$, and $\nu'$ has exerted a relatively minor influence on $K_i$. As a result, typical values of Poisson’s ratios can be used in determining the other elastic constants.

3. Young’s modulus, $E$, of transversely isotropic rock can still be determined quite accurately using the same procedure for determining $E$ of isotropic rock as long as the angle between a perpendicular to borehole’s axis and the line of dip measured in the plane of both the borehole’s axis and the line of the dip $\hat{\theta} \leq 25^\circ$.
Table 9-1: Parameters variation for the FEA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Base Line Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>5-50</td>
<td>15</td>
</tr>
<tr>
<td>E' (GPa)</td>
<td>0.5-50</td>
<td>5</td>
</tr>
<tr>
<td>G' (GPa)</td>
<td>0.25-20</td>
<td>2</td>
</tr>
<tr>
<td>ν</td>
<td>0.05-0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>ν'</td>
<td>0.005-0.3</td>
<td>0.15</td>
</tr>
<tr>
<td>θ</td>
<td>0°-90°</td>
<td>45°</td>
</tr>
</tbody>
</table>

*Baseline value is the reference value in the parametric study, when only one parameter is varied while keeping other parameters at the baseline value

Table 9-2: Sensitivity analysis of K_i

| Parameter | $K_{i\min}$ (10^6) | $K_{i\max}$ (10^6) | $|K_{i\max} - K_{i\min}|$ (10^6) | $\frac{|K_{i\max} - K_{i\min}|}{\sum|K_{i\max} - K_{i\min}|} \times 100\%$ |
|-----------|---------------------|---------------------|----------------------------------|----------------------------------|
| E         | 3117                | 10011               | 6894                             | 30.9                             |
| E'        | 4007                | 7987                | 3980                             | 17.9                             |
| G'        | 1173                | 8129                | 6956                             | 31.2                             |
| θ°        | 3885                | 7586                | 3701                             | 16.6                             |
| ν         | 6839                | 7446                | 607                              | 2.7                              |
| ν'        | 7054                | 7210                | 156                              | 0.7                              |

$\Rightarrow \sum \quad 22294 \quad 100$
Table 9-3: Effect of inclination $\theta$ of the pressuremeter to schistosity in the accuracy of the determination of $E$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>3%</td>
</tr>
<tr>
<td>15°</td>
<td>5%</td>
</tr>
<tr>
<td>20°</td>
<td>7%</td>
</tr>
<tr>
<td>25°</td>
<td>10%</td>
</tr>
<tr>
<td>30°</td>
<td>17%</td>
</tr>
</tbody>
</table>

Table 9-4: Effect of varying Poisson’s ratio $\nu$ on the accuracy of Equation (9-7) and Equation (9-9).

<table>
<thead>
<tr>
<th>Effect on</th>
<th>Relative error associated with $\nu \pm 10%$</th>
<th>Relative error Associated with $\nu \pm 20%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>2.8%</td>
<td>5.7%</td>
</tr>
<tr>
<td>$G'$</td>
<td>2.7%</td>
<td>5.6%</td>
</tr>
</tbody>
</table>
Table 9-5: comparing measured elastic moduli \((E, E', G')\) to estimated ones according to this method proposed in this study

<table>
<thead>
<tr>
<th>Material</th>
<th>Measured values * (GPa)</th>
<th>Estimated values this study (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\nu)</td>
<td>(\nu')</td>
</tr>
<tr>
<td>Indiana limestone</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td>Loveland sandstone</td>
<td>0.18</td>
<td>0.1</td>
</tr>
</tbody>
</table>

*Measured values are taken from Talesnick et al. (1999)
Figure 9-1 Typical pressuremeter records (Pressure vs. Volume change)
Figure 9-2 Transversely isotropic rock Cartesian coordinate system with X and Y axes laying on the isotropy plane and Z axis normal to it (axis of symmetry).
Figure 9-3 FE meshes of pressuremeter in rock media.
Figure 9-4 Cavity under uniform external pressures $p_o$ and incremental internal pressure $p_o + \Delta p$ (r=a, and d=b)
Figure 9-5 Validation of FE model for (a) Normalized radial stress versus radial distance and (b) Radial displacement versus Radial distance.
(a) \[ \bar{K}_i = 10.2(1 - e^{-0.03E}) \]

(b) \[ \bar{K}_i = 7.79(e^{-0.4E'}) \]
(c) \[ \bar{K}_i = 1.85 \bar{G}^{0.5} \]

(d) \[ \bar{K}_i = 7.5e^{-0.1667\nu} \]
Figure 9-6 Effects of transversely isotropic parameters on normalized Ki. (a) $E$ effect (b) $E'$ effect (c) $G'$ effect (d) Poisson’s ratio $\nu$ effect (e) Poisson’s ratio $\nu'$ effect (f) $\theta$ effect.
Figure 9-7 Comparison of FEM predictions and empirical predictions for $K_i$
Figure 9-8 Arrangement of a borehole to determine the modulus E from dilatometer tests (Borehole sunk perpendicular to the schistosity)
(a) $\theta = 15^\circ$
(b) $\theta = 30^\circ$
(c) $\theta = 45^\circ$
Figure 9-9 Design charts for estimating $\bar{E}$ and $\bar{E}'$ for different $\theta$ and different Poisson’s ratios $\nu$. 

(d) $\theta = 60^\circ$
Figure 9-10 FE Simulation results for (a) Indiana Limestone (b) Loveland Sandstone.

IX-33
CHAPTER X: STATISTICAL CROSS CORRELATION STUDY OF
ELASTIC CONSTANTS AND DEPENDENCY OF SHEAR MODULUS G’
FOR TRANSVERSELY ISOTROPIC ROCKS

10.1 INTRODUCTION:

10.1.1 General

Sedimentary rocks, such as shale and limestone, exhibit transversely isotropic elastic stress strain behavior. As illustrated in Figure 10-1, the elastic properties of rocks in the horizontal plane (x-y plane) have no preferred direction and are symmetrical about the perpendicular z axis. For such rocks, the elastic constrains are usually characterized by the following independent constants: E, Young’s modulus in the plane of transverse isotropy (x-y) plane; E’, Young’s modulus in a direction normal to plane of transverse isotropy (z direction); ν, the Poisson’s ratio which characterizes the lateral strain response in the plane of isotropy due to the on-plane applied stress; ν’, the Poisson’s ratio characterizing the lateral strain in the plane of isotropy due to stress applied perpendicular to the plane, and G’, the shear modulus for the planes normal to the plane of isotropy.

The determination of these five elastic constants is often a challenging task, in particular, the determination of shear modulus G’. Considerable research effort targeted investigating the potential correlations between these elastic constants using analytical methods or experimental data. Yet, there have been no conclusive studies to assert existence of strong correlations due to the lack of experimental data. However, for
practical engineering applications, there remains strong needs for evaluating and developing any possible correlation between these five elastic constants; in particular, the determination of $G'$ from other elastic constants.

10.1.2 Objectives and Scope of the work:

The Objective of this study is to compile the available large data base using statistical analyses to examine the possible relations between these elastic constants for practical engineering applications. Additionally, the relations between the elastic constants will be interpreted statistically using the comprehensive and well documented data base collected exclusively from the pertinent literature with experimental measurements of the elastic constants for different transversely isotropic rocks. A new empirical formula will be developed from the compiled database for estimating $G'$ using other elastic constants.

10.2 THEORETICAL BACKGROUND

It is well established in theory that the five elastic constants for characterizing the elastic stress strain relationship of a transversely isotropic material are independent. The generalized Hooke’s law of a linearly elastic, transversely isotropic medium is given in Equation (9-1).

Values of the five elastic constants should follow the following inequalities (Equations (10-1a) through (10-1c)) due to the requirement that the strain energy should be positive definite (thermodynamic constraints).
\[ E, E', G' > 0 \]  
\[ -1 < \nu < 1 \]  
\[ -\sqrt{\frac{E}{E'} \frac{(1-\nu)}{2}} < \nu' < \sqrt{\frac{E}{E'} \frac{(1-\nu)}{2}} \]

Determination of these elastic constants using laboratory or in-situ tests can be laborious. In particular, the test procedure for determining \( G' \) can be quite cumbersome due to the need for a particular testing apparatus and special sample preparation procedure. For instance, the elastic constants of the transversely isotropic rock cannot be determined with only one angle of inclination \( \theta \). The five elastic constants may be derived when specimens are tested at different isotropic plane inclinations. Determination of shear modulus \( G' \) requires the specimens be prepared with isotropic plane at \( 45^\circ \) from the uniaxial loading direction.

The shear modulus \( G' \) may also be determined in a torsion shear test, in which the isotropic plane of a cylindrical specimen needs to be parallel to the direction of the applied torsion loading. It is also important to note that the tests should be conducted in a small shear stress level to ensure that the test specimen remains in the elastic range. With the experimental difficulties outlined, it is therefore desirable and practical to reduce the number of unknown elastic constants by relating the shear modulus \( G' \) to other elastic constants. Most of the past literature on the study of empirical equations for determining \( G' \) was mainly centered on the classic Saint Venant’s (1863) expression:

\[ G' = \frac{EE'}{E + E' + 2\nu E'} \]  

\[ (10-2) \]
In a survey of elastic constants of rocks exhibiting anisotropy, Amadei (1996) concluded that the majority of the available experimental data support, to some extent, the validity of Saint Venant’s equation. In a recent publication, Talesnick and Ringle (1999) suggested a modification of Saint Venant’s equation to include a multiplication factor that would take into account the relative difference between E and E’. The modified G’ takes the following form:

\[
G' = \frac{EE'}{E + E' + 2\nu E'} \left[ \frac{2E - E'}{E} \right] \quad (10-3)
\]

It should be pointed out that both Equations (10-2) and (10-3) are reduced to an isotropic form when the deformation moduli E and E’ are identical:

\[
G = \frac{E}{2(1+\nu)} \quad (10-4)
\]

In addition to Saint Venant’s empirical expressions, other empirical expressions were also proposed for estimating G’. For instance, Wittke (1990) suggested that G’ might be obtained from an analogous relationship to Equation (10-4) as follow:

\[
G' = \frac{E'}{2(1+\nu')}
\]

By differentiating between the estimates of G’ either by assuming a plane stress or a plane strain condition, Exadakylos (2001) proposed a relationship for G’. Also, Cauwelaert (1977) and Kiehl (1980) stated that empirical approximate expressions for G’ based on other elastic constants were feasible. In Chapter III, an empirical relationship for estimating G’ was proposed based on experimental data from laboratory testing.
However, that equation could not reduce to isotropic relationship when elastic moduli $E$ and $E'$ are the same. A summary of common empirical relationships for $G'$ is presented in Table 10-1.

### 10.3 ELASTIC CONSTANTS DATABASE

Through extensive review of the available literature, the authors have compiled the results of 113 data sets for the five elastic constants for different transversely isotropic rock types. The compiled database is presented in

Table 10-2, which contains the results of both static and dynamic tests of rocks. In preparing this table, only the data that include the five elastic constants have been used. Since the database was collected from the literature; the authors exercised judgment in excluding some of the data sets that deemed to be incomplete, missing one or more of the elastic parameters, or lacking sufficient documentation. Nevertheless, every attempt was made to include every set of data and as many contributions to the database as possible.

The information contained in Table 10-2 includes the following: (1) reference of the source of data set, (2) rock type, and (3) values of the reported elastic constants. Sources for the data sets in

Table 10-2 include publications by Amadei (1996), Talesnick and Ringle (1999), Batugin and Nierenburg (1972), Chen et al. (1998), Essia (1980), Gerrad (1975), Hakala et al. (2007), Homand et al. (1993), Johnston and Christensen (1994), Liao et al. (1997), Lo et al. (1986), and Wong (2008). Rock types presented in these references cover the
majority of rocks that may be regarded as transversely isotropic, including different Sandstone, Argillite, Slate and Chlorite Slate, Shale and Chicopee Shale, Chelmsford Granite, Siltstone, Phyllite, Basalt, Periodotite, Sylvinite, Limestone, Marble, Magmatite, Grano Diortite, Amphibolites, Khibinite, Chalk, Aircraft Aluminum, Mica Gneiss, Schist and Muscovite Mica Schist. In fact, Worotnicki (1993) categorized anisotropic rocks into four major groups: (1) Quartzofeldspatic rocks such as granites and sandstones, (2) Basic/Lithic rocks such as Basalt and Amphibolites, (3) Peletic (clay) and Peltic (micas) rocks such as slates and schist, and (4) Carbonate rocks such as limestone and marbles.

10.4  CORRELATIONS BETWEEN ELASTIC CONSTANTS:

The possible values and their variations of five elastic constants for the transversely isotropic rock were investigated by Gerrard (1975) and later discussed in depth by Amadei (1987) and Amadei et al. (1996). However, no effort has been reported in the literature to discuss the possible cross-correlations among these five elastic constants.

10.4.1 Correlation between E and E’

Using the data contained in Table 10-2, the elastic Young’s modulus E in the isotropy plane was plot against the elastic Young’s modules E’ in a plane normal to it, as shown in Figure 10-2. One can see that in most rock types, the Young’s modules E tends to be higher than Young’s modules E’, indicating that the higher stiffness is associated with the direction of plane of isotropy in the transversely isotropic rock. In terms of the degree of anisotropy, μ, defined as E/E’, the analysis shows that the mean value of μ is approximately 1.38 with a standard deviation of approximately 0.43. Moreover, it was
found that the degree of anisotropy $\mu$ has a tendency to increase as the rock moduli increases, indicating that the stiffer the rock, the more the possibility of higher degree of anisotropy.

The best correlation between the increase of anisotropy degree and modulus was found to be the second order polynomial with R square equal to 0.7. As seen from Table 10-2, the anisotropy degree was not greater than 3.2. Few cases were reported in which the anisotropy degree fell below unity, but not less than 0.74; they were noted mainly for some Sandstone and Slate. These findings seem to be in agreement with the conclusions drawn by Amadie (1987).

10.4.2 Correlation between $G$ and $G’$

Results of the statistical analysis between $G’$, the shear modulus for the plane normal to the plane of isotropy, and $G$, the shear modulus in the plane of isotropy, are presented in Figure 10-3. Here, a second order polynomial seems to provide a good fit for the data sets with R square value of about 0.61. For the majority of rock types, the shear modulus $G$ tends to be higher than $G’$, and the shear moduli ratio $G’/G$ is less than unity with a mean value and a standard deviation in the order of 0.82 and 0.22, respectively. An exception to this was mainly observed in the case of some Sandstone as well as Indiana Limestone, where $G’/G$ was equal to 1.1. It can also be noticed that the increase of $G’/G$ is associated with a decrease in $E/E’$, providing some insight for possible relationships between these two ratios. The nature of such a relationship will be discussed later in this
10.4.3 Correlation between Poisson’s Ratios v and v’

Most values of reported v lie within 0.075 to 0.33, while most values of v’ lie within 0.1 to 0.35. A plot of v versus v’ is presented in Figure 10-4, with highly scattered data not exhibiting any trend. However, it can be seen from Figure 10-4 that the majority of the values are spinning around the equality line indicating relatively close values. Also, the ratio (v/v’) were found to slightly depart from the unity, with the mean value of approximately 0.98 and the standard deviation of about 0.38. Most of the values (93% of them) were found to be greater than 0.5 and less than 2.

10.5 DEVELOPMENT OF EMPIRICAL EXPRESSION FOR G’

10.5.1 Statistical Basis

The database presented in Table 10-2 and the statistical parametric study presented earlier form the base for the development of empirical expressions for the shear modulus G’ as a function of other elastic constants. The correlations discussed earlier showed possible relationship between the degree of anisotropy, μ=E/E’, and the shear moduli ratio, G/G’. The illustration for such a relationship is presented in Figure 10-5, which excludes the data points that could be outliers determined by their values greater or less than two standard deviations from the mean values of ratios E/E’ and G/G’. Figure 10-5 shows relatively good correlation between G/G’ and the degree of anisotropy μ, represented by an exponential expression with R square equal to 0.76. However, it should be noted that the exponential correlation
becomes relatively weak as the degree of anisotropy increases, indicating the need for better correlation.

10.5.2 Regression Analysis of the Transverse Isotropy Constants

For the purpose of the regression analysis, the elastic constants of the transversely isotropic rocks presented in

Table 10-2 were analyzed by using a statistical analysis software SPSS (2003) program. A simple regression analysis was resulted in the following correlations for the normalized shear modulus $G'/G$:

$$\frac{G'}{G} = A \cdot f(\nu) \cdot f(\nu_{ave}) \cdot f(\mu)$$

(10-6)

where $A$ is the regression constant and $\nu_{ave}$ is the mean value of the Poisson’s ratios. i.e., $(\nu+\nu')/2$.

The built-in Levenberg-Marquardt algorithm (LMA) in the SPSS was used to perform the curve fitting to find various functions in Equation (10-6). The best solution obtained from the regression analysis is presented in the following equation:

$$G' = 0.2 \cdot \frac{E(1 + \nu_{ave})}{(1 + \nu)^2 (1 - e^{-0.5\nu})}$$

(10-7)

The result of equation (10-7) was compared to the experimentally measured shear modulus $G'$ in Figure 10-6. Referring to this figure, it can be seen that Equation (10-7) provides good estimate, with relatively high correlation with $R$ square equal to 0.92. In fact, it seems that the trend line of the data set perfectly matches the equality line and the data points are not deviated from the equality line, indicating high correlation.
The contribution of each elastic constant to equation (10-7) was also plotted in Figure 10-7 for the effects of $E$, $E'$, $\nu$, and $\nu'$. Based on this figure, $G'$ is shown to be directly proportional to $E$, $E'$, and relatively directly proportional to $\nu'$ and inversely proportional to $\nu$. The relative importance (or sensitivity) of each single input parameter on the $G'$ is calculated by dividing the absolute value, $|G'_{\max} - G'_{\min}|$, of each individual parameter by the summation of the absolute of all values, $\sum|G'_{\max} - G'_{\min}|$. The results are also summarized in Table 10-3. The order of importance of the investigated parameters affecting $G'$ can be ranked from high to low as follows: $E$, $E'$, $\nu$, and $\nu'$.

To better estimate the shear modulus $G'$ of a transversely isotropic rock with a simpler expression, another set of regression analysis was conducted for the data sets where the following inequities are satisfied:

\begin{align}
1 & \leq \mu \leq 2 \\
\frac{2}{3} & \leq \frac{\nu}{\nu'} \leq \frac{3}{2}
\end{align}

(10-8-a)

(10-8-b)

Using a statistical analysis software SPSS (2003), the shear modulus $G'$ can be expressed in the following equation:

$$G' = 0.2 \frac{E}{(1 + \nu_{ave})(1 - e^{-0.51\mu})}$$

(10-9)

The shear Modulus $G'$, estimated from Equation (10-9) was compared to the experimentally measured shear modulus $G'$ in Figure 10-7, which clearly shows that $G'$ values estimated from Equation (10-9) correlates well with the experimentally measured
values, with an R square of 0.95.

The above empirical formulae (10-7) and (10-9) can degenerate to the isotropic form Equation (10-4) when the moduli E and E’ are identical. It should be pointed out that the development of the above two empirical equations for estimating G’ was for practical engineering approximation purpose. It is not to refute the theoretical soundness of the independence of the five elastic constants of a transversely isotropic rock.

10.6 COMPARISONS WITH OTHER MODELS

10.6.1 Statistical Comparative Study

In this comparative study, the computed shear modulus G’ using the empirical equations by Saint Venant (1863), Talesnick and Ringle (1999), Wittke (1990), and Kiehl (1980) are compared with the measured in Table 10-4. The statistical comparisons are presented in one of the following three forms:

(1) mean value of Bias, which is the experimentally measured G’ divided by the empirically predicted G’

(2) the standard deviation of the bias, and

(3) the statistical measure of goodness of fit, R square.

These statistical comparisons are summarized in Table 10-4, based on which, the best estimate of G’ is shown to be associated with the use of empirical equations developed in this chapter. Table 10-4 also shows that the mean value of the bias equal to 0.99 is associated with the use of Equation (10-7) with a standard deviation of no more than 12% and R square in the order of 0.92. When experimental measurements of G’ are compared
with the estimated values using Equation (10-2), then mean of the bias, the standard
deviation, and R square are 0.91, 20%, and 0.78, respectively. Also, it can be noted that
the lowest R square is associated with the use of Talesnick and Ringle (1999) expression
(Equation (10-3)).

10.6.2 Confidence Interval Limits Comparisons

The level of confidence of estimated values from Equations (10-2) and (10-7) as well
the illustrations presented in Figure 10-9 and Figure 10-10 is described by comparisons
with the experimentally measured shear modulus $G'$ for which the only data sets of
measured and Estimated $G'$ that lies within two standard deviations from the bias, 95%
confidence interval, are considered. Figure 9-9 shows a relatively equal absolute upper
and lower bounds that do not deviate nor bias from the equality line, while Figure 10-10
reveals, in general, the tendency of shear modulus $G'$ estimated from Equation (10-2) to
overestimate the experimentally measured shear modulus $G'$. The absolute difference
from the equality line in Figure 10-10 shows upper bound of almost three times the lower
bound. This reinforces the assertion regarding the tendency of Equation (10-2) to
overestimate the actual shear modulus of the transversely isotropic rocks.

10.7 SUMMARY AND CONCLUSIONS

In this chapter, a large database was collected for evaluating statistically the possible
cross-correlations between the five elastic constants of the transversely isotropic rocks.
Two different empirical equations for estimating the shear modulus $G'$ in terms of the
other elastic constants were presented. The comparisons between experimentally

X-12
measured and the predicted shear modulus from other existing empirical equations provided the following observations.

1. Although the five elastic constants of the transversely isotropic rocks are theoretically independent, this study showed the existence of empirical relations among these constants.

2. The newly developed shear modulus prediction equations presented in Equation (10-7) can predict accurately the experimentally measured shear modulus $G'$. Furthermore, Equation (10-7) can predict almost perfectly $G'$ when the inequalities (Equations 10-8-a, and 10-8-b) are satisfied.

3. Among the existing empirical equations for estimating the shear modulus $G'$, the equations presented in this study showed the best prediction capability compared to other equations.

In closing, developing empirical equations for predicting $G'$ from other elastic constants of the transversely isotropic rock should be viewed as a practical engineering approach to alleviate the burden of carrying out often difficult and tedious laboratory tests.
Table 10-1: Empirical Equations for estimation of the shear modulus $G'$

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<tr>
<th>Author</th>
<th>Empirical Correlations</th>
<th>Remarks</th>
</tr>
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<tr>
<td>Saint Venant [1]:</td>
<td>$G' = \frac{EE'}{E + E' + 2\nu E}$</td>
<td></td>
</tr>
<tr>
<td>Talesnick and Ringle [3]</td>
<td>$G' = \frac{EE'}{E + E' + 2\nu E'}\left[\frac{2E - E'}{E}\right]$</td>
<td></td>
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<tr>
<td>Wittke [4]</td>
<td>$G' = \frac{E'}{2(1 + \nu')}$</td>
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</tr>
<tr>
<td>Exadaktylos [5]</td>
<td>$G' = \frac{\sqrt{2\nu'}(1 + \nu)}{2\nu'(1 + \nu) + \left[\frac{(1 - \nu'^2)E/E'}{E'} - \frac{(1 - \nu^2)}{E}\right]}$</td>
<td>(plane strain)</td>
</tr>
<tr>
<td>Cauwelaert [6]</td>
<td>$\frac{1}{G'} = \left[\frac{1}{E} + \frac{1}{E'} + 2k\right]$</td>
<td>$k = \nu'/E'$</td>
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<td>kiehl [7]</td>
<td>$G' \leq \frac{E'}{2[\nu'(1 + \nu) + (E'/E - \nu'^2)(1 - \nu^2)]}$</td>
<td>(upper limit)</td>
</tr>
<tr>
<td>This Study (Chapter III) [8]</td>
<td>$G' = \frac{0.032 E}{(1 + \nu)(1 - e^{-0.06 E'/E})}$</td>
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* Average of 6 Specimens
** Average of 2 Specimens
*** Average of 19 Specimens
## Table 10-3: Sensitivity Analysis of Ki

| Parameter | $G'_\text{min}$ (MPa) | $G'_\text{max}$ (MPa) | $\frac{|G'_\text{max} - G'_\text{min}|}{(10^6)}$ | $\frac{\sum |G'_\text{max} - G'_\text{min}|}{100}$ |
|-----------|-----------------------|-----------------------|----------------------------------|----------------------------------|
| $E$       | 15.4                  | 26.4                  | 11.0                             | 31.0                             |
| $E'$      | 11.5                  | 27.8                  | 16.3                             | 45.7                             |
| $\nu$     | 15.1                  | 21.4                  | 6.3                              | 17.6                             |
| $\nu'$    | 17.3                  | 19.3                  | 2.0                              | 5.7                              |

$\Rightarrow \sum$ | 35.6 | 100
Table 10-4: Comparing experimentally measured G’ to Estimated G’ with different models in terms of mean of bias $\bar{X}$, standard deviation $\sigma$, and measure of goodness $R^2$.

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<th>Standard deviation of the Bias $\sigma$</th>
<th>Measure of Goodness $R^2$</th>
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* Values presented are the upper bound ones
+ Values presented are based on database satisfy: $1 \leq \mu \leq 2$, $3/2 \leq \nu/\nu' \leq 2/3$. 
Figure 10-1 Transversely isotropic rock Cartesian coordinate system with X and Y axes laying on the isotropy plane and Z axis normal to it (axis of symmetry).
Figure 10-2 Plot of the elastic modulus $E$ in the isotropy plane against the elastic modules in a plane normal to it $E'$. 

$$y = -0.0035x^2 + x$$

$R^2 = 0.7$
Figure 10-3 Plot of the shear modulus in the plane of isotropy $G$ against the shear modulus for planes normal to the plane of isotropy $G'$. 

$y = -0.01x^2 + 1.1x$

$R^2 = 0.61$
Figure 10-4 Plot of elastic parameters of transverse isotropy Poisson’s ratio $\nu$ against Poisson’s ratio $\nu'$.

Figure 10-5 Plot of Degree of anisotropy ($\mu=E/E'$) against the shear ratio $G/G'$. 

$G/G' = 0.40 e^{0.71\mu}$
$R^2 = 0.76$
Figure 10-6 Comparison plot of experimentally measured shear modulus $G'$ and estimated shear modulus from this study (Equation (10-7)).
Figure 10-7 Effects of transversely isotropic elastic constants on $G'$ from Equation (10-9). (a) effect of $E$, (b) effect of $E'$, (c) effect of $\nu$, (d) effect of $\nu'$. 
Figure 10-8 Comparison plot of experimentally measured shear modulus $G'$ and estimated shear modulus from this study (Equation (10-9)).

Figure 10-9 Experimentally measured $G'$ versus $G'$ Estimated from Equation (10-7), upper and lower bounds based on 95% confidence interval.
Figure 10-10 Experimentally measured $G'$ versus $G'$ Estimated from Equation (10-2), upper and lower bonds based on 95% confidence interval.
CHAPTER XI: TRANSVERSELY ISOTROPIC p-y CRITERION INDUCED FROM PRESSUREMETER TEST

11.1 INTRODUCTION

Several research efforts have been carried out in the past for the design of laterally loaded drilled shafts using pressuremeter measurements, Braiud et al. (1983), Baguline et al. (1978), Robertson et al. (1983), and Baguelin (1982). Most of these methods were developed based on pressuremeter tests in soils. However, the pressuremeter has advanced significantly to widen its application to the in situ testing of several geomaterials, including most types of rock.

Due to the inherent mineral grain orientation and the presence of bedding planes of parallel sets of joints, rock mass often exhibits transversely isotropic stress strain behavior. Up till recently, only little research on the application of the pressuremeter in a transversely isotropic rock were carried out and almost no attempt was documented for the application of the pressuremeter as an is-situ method for the design of laterally loaded drilled shafts socketed into the transversely isotropic rock mass. In addition, there are several methods available for developing p-y curves from pressuremeter test (PMT). Yet, the application of these methods for pressuremeter tests in the transversely isotropic rock has not been verified yet, and there was no specific method to predict the behavior of the laterally loaded drilled shaft in a transverse isotropy media. This chapter presents a new methodology for deriving a transversely isotropic rock p-y criterion from the pressuremeter tests.
Provided that the pressuremeter was installed to model the geomaterial disturbance during drilled shaft installation, it is practical to assume that the geometric shape of the pressuremeter test results from the pressuremeter test would be analogous to the p-y curve. Generally, the laterally loaded drilled shafts have exhibited limiting geomaterial reactions that are higher than those for a radially expanding pressuremeter. Baguelin et al. (1978) and later Hughes et al. (1979) and Robertson et al. (1983) suggested that the pressuremeter curves should be increased by some factor ($\eta$) to yield the p-y curves for the laterally loaded drilled shaft (Figure 11-1).

Baguelin et al. (1978) suggested that ($\eta$) factor to be between 0.33 to 3. Hughes et al. (1979) and Robertson et al. (1983) suggested that the multiplication factors are $\eta=2$ for cohesive soils and $\eta=1.5$ for cohesionless soils. Regardless the unsuitable assumption of one single value for the wide variety of cohesive or cohesionless soils, no attempt was made to find this multiplication factor for the transversely isotropic rock. Thus, the research effort presented in this chapter focuses on the determination of a suitable lateral resistance factor $\eta$ for the transversely isotropic rock media.

The objective of this chapter is to present a methodology for utilizing a unified hyperbolic mathematical formulation for p-y criterion in a transversely isotropic rock by way of the in-situ pressuremeter test results. To estimate the parameters of the hyperbolic p-y formulation, (initial tangent to p-y curve, $K_T$, and ultimate resistance, $p_u$), two approaches have been discussed: (1) based on theoretical derivations to estimate $K_T$ and (2) based on a 3-D results of a pressuremeter test in transversely isotropic media to
deduce the relevant parameters for estimating ultimate resistance, $p_u$. More interestingly, the full-scale fully instrumented lateral load tests results at Dayton, WAR-48, and JEF-152 sites were used to validate the proposed p-y criterion in predicting the load deflection and the bending moment distribution of the drilled shafts under the applied lateral loads.

11.1.1 General Background

The design and analysis of the laterally loaded drilled shaft behavior using the p-y approach has been the most used in recent years (Matlook and Reese, 1962) and (Reese et al., 1974). However, after a careful review of literature, it appears that only recently that a few researchers have devoted their attention to the proper analysis method for analyzing the laterally loaded drilled shafts socketed in a rock mass that exhibits the transversely isotropic elastic behavior. Chapter IV presents a p-y criterion for the transversely isotropic rock mass for analyzing the rock socketed drilled shafts under the lateral loads.

Obviously, pressuremeter test has become an efficient in-situ test method in the design of the drilled shafts subjected to horizontal loads, due to the noticeable analogy between the cylindrical expansion of the PMT probe and the horizontal movement of a laterally loaded drilled shaft segment. However, this analogy requires several steps to extrapolate from the pressuremeter expansion curve to the p-y curve for the drilled shaft. Existing methods to clarify analogy of pressuremeter expansion curve and the p-y curve differ in these extrapolation steps.

Briaud, (1986) summarized the available nine (9) methods for designing piles under horizontal load using the pressuremeter test results. Only a few of these methods are discussed herein where we believe they are of interest to this study.
Menard et al. (1969) presented a method for deriving the p-y curves from preboring pressuremeter test. It considers that the p-y curve to be bilinear elastic and perfectly plastic. Based on Menard’s analysis on settlement of a strip footing, the value of the first slope of p-y curve, K, was proposed as follows.

\[
\frac{1}{K} = \frac{2D_o}{9E} \left[ \frac{2.65D}{D_o} \right]^\alpha + \frac{\alpha D}{6E}
\]

(D>0.6M) \hspace{1cm} (11-1)

\[
\frac{1}{K} = \frac{D}{E} \left[ \frac{4(2.65)^\alpha + 3\alpha}{18} \right]
\]

(D>0.6M) \hspace{1cm} (11-2)

where \( D \) is the diameter of piles; \((D_0 = 0.6 \, \text{m})\) is the reference diameter; \( E \) is the modulus of soils from the test; \( \alpha \) is a rheological factor, which is dependent on the soil type and the ratio \( E / p_i \) (\( p_i \) is the net limit pressure). The slope of the second linear part of the p-y curve is assumed to be half of the first slope \((K_2=0.5 \, \text{K})\).

According to Baguelin, et al. (1978), the p-y curve at a depth \( z \) for the pile is obtained from the pressuremeter expansion curve at same depth \( z \) as follow:

\[
p = \eta P^* D
\]

\[y = \frac{1}{2} \frac{\Delta V}{V_o} R
\]

(11-3) \hspace{1cm} (11-4)

where \( p \) is the soil resistance on the pile expressed as a force per unit length of pile, \( y \)
is the pile horizontal displacement, $\eta$ is the lateral resistance factor varying from 0.33 to 3, $P^*$ is the net pressure ($P - P_s$) in the pressuremeter curve, $R$ is the pile radius, $V_o$ is the initial volume of the probe, $\Delta V$ is the volume of fluid injected into the probe, $P$ is the membrane pressure, and $P_s$ is the in-situ at-rest horizontal stress.

Based on the similarity of pressuremeter test and the lateral loading of a pile, Robertson et al. (1983) derived p-y curves directly from pressuremeter tests. The following procedure was proposed by Robertson et al. (1983).

$$p = \eta P^* D$$

(11-5)

$$y = \frac{1}{4} \frac{\Delta V}{V_o} D$$

(11-6)

where $D$ is the diameter of piles, $\eta$ is 2 for clays and 1.5 for sands (Robertson et al., 1986). This method is similar to Baguelin et al. (1978) method except the value of coefficient $\eta$ is different. However, further evaluation of $\eta$ factors is still required to account for the different geomaterials.

Briaud, et al (1983) developed a method for deriving p-y curves from the results of pressuremeter test. Briaud method considers that a p-y curve is made of a front resistance $Q$-y curve and a friction resistance $F$-y curve. This method is also applicable to dilatometer test results.

The Q-y and F-y curves can be obtained point by point from the pressuremeter curve as follows:
where \( Q \) = the frontal soil resistance on the pile, \( D \) = pile diameter or width, \( P^* \) is the net pressure \( (P - P_c) \), \( SQ \) = shape factor for pressure reaction and is equal to \( \pi/4 \) for circular piles and 1.0 for square piles.

\[
Q = (SQ)(P^*)(D) \quad (11-7)
\]

in which, \( F \) = the frictional soil resistance on the pile, \( SF \) = shape factor for shear reaction and is equal to 0.79 for circular piles and 1.76 for square piles, \( \Delta P^* \) is the increase of net pressure, and

\[
F = (SF)(D)(X)(1 + X) \frac{\Delta P^*}{\Delta X} \quad (11-8)
\]

\( X = \Delta V / V_o \) where, \( V_o \) is initial volume of the probe and \( \Delta V \) is the volume of the fluid injected into the probe from the start point of the test.

\[
y = \frac{1}{4} \frac{\Delta V}{V_o} D \quad (11-9)
\]

Where \( y \) = the horizontal displacement of the pile.

The above reviewed methods for deriving p-y curves from pressuremeter test results were developed for applications in soils. Briaud et al. (1983) method does not require the estimation of the yield pressure \( p_y \) and the limit pressure \( p_l \) where the yield pressure \( p_y \) is the pressure measured from the start of the test to the end of the straight line portion of the pressuremeter curve. However, with the development of the computer software as being efficient method to simulate the geotechnical problems, estimation of yield and limit pressures becomes possible using numerical simulations.
Based on a methodology for estimating p-y criterion for transversely isotropic rock based on theoretical derivations and numerical (finite element) parametric analysis results presented in Chapter VII and the work conducted by Nusairat et al. (2006) it was concluded that a hyperbolic mathematical representation can offer the best fit to the data set obtained in a transversely isotropic media. Therefore, it was decided to develop a hyperbolic a p-y method approach for a transversely isotropic rock media using Baguelin et al. (1978) method as a base to deduce the hyperbolic p-y curve parameters $K_T$ and $p_u$. A numerical FE simulation for the pressuremeter test in a transversely isotropic media was used in generating pertinent data for the development efforts.

11.2 GENERAL SHAPE OF P-Y CURVES (HYPERBOLIC CURVES)

As stated earlier, a hyperbolic equation was believed to be appropriate for mathematical modeling of the p-y curves for the transversely isotropic rock. However, the key controlling parameters in characterizing the hyperbolic curve are: (1) the initial tangent slope corresponds to the subgrade modulus, $K_T$ and (2) the asymptote corresponds to the ultimate resistance, $p_u$. The hyperbolic p-y relationship can be written as:

$$p = \frac{1}{1 + \frac{1}{yK_T + \frac{1}{p_u}}}$$

(11-10)

where

$p =$ force per unit shaft length (F/L); $y =$ lateral displacement of shaft (L); $K_T =$ initial
subgrade reaction modulus of the soil/rock (F/L^2); \( p_u \) = the ultimate lateral resistance (F/L).

11.3 ESTIMATION OF SUBGRADE MODULUS \( K_T \) FROM PRESSUREMETER TESTS

Considering the following Equation (11-11) for the initial tangent of the pressuremeter curve, (i.e., \( K_i \)), obtained from the pressuremeter test results (\( \Delta p, \Delta V / V_o \)).

\[
P' = K_i \frac{\Delta V}{V_o}
\]

(11-11)

Further extending the method proposed by Baguelin et al. (1978) and rearranging Equation (11-4) and plugging it into Equation (11-11) and further substituting into Equation (11-3), the force per unit length \( p \) can be expressed as:

\[
p = 4\eta K_i y
\]

(11-12)

where \( K_i \) is the initial tangent to the linear portion of (\( \Delta p, \Delta V / V_o \)) curve obtained from the pressuremeter test.

By differentiating Equation (11-12) at the early portion of the p-y curve,

\[
\frac{dp}{dy} = K_T = 4\eta K_i
\]

(11-13)

\( K_T \) is the initial tangent to the p-y curve (subgrade modulus), \( \eta \) is the lateral resistance
factor for a specific geomaterial.

From Equation (11-13), it is obvious that in order to determine the initial tangent to the $p$-$y$ curve (i.e., $K_T$) from the pressuremeter test, we need to identify the lateral resistance factor ($\eta$) for the transverse isotropic medium.

11.3.1 Determination of Lateral Resistance Factor of the Transversely Isotropic Rock

Referring to Equation (11-13), it appears that the lateral resistance factor $\eta$ for a transversely isotropic rock is a function of the initial tangent to the $p$-$y$ curve, $K_T$, and the initial tangent, $K_i$, to the linear portion of $(\Delta p, \Delta V/V_0)$ curve obtained from the pressuremeter test. Based on two separate numerical FE parametric studies presented earlier in the report, a method for estimating the initial tangent $K_T$ to the $p$-$y$ curve for a transversely isotropic medium was developed and an empirical relationship between the five elastic constants of a transversely isotropic rock and the initial tangent to the pressuremeter curve $K_i$ was proposed. A parametric study including all the possible factors that may affect the prediction of the lateral resistance factor, $\eta$, was conducted in this research. Parameters studied in this work are summarized in Table 11-1 and, further, the lateral resistance factor is assumed to vary in accordance with the following expression;

$$\eta \propto f(E, E', G', \nu, \nu', \theta, D, \gamma_{rock})$$

where $E$ and $\nu$ are Young’s modulus and Poisson’s ratio in the plane of transverse isotropy, respectively; $E'$, $G'$, and $\nu'$ are Young’s, shear modulus, and Poisson’s ratio in
the perpendicular plane (Figure 11-2); \( \theta \) is the dip angle (i.e., \( \theta = \) the orientation of the plane of anisotropy in relation to the axis of pressuremeter membrane or borehole’s axis of symmetry); and \( \gamma_{\text{rock}} \) is the unit weight of the transversely isotropic rock.

The effect of varying each single input parameter on \( \eta \) is calculated by dividing the absolute value, \( |\eta_{\text{max}} - \eta_{\text{min}}| \), as affected by each individual parameter, by the summation of the absolute of all values, \( \sum |\eta_{\text{max}} - \eta_{\text{min}}| \), as affected by all parameters. The results are summarized in Table 2-3. The order of importance of the parameters affecting \( \eta \) (i.e., a lateral resistance factor of the transversely isotropic rock) can be ranked from high to low as follows: \( G' \), \( E' \), \( D \), \( E \), \( \theta \), \( \nu' \), \( \nu \), and \( \gamma_{\text{rock}} \). It should also be noted that the unit weight of the rock exerted relatively no effect, with importance percentage approaches \( \approx 0.0\% \). Using curve fitting techniques, the parametric results for the relationships between the lateral resistance factor \( \eta \) and the important variables affecting its values are plotted in Figure 11-3(a through e) for the effects of five elastic constants and in Figure 11-13(f and g) for the effect of the dip angle (\( \theta \)) presented as \( \sin \theta \), and the effect of the shaft diameter is shown in Figure 11-3(h). Also, it should be pointed out that the values of \( E \), \( E' \), and \( G' \) plotted in Figure 11-3 are normalized values defined as follows:

\[
E_n = \frac{E(kPa)}{10^4 P_\sigma(kPa)} \quad (11-14-A)
\]

\[
E' = \frac{E'(kPa)}{10^4 P_\sigma(kPa)} \quad (11-14-B)
\]

\[
G' = \frac{G'(kPa)}{10^4 P_\sigma(kPa)} \quad (11-14-C)
\]
where \( p_a \) is the absolute atmospheric pressure and equals to 101.3 kPa.

From Figure 11-3(a through c), it can be seen that \( \eta \) increases with the increase of E, E', G'. However, this general trend becomes less noticeable as the rock becomes stiffer (high values of E and E'). As shown in Figure 11-3(d) and Figure 11-3(e), \( \eta \) decreases slightly with increasing Poisson’s ratios \( v \) and \( v' \). The effect of \( \theta \) on \( \eta \) is shown in Figure 11-3(f and g). It appears that \( \eta \) increases with increasing \( \sin \theta \) up to 0.76 (\( \theta = 50^\circ \)). Thereafter, it can be seen that \( \eta \) decreases with increasing \( \sin \theta \) for the range between 0.76 and 1 (\( \theta \) between 50° and 90°). Finally, it can be seen that the shaft diameter can have an effect on the lateral resistance factor. The larger the shaft diameter, the higher the lateral resistance factor \( \eta \). This increasing trend is best approximated by an exponential curve as can be seen in Figure 11-3(h).

The computational results of the parametric study were analyzed by using a statistical analysis software SPSS (SPSS, 2003). From the regression analysis, the empirical equation for estimating \( \eta \) from the parameters in Table 11-1 is best given in Equation (11-15).
\[ \eta = 1.5\eta_1(E_n, G_n', D)\eta_2(E'_n, \nu, \nu')\eta_3(\theta) \]  

(11-15)

\[ \eta_1 = (e^{0.025D/D_{ref}})(1.6 - 0.002E_n')\left(\frac{G_n'}{7.5 + G_n'} + e^{-0.7G_n'}\right), \]  

(11-15-A)

where \( D_{ref} = 1m \)

\[ \eta_2 = 9e^{-(0.6\eta + 1.5\nu')}(4/3 - e^{-0.35(\eta')^{0.68}}) \]  

(11-15-B)

\[ \eta_3 = 0.18 + 45e^{2\theta} \quad \text{FOR } \theta \leq 50^\circ \]  

(11-15-C1)

\[ \eta_3 = 0.35e^{\sin\theta} - 2/3 \quad \text{FOR } \theta > 50^\circ \]  

(11-15-C2)

A comparison between the parametric results for \( \eta \) and the empirical predictions of \( \eta \) using Equation (11-15) was statistically examined. The comparison showed a good match between the predicted and the measured \( \eta \) values, and the measure of goodness of fit R-squared was found to be 0.96 as shown in Figure 11-4.

### 11.4 ESTIMATION OF THE ULTIMATE RESISTANCE, \( p_u \)

The second essential parameter to construct the hyperbolic p-y curve is the determination of the ultimate resistance \( p_u \). Using the pressuremeter tests results, Menard et al. (1969) figured out that in order to estimate the ultimate lateral resistance, \( p_u \), it is convenient to presume the \( p_u \) linked with \( p_l \)

\[ p_u = p_lD \]  

(11-16)
where $P_l$ is the limit pressure of the pressuremeter tests and is defined to be the pressure needed to double the initial volume of the cavity. However, doubling the initial volume of the cavity is not always reachable during a field pressuremeter test. It is possible, through correlations, to determine the limit pressure $P_l$ with earlier pressures such as $p_{20}$ (pressure in the probe at 20% volumetric strain), or $p_5$ (pressure in the probe at 5% volumetric strain). Usually correlations of this nature require recognition of the rock identification number $\beta$, which is typically given by the following equation:

$$\beta = \frac{p_{20} - p_5}{p_{20} - p_0}$$  \hspace{1cm} (11-17)

where: $p_{20}$ and $p_5$ are defined earlier and $p_0$ is soil pressure at rest.

To achieve these correlations for a transversely isotropic rock, a numerical finite element simulation of the pressuremeter test was conducted using FE ABAQUS program (2006).

11.4.1 FE Numerical simulation

In this study, the finite element method was used to simulate the pressuremeter test in a transversely isotropic media. In essence, it was to simulate a 3-D expansion of a finite length pressuremeter membrane in an elasto-plastic, homogenous, transversely isotropic rock media. The ABAQUS software was used to simulate the pressuremeter test in this work where the transversely isotropic elastic constants were used. The plasticity was described on the basis of strain ratios (Lankford’s r-values defined as the ratio of width to thickness strain). These r-values ($r_x, r_y$) were set to be 1 to recover the Mises isotropic plasticity model used in this FE study.
The 3-D finite element mesh (Figure 11-5) was generated from 8-node hexahedral brick elements. Because of symmetry, only half of the transverse isotropic media needs to be modeled. The radius of the pressuremeter membrane was set to be one unit with the length being 10 units to achieve the length to diameter ratio (l/d) of 5. The length of the mesh in the radial direction was set to be 25 units and in the axial (i.e., vertical) direction 15 units above and below the center of the pressuremeter membrane. A schematic diagram of the mesh that exemplifies this is shown in Figure 11-6. Also shown in Figure 11-6 is the schematic for boundary condition, rotations (R_y, R_z) and the displacements (U_x, U_y, and U_z), which were all set to zero. The pressuremeter membrane was set to be in the middle of the mesh, and the mesh domain in all directions was set to be sufficiently large so that the outside boundaries have minor influence on the numerical results. Also, the mesh was designed so that the density of elements is greatest in the regions of high stresses. Generating the adequate mesh for this model required a total of a 37,120 elements with a total of 41,123 nodes. Analysis carried out with coarse and fine meshes shows that the mesh shown in Figure 11-5 provides a sensible degree of both sensitivity and computational efficiency. The pressuremeter was assumed to be always in contact with the surrounding transversely isotropic rock mass.

The origin of the test was assumed to be the in-situ horizontal stress \( p_x \). From this point, the pressure increments were applied uniformly to the mesh elements which represent the rock adjacent to the pressuremeter membrane until the required degree of expansion was attained.
11.4.2 Finite Element Parametric Study Results and Discussions

In a manner analogous to the soil identification coefficient rules presented by Baguline (1982), the transversely isotropic rock identification coefficient \( \beta \), as defined in equation (11-17), was assumed to be related to the nature of the rock. As a result, the shape of the corrected pressuremeter curve may be characterized by the transversely isotropic rock identification coefficient \( \beta \).

Therefore, as a basis for the FE parametric study to establish possible correlations between the limit pressure \( p_l \) and \( p_{20} \) (i.e., pressure at 20% volumetric strain), the five elastic constants, \( E, E', \nu, \nu' \), and \( G' \) and a parameter \( \theta \) characterizing the orientation of the plane of anisotropy in relation to the axis of membrane (or borehole’s axis of symmetry) were also included. The range of each parameter studied was the same as the one used to determine \( \eta \) (lateral resistance factor) and can be documented in Table 11-1.

The parametric study results were plotted in Figure 11-7 and Figure 11-8. Figure 11-7 shows that the ratio \( p_l / p_{20} \) tends to increase as the transversely isotropic rock identification coefficient number (\( \beta \)) increases. Also, the existence of a certain relationship between the studied parameters can be seen in Figure 11-7. The nature of this relationship is further examined by using some curve fitting techniques and is found to be best represented by the following empirical equation:

\[
 p_l = \frac{\frac{4}{5} p_{20}}{0.9 - \frac{7}{5} \beta^{3.5}} 
\]  

(11-18)
In the same manner, the possible relation between the $p_i$ and the $p_s$ (pressure at 5% volumetric strain) was further examined and the results are shown in Figure 11-8. The best equation describing such a relationship is given by;

$$p_i = \frac{\frac{5}{6} p_s}{1 - 6/5\beta}$$

(11-19)

By adopting equation (11-16) in combination with either Eq. (11-18) or Eq. (11-19) for the estimation of the ultimate resistance of the transversely isotropic rock, the estimation of the main ingredients of a p-y criterion (i.e. $K_T$, and $p_u$) can be accomplished for the rock mass that exhibits the transverse isotropy behavior through extrapolation of the in-situ pressuremeter test results.

11.5 EFFECT OF CRITICAL DEPTH, $Z_C$

The approach described above was applied to the pressuremeter test conducted at some depth below the ground surface. However, close to surface, the displacements of both the laterally loaded drilled shaft and the expanding pressuremeter can be affected by the surface effect. When a shaft is loaded laterally to failure, the zone just below the surface suffers a resistance reduction due to the lack of constraint cased by stress free ground surface. This reduction extends to the critical depth $z_c$ below which the surface effect becomes negligible. The critical depth $z_c$ was found to be in the order of 2D for cohesive soils and 4D for granular soils. In this study, the critical influence zone as well as the associated reduction factor for the transversely isotropic rock was studied by estimating
the ultimate resistance $p_u$ at different depth to diameter ratio ($Z/D$). The study revealed that the ultimate resistance $p_u$ should be reduced in order to account for the surface effect from the ground surface to a depth equal to the 1.2 shaft diameter ($z_c = 1.2D$) as can be illustrated in Figure 11-9. Within the critical depth, $z_c$, the corrected ultimate resistance $(p_u)_c$ is less than the ultimate soil resistance $p_u$ and the best correlation for this difference is given in the following Equation;

$$(p_u)_c = (0.82z_c/z) p_u \quad \text{for } z \leq 1.2D \quad (11-20)$$

where $(p_u)_c$ is the corrected ultimate resistance, $z$ is the depth below the ground surface, and $D$ is the shaft diameter.

The critical influence zone assumed to have the same effect on the initial tangent to the p-y curves (i.e., subgrade reaction $K_T$) and a reduction correction correlation similar to the correction correlation of the ultimate resistance is suggested as follow;

$$(K_T)_c = (0.82z_c/z) K_T \quad \text{for } z \leq 1.2D \quad (11-21)$$

where $(K_T)_c$ is the corrected initial tangent to the p-y curves.
11.6 TRANSVERSELY ISOTROPIC ROCK IDENTIFICATION NUMBER $\beta$

As part of this work, the finite element parametric study result was further used to establish a possible systematical empirical correlation that is capable of describing the transversely isotropic rock identification number ($\beta$) as a function of the transverse isotropy rock prosperities. Mainly, the five elastic constants ($E$, $E'$, $\nu$, $\nu'$, and $G'$) and the dip angle ($\theta$) presented as $\sin \theta$, as shown in the following expression,

$$\beta \propto f(E, E', G', \nu, \nu', \theta)$$

The effect of varying each single parameter on $\beta$ is presented in Figure 11-10(a through f). From Figure 11-10(a & b) it can be illustrated that $\beta$ value decreases as $E$ and $E'$ are increased and slightly decreases as the Poisson’s ratios increase as shown in Figure 11-10 (d & e). The increase in $\beta$ is noticed with the increase of the shear modulus $G'$ (Fig. 11-10-c) and with the increase of dip angle ($\theta$) up to almost $\theta = 65^\circ$ ($\sin \theta = 0.9$). Thereafter, it can be seen that $\beta$ decreases with increasing $\sin \theta$ for the range between 0.9 and 1 ($\theta$ between 65° and 90°) as shown in Figure 11-10-f.

The relative importance calculated by dividing the absolute value, $|\beta_{\text{max}} - \beta_{\text{min}}|$, as affected by each individual parameter, by the summation of the absolute of all values, $\sum |\beta_{\text{max}} - \beta_{\text{min}}|$, as affected by all parameters are summarized in Table 11-2. The order of importance of the parameters affecting $\beta$ can be ranked from high to low as follows: $E'$, $G'$, $\theta$, $E$, $\nu'$, and $\nu$. 

XI-18
It is essential to indicate that the values of $E$, $E'$, and $G'$ plotted in Figure 11-10 are normalized values as defined in Equation (11-14).

The computational results of the parametric study were analyzed by using a statistical analysis software SPSS. From the regression analysis, the empirical equation for estimating $\beta$ is best given by Equation (11-22).

$$
\beta = \frac{1}{3} \left( \left( \frac{0.65 + 0.15 \sin \theta}{1 - 0.23 \sin \theta} \right) \left( \frac{E_n^{0.001}}{0.55 + 0.2 e^{-0.75 G_n}} \right) \left( \frac{e^{-0.15(v'+v')}}{0.75 + 0.015(E_n')^{3/4}} \right) \right)
$$

Equation (11-22)

A comparison between the parametric results for $\beta$ and the empirical prediction of $\beta$ using Equation (11-22) was statistically examined and plotted in Figure 11-11. From Figure 11-11 it can be seen that the data are banded in a good manner around the equality line and a measure of goodness R-squared was found to be in the order of 0.92. An interesting illustration from Figure 11-11 is that most of the $\beta$ values, (72%), are within the range of $60% \leq \beta \leq 67%$ which is within the range of very dilatant sand stated earlier by Baguelin (1982). In addition, within these ranges of $\beta$ values, the magnitude of the plastic strain as can be illustrated from the contour plots in Figure 11-12 do not exceed 0.01 at $p_{20}$ (the pressure corresponds to a volumetric strain of $20\%$). The indication of this is that the rock plasticity has minor effect on $\beta$ and that a reasonable estimate of rock identification value ($\beta$) can be obtained from Eq. (11-22). It is also worthy to articulate that theoretically the maximum characteristic values of $\beta$ can't exceed 75% where the general shape of the corrected pressuremeter curve, ($\Delta p, \Delta V/V_0$) curve, is represented by a linear relationship (elastic behavior).
11.7 STEPS FOR CONSTRUCTING HYPERBOLIC P-Y CURVE FROM PRESSUREMETER TEST

As stated earlier, constructing p-y curve based on a hyperbolic mathematical representation is a demanding process. It necessarily requires the determination of $K_r$ (i.e., subgrade modulus) and the ultimate resistance $p_u$.

Based on the work presented in this study, the following “step by step” procedure is proposed to obtain a hyperbolic p-y curve for a drilled shaft socketed into a transversely isotropic rock median.

Step 1. Given the five elastic constants of the transverse media and the dip angle as well, determine the lateral resistance factor $\eta$ using Equation (11-15).

Step 2. Estimate $K_r$ as stated in Equation (11-13) and do the correction if $Z<1.2D$ as stated in Eq. 11-21.

Step 3. Conduct a pressuremeter test at the desired.

Step 4. Based on the availability of data obtained from step 3, estimate the limit pressure $p_l$ from the pressuremeter test result. If not possible then use Equation (11-18) and if not reachable, (rarely possible), then use Equation (11-19) after estimating $\beta$ value from Equation (11-22).

Step 5. Estimate ultimate pressure $p_u$ using Equation (11-16).

Step 6. Construct the hyperbolic p-y curve.
11.8 VALIDATION OF THE PROPOSED METHOD THROUGH COMPARISONS WITH CASE STUDIES:

11.8.1 Dayton Test Results

For the validation purpose, the relevant data related to an actual full-scale lateral load test at the Dayton site documented in SJN 134137 (Nusairat et al. 2006) is used. Some of the relevant information associated to this site test are summarized herein. The Young modulus of the tested drilled shaft is estimated to be 27.48 GPa with the Poisson’s ratio of 0.25 and moment of inertia $I_p = 0.549 \text{ m}^4$. The test shaft is 1.82 m in diameter with socketed length to diameter (Ls/D) of 3. The rock is classified as Ohio shale. The unconfined compressive strength for the intact core of this shale was estimated to be 39.08 MPa and the elastic modulus of the shale was estimated to be 4.1 GPa.

To signify the assumed transversely isotropic of the rock mass, some judgments are needed to interpret the required input parameters. For these analyses, the modulus of elasticity in the plane of transverse isotropy is taken to be equal to the elastic modulus of the gray shale ($E=4.1 \text{ GPa}$), while $E/E'$ ratios is estimated to be 1.2 based on typical values of $E/E'$ of Shale documented in Chapter X. Also, a typical value of Shale Poisson’s ratio was used ($\nu = 0.25$, and $\nu' = 0.18$). Shear modulus $G'$ was estimated to be 1.5 GPa using Equation (10-10).

The orientation of plane of transversely isotropy (i.e., $\theta$) was set to be $\theta=30$. Using the above educated assumptions as input parameters in the finite element simulation, the pressuremeter test results required to generate the p-y curves were obtained from this
simulation and summarized in Table 11-4.

Using the initial modulus of subgrade reaction, and the ultimate lateral resistance summarized in Table 11-4, the hyperbolic p-y curves at two different depths ($Z_1=0.9$ m, and $Z_2=3.3$ m) are generated as shown in Figure 11-13. The generated p-y curves are fed into LPILE computer program (Reese, et al., 2004) to compute the response of the test shaft under the applied lateral loads.

The predicted load-deflection curve at the shaft head is compared with the measured in Figure 11-14. Although there is some difference between the two curves particularly at the small load levels, in general a good agreement between the measured and the predicted can be observed. As revealed in Figure 11-15, a good match of the maximum moments in the drilled shaft at various lateral load levels applied at shaft head is achieved between the measured and computed.

11.8.2 WAR-48-2102 Test Results

The relevant information associated with the full-scale lateral load test conducted at this site are taken from SJN 134137 (Nusairat et al. 2006) and summarized herein. The Young modulus of the tested drilled shaft is estimated to be 27.58 GPa with Poisson’s ratio of 0.25 and moment of inertia $I_p = 0.11$ m$^4$. The test shaft is 1.22 m in diameter with rock socketed length $L_s$ of 7 m. The rock is classified as Ohio Shale. The unconfined compressive strength for the intact core of this Shale was estimated to be 1.45 MPa and the elastic modulus of the Shale was estimated to be 0.25 GPa based on three tested samples.
For these analyses, the modulus of elasticity in the plane of transverse isotropy is taken to be equal to the maximum elastic modulus of the three Shale samples tested (E=0.4 GPa), while E’ is assumed to be the minimum elastic modulus of the three Shale samples tested (E’=0.16 GPa). These assumptions were made to represent the assumed transversely isotropic of the rock mass. Also, Poisson’s ratio \( \nu = 0.39 \), based on laboratory testing, was used and \( \nu / \nu' \) was assumed to be equal to \( E/E' \) which gives \( \nu' = 0.15 \). Shear modulus \( G' \) was estimated to be 0.08 GPa using the empirical Equation (10-10) and the orientation of plane of transversely isotropy (i.e., \( \theta \)) was set to be \( \theta = 30^\circ \).

Using the above educated assumptions in conjunction with the three pressuremeter test result conducted on site at depths of 6.2 m, 7.6 m, and 8.5 m as summarized in Table 11-5, the p-y curves were generated accordingly. The p-y curves deduced from these pressuremeter tests are illustrated in Figure 11-16. The generated p-y curves are fed into LPILE computer program (Reese, et al., 2004) to compute the response of the test shaft under the applied lateral loads.

The predicted load-deflection curve at the shaft head is compared with the measured in Figure 11-17 for tested shaft #31. Regardless the minor discrepancy at the middle of the curve, a good match at the start and at the end of the test can be seen from Figure 11-16.

11.8.3 JEF-152-1.3 Test Results

The relevant data related to an actual full-scale lateral load test at Jefferson-152 site reported in SJN 134137 (Nusairat et al. 2006) is used. Some of the relevant information associated to this site test are summarized herein. The Young modulus of the tested
drilled shaft is estimated to be 27.50 GPa with Poisson’s ratio of 0.25 and moment of inertia \( I_p = 0.064 \text{ m}^4 \). The test shaft is 1.07 m in diameter with socketed length (Ls) of 4.5 m. To isolate the overburden soil and to fully mobilize the rock-shaft interaction, a 1.83 m diameter casing was used to form a space between the tested drilled shaft and the soil above the rock. The rock is classified as Mudstone and the unconfined compressive strength for the intact core of this Mudstone was estimated to be 0.69 MPa. The elastic modulus of the Mudstone was estimated to be 0.1 GPa.

To represent the assumed transversely isotropic of the rock mass, some judgments are needed to interpret the required input parameters. For these analyses, the modulus of elasticity in the plane of transversely isotropy is taken to be equal to the elastic modulus of the gray Shale (E=0.1 GPa), while E/E’ ratios is estimated to be 1.0 similar to isotropic-like rock situation. Also, a typical value of Mudstone Poisson’s ratio was used (\( \nu \) and \( \nu' = 0.25 \), to indicate isotropic rock). Shear modulus \( G' \) is estimated to be 0.05 GPa using Equation (10-00).

Using the two pressuremeter test results performed on site at depths of 8 m and 9.6 m summarized in Table 11-5, the p-y curves at JEF-152-1.3 were generated according to the procedure outlined. The p-y curves deduced from these pressuremeter tests are illustrated in Figure 11-18. The generated p-y curves were fed into LPILE computer program (Reese, et al., 2004) to compute the response of the test shaft under the applied lateral loads.

The predicted load-deflection curve at the shaft head is compared with the measured as shown in Figure 11-19. From Figure 11-19 it can be seen that a good agreement is
achieved between the predicted and the measured load-deflection curves, particularly at the small load levels. At a higher load level, the predicted deflection at the top of the pile is larger than the measured deflection for the same load level. Still, the predicted deflection is within a reasonable tolerance where the bias, which is the experimentally measured deflection at a particular load level divided by the predicted deflection at the same load level, a does not go below 0.82 at the maximum applied load of 700 kN.

11.8.4 Illustrative Example

A case study presented in Chapter VII which was solved with FE numerical simulation for the drilled shaft socketed into transversely isotropic rock was used here to demonstrate the use of the developed method for deriving the p-y curve of transversely isotropic rock. For this case, the Young modulus of the tested drilled shaft shown in Figure 11-20 is assumed to be 10 GPa with Poisson’s ratio of 0.25. The test shaft is 1 m in diameter with 5 m rock socket length.

For the transversely isotropic rock properties assumed for this hypothetical case, the five elastic constants used are (E=4.9 GPa, E’=4.6 GPa, ν=0.25, ν’=0.24, G’=1 GPa). Numerical FE simulations for the pressuremeter of this case were carried out. The findings of the pressuremeter simulation carried out in this study and the findings of drilled shaft simulation carried in Chapter VII are summarized in Table 11-6.

The hyperbolic p-y curves of this case study are generated and plotted in Figure 11-21. The generated p-y curves were input into LPILE computer program to compute the response of the test shaft under the applied lateral loads. The predicted load deflection
curve at the top of the shaft and the maximum moment versus the applied load are compared in Figure 11-22 and Figure 11-23, respectively. From these comparison figures, a good match between the two approaches can be clearly seen.

11.9 SUMMARY AND CONCLUSIONS

In this chapter, a hyperbolic p-y criterion for transversely isotropic rock that can be deduced from the pressuremeter test was developed. The methods for determining the necessary parameters required to satisfy steps to construct the p-y curves (i.e., $K_r, p_u$) were presented in this chapter. As part of estimating $K_r$ (Subgrade Modulus) using the pressuremeter test, a semi-empirical method for estimating the lateral resistance factor of a transversely isotropic rock (i.e., $\eta$) was also presented in this study. Based on FE parametric study of pressuremeter test in the transversely isotropic rock media, the method for estimating parameters for the determination of ultimate resistance $p_u$ from pressuremeter test (i.e., $p_r$) was empirically developed. Also, based on the FE parametric study, an empirical estimation for the transversely isotropic Identification Index $\beta$ in terms of the five elastic constants and the dip angle $\theta$ was presented in this study. In addition, a step by step procedure for constructing the hyperbolic p-y curves from the pressuremeter test was summarized herein. To validate the proposed methodology, four different case studies were conducted. The measured load test data at Dayton, Warren (WAR-48), and Jefferson (JEF-152-1.3) test sites were compared to the predicted shaft response based on the p-y curve criterion and the estimate of the on-site rock properties. Also, the outcome of the hypothetical case study presented in Chapter VII was compared.
with the results obtained in this chapter. The comparisons of the outcome support the applicability of the proposed p-y curve criterion. On the basis of the work presented in this chapter, the following conclusions may be drawn:

1. Transverse isotropy has a major influence on the lateral resistance factor of a transversely isotropic rock. The larger the elastic constants (i.e., $E$, $E'$, and $G'$), the stiffer the rock and the larger the lateral resistance factor $\eta$.

2. For the initial tangent to the p-y curves ($K_T$), the larger the resistance factor $\eta$ the greater the initial tangent to the p-y curve.

3. The pressuremeter limit pressure $p_l$, may be reasonably predicted from the earlier pressure states (i.e., $p_{20}$, and $p_5$) in conjunction with the rock identification number $\beta$. The higher the $\beta$, the higher the limit pressure $p_l$.

4. Varying the elastic constants of the transversely isotropic rock would influence the rock identification number $\beta$. In addition, the majority of $\beta$ values of the transversely isotropic rock are within $60\% \leq \beta \leq 67\%$.

5. Based on the case studies, it can be concluded that the proposed methodology for p-y curves gives a good prediction of the behavior of the laterally loaded shafts socketed into a transversely isotropic rock.
Table 11-1: Parameters variation for the determination of lateral resistance factor $\eta$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base Line Value*</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (GPa)</td>
<td>15</td>
<td>5-50</td>
</tr>
<tr>
<td>$E'$ (GPa)</td>
<td>10</td>
<td>0.5-50</td>
</tr>
<tr>
<td>$G'$ (GPa)</td>
<td>2</td>
<td>0.5-20</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>0.05-0.5</td>
</tr>
<tr>
<td>$\nu'$</td>
<td>0.15</td>
<td>0.005-0.3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>45</td>
<td>$0^\circ$-$90^\circ$</td>
</tr>
<tr>
<td>$D/D_{ref.*}$</td>
<td>1</td>
<td>0.25-4.0</td>
</tr>
<tr>
<td>$\gamma_{rock}$ (kN/m³)</td>
<td>18</td>
<td>10-25</td>
</tr>
</tbody>
</table>

$D_{ref.} = 1.0$ (m)

Table 11-2: Importance ranking of factors affecting lateral resistance factor $\eta$

| PARAMETER | $|\eta_{max} - \eta_{min}|$ | $\left(\frac{|\eta_{max} - \eta_{min}|}{\sum|\eta_{max} - \eta_{min}|}\right) \times 100\%$ |
|-----------|-----------------------------|--------------------------------------------------|
| $E$       | 0.81                        | 11.2                                             |
| $E'$      | 1.26                        | 17.4                                             |
| $G'$      | 2.03                        | 28.0                                             |
| $\Theta^\circ$ | 0.98                        | 13.5                                             |
| $N$       | 0.43                        | 5.9                                              |
| $N'$      | 0.62                        | 8.6                                              |
| $D$       | 1.12                        | 15.4                                             |
| $\gamma_{rock}$ | $\approx 0.00$            | 0.00                                             |

$\Rightarrow \Sigma$ 7.25 100
Table 11-3: Importance ranking of factors affecting transversely isotropic rock identification number $\beta$

| PARAMETER | $|\beta_{\text{max}} - \beta_{\text{min}}|$ | $\frac{\sum|\beta_{\text{max}} - \beta_{\text{min}}|}{100\%}$ |
|-----------|--------------------------------|----------------------------------|
| $E$       | 0.018                         | 2.3                              |
| $E'$      | 0.316                         | 40.1                             |
| $G'$      | 0.234                         | 29.7                             |
| $\Theta^\circ$ | 0.196                   | 24.9                             |
| $N$       | 0.011                         | 1.4                              |
| $N'$      | 0.013                         | 1.6                              |
| $\Rightarrow \sum$ | 0.788                    | 100                              |

Table 11-4: Summary of the parameters induced from the Dayton site

<table>
<thead>
<tr>
<th>Case</th>
<th>Depth</th>
<th>$\eta$</th>
<th>$\beta$</th>
<th>$p_{20}$ (MPa)</th>
<th>$p_{t}$ (MPa)</th>
<th>$K_T$ (MPa)</th>
<th>$p_u$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.36</td>
<td>52</td>
<td>32.85</td>
<td>57.78</td>
<td>1005.2</td>
<td>42641.6</td>
</tr>
<tr>
<td>2</td>
<td>3.3</td>
<td>0.36</td>
<td>52</td>
<td>32.89</td>
<td>57.78</td>
<td>2396.5</td>
<td>105159.5</td>
</tr>
</tbody>
</table>
Table 11-5: Summary of pressuremeter/Dilatometer test results at Jefferson (JEF-152) and Warren (WAR-48) sites and the p-y parameters induced

<table>
<thead>
<tr>
<th>Site</th>
<th>Depth (m)</th>
<th>$K_i$ (MPa)</th>
<th>$p_i$ (MPa)</th>
<th>$K_T$ (MPa)</th>
<th>$p_u$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JEF-152</td>
<td>8.0</td>
<td>52.093</td>
<td>4.895</td>
<td>62.512</td>
<td>5218</td>
</tr>
<tr>
<td></td>
<td>9.6</td>
<td>45.862</td>
<td>6.239</td>
<td>55.034</td>
<td>6675</td>
</tr>
<tr>
<td>WAR-48</td>
<td>6.2</td>
<td>30.400</td>
<td>12.824</td>
<td>51.072</td>
<td>15646</td>
</tr>
<tr>
<td></td>
<td>7.6</td>
<td>32.577</td>
<td>11.721</td>
<td>54.729</td>
<td>14299</td>
</tr>
<tr>
<td></td>
<td>8.5</td>
<td>53.428</td>
<td>15.921</td>
<td>89.760</td>
<td>19423</td>
</tr>
</tbody>
</table>

Table 11-6: Summary of the illustrative example findings

<table>
<thead>
<tr>
<th>Case</th>
<th>$\eta$</th>
<th>$\beta$ %</th>
<th>$p_{20}$ (MPa)</th>
<th>$K_i$ (GPa)</th>
<th>$K_T$ (GPa)</th>
<th>$p_u$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This Study</td>
<td>1.11</td>
<td>50</td>
<td>14801.9</td>
<td>0.205</td>
<td>0.91</td>
<td>25424.5</td>
</tr>
<tr>
<td>Shatnawi (2008)</td>
<td>0.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23615.6</td>
</tr>
</tbody>
</table>
Figure 11-1 Schematic Representation to Illustrate the Similarity of Drilled Shaft p-y Curves and the Pressuremeter Curves.
Figure 11-2 Section in a Transversely Isotropic Rock with the Directional Deformation Constants in a Cartesian Coordinate System.
\[ \eta = 2 - 0.0038 E_n - \frac{22}{E_n} \]

\[ E_n = \frac{E(kPa)}{10^4 \rho_n (kPa)} \]

\[ \eta = 2.3 - \frac{4.6}{e^{1.5(E_n)^{0.5}}} \]

\[ E_n = \frac{E(kPa)}{10^7 \rho_n (kPa)} \]
\( \eta = \frac{2.85 G''}{7.1 + G''} \)

\( G'' = \frac{G' (kPa)}{10^{-5} p_e (kPa)} \)

\( \eta = \frac{2.18}{e^{0.48 \nu}} \)
\[ \eta = \frac{2.26}{e^{1.32\nu'}} \]

\[ \eta = 0.91 e^{\sin\theta} \]
Figure 11-3 Effects of transversely isotropic parameters on lateral resistance factor $\eta$.

- (a) $E_n$ effect
- (b) $E_n'$ effect
- (c) $G_n$ effect
- (d) Poisson's ratio $\nu$ effect
- (e) Poisson's ratio $\nu'$ effect
- (f & g) $\theta$ effect
- (h) Shaft diameter effect.

Mathematical expressions:
- $\eta = 2 - 33.7e^{\frac{4}{6\sin\theta}}$
- $\eta = 0.7e^{0.24D}$
Figure 11-4 Comparison of parametric study predicted $\eta$ and empirical predicted from Equation (11-15)
Figure 11-5 FE mesh of pressuremeter in rock media.
Figure 11-6 Schematic of the FE mesh and the boundary conditions (a) Front view (b) Top view
Figure 11-7 A plot of rock identification number, $\beta$, vs. the ratio $p_I / p_{20}$. 

The equation for the plot is:

$$p_I = \frac{4/5 p_{20}}{0.9 - 7/5 \beta^{3.5}}$$
Figure 11-8 A plot of rock identification number, $\beta$, vs. the ratio $p_1/p_5$. 

\[ p_1 = \frac{\frac{5}{6} p_5}{1 - 6/5 \beta} \]
Figure 11-9 Drilled shaft correction factor diagram

Reduction Factor, \( \left( \frac{p_u}{p_u} \right)_c \)
\[ E_n = \frac{E_n(kPa)}{10^4 p_n(kPa)} \]

\[ \beta = 0.65 E^{0.005} \]

\[ \beta = \frac{1}{1.5 + 0.0035 E^{11.54}} \]
\[ \beta = \frac{0.62}{1 - 0.05e^{-1.71G_n}} \]

\[ G_n = \frac{G'(kPa)}{10^5 p_c(kPa)} \]

\[ \beta = 0.65 - 0.025\nu \]
Figure 11-10 Effects of transversely isotropic parameters on rock identification number

$\beta$ (a) $E_n$ effect (b) $E'_n$ effect (c) $G_n$ effect (d) Poisson’s ratio $\nu$ effect (e) Poisson’s ratio $\nu'$ effect (f) $\theta$ effect.
Figure 11-11 Comparison of the FE parametric prediction of $\beta$ and the empirically predicted $\beta$ from Equation (5-18)
Figure 11-12 Plot of magnitude of plastic strain contours corresponds to $P_{20}$. 
Figure 11-13 Hyperbolic p-y curves of Dayton site
Figure 11-14 Comparison of load-Top shaft deflection of tested shaft at Dayton site.
Figure 11-15 Comparison of load-Maximum moment of test shaft #4 at Dayton load test
Figure 11-16 Hyperbolic p-y curves of Warren (WAR-48-2102 shaft# 31) site induced from pressuremeter tests.
Figure 11-17 Comparison of load-Top shaft deflection of tested shaft #31 at Warren county (WAR-48-21.02).
Figure 11-18 Hyperbolic p-y curves of Jefferson (JEF-152-1.3) site induced from pressuremeter tests
Figure 11-19 Comparison of load-Top shaft deflection of tested shaft #1 at Jefferson site.
D = 1m, H = 7 m

E = 30 GPa
E' = 20 GPa
G' = 10 GPa

ν = 0.3
ν' = 0.2

Figure 11-20 Schematic of the illustrative example
Figure 11-21 Comparison of p-y curves induced from pressuremeter (PM) based criteria with the one based on laboratory testing documented in Chapter VII for the illustrative hypothetical example.
Figure 11-22 Comparison of load-Top shaft deflection of the illustrative example.
Figure 11-23 Comparison of load-Maximum moment of the illustrative example.
CHAPTER XII: VERIFICATION OF p-y CRITERION FOR WEAK ROCK
PROPOSED IN SJN 134137

12.1 INTRODUCTION

During the course of this research project, verification of the p-y criterion proposed in SJN 134137 was conducted. One lateral load test conducted on short shafts in limestone was conducted by Kansas Department of Transportation (KDOT) in 2009. The lateral load test details and results are presented in the following section. The hyperbolic p-y criterion proposed in SJN 134137 was used to predict the movement at the top of the shaft. The p-y criterion seems to predict the load-deflection behavior of the shaft reasonably.

12.2 KDOT K-TRAN: KU-09-6 LATERAL LOAD TEST

Kansas Department of Transportation (KDOT) conducted full scale lateral load testing on two short rock socketed shafts in limestone, for the purpose of the development of recommendations for p-y analysis using those results.

12.2.1 Test Site

The shafts were constructed in the northeast quadrant of the intersection of I-70 and I-435 in Wyandotte County, Kansas. The shafts were constructed in the fall of 2007 and tested in the summer and fall of 2009. The shafts were set in the Plattsburg Limestone and spaced 144 inches apart center to center.

Borings were taken near the shaft locations on July 11, 2007. Rock cores were
inspected for RQD and samples were tested for unconfined compressive strength. The site geology consisted of minimal to no soil overburden, 1.5-2.5 feet of weathered to hard sandstone over hard limestone. The overburden and sandstone were removed so the sockets were entirely in limestone. Table 12-1 documents the rock testing data at elevations relevant to the test shafts.

Table 12-1: Rock Core Test Data used for Analysis

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Depth (Ft)</th>
<th>Unconfined Compression $q_u$ (psi)</th>
<th>Elastic Modulus $E$ (ksi)</th>
<th>Dry Density $y_d$ (pcf)</th>
<th>Moisture Percent w%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upper Layer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14-1-2</td>
<td>3.33</td>
<td>799</td>
<td>292</td>
<td>154.4</td>
<td>2.7</td>
</tr>
<tr>
<td>15-1-3</td>
<td>3.33</td>
<td>2701</td>
<td>448</td>
<td>149.5</td>
<td>4.3</td>
</tr>
<tr>
<td><strong>Lower Layer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14-2-1</td>
<td>4.05</td>
<td>4458</td>
<td>958</td>
<td>150.6</td>
<td>3.7</td>
</tr>
<tr>
<td>14-2-2</td>
<td>7.03</td>
<td>7778</td>
<td>1333</td>
<td>157.5</td>
<td>2.1</td>
</tr>
<tr>
<td>15-2-1</td>
<td>4.30</td>
<td>5979</td>
<td>1118</td>
<td>156.9</td>
<td>2.7</td>
</tr>
<tr>
<td>15-2-2</td>
<td>5.30</td>
<td>5056</td>
<td>1042</td>
<td>156.0</td>
<td>3.1</td>
</tr>
<tr>
<td>15-2-3</td>
<td>6.95</td>
<td>4778</td>
<td>660</td>
<td>152.2</td>
<td>4.6</td>
</tr>
</tbody>
</table>

*Information from KDOT Report

12.2.2 Shaft Details

The shafts were 42 inches in diameter and cast in sockets approximately six feet deep for the north shaft and seven feet deep for the south shaft. Shaft reinforcement consisted of twelve #11 longitudinal bars and hoops made of #5 bars on with one foot spacing within the socket and a spacing of approximately 6 inches above ground at the point of load application. The load was applied approximately one foot above ground level. Concrete was KDOT standard drilled shaft mix with a compressive strength of 7500 psi.

12.2.3 Testing

Load was measured using two separate systems, load cells and hydraulic pressure. The
hydraulic pressure was monitored by gauge and by pressure transducer. The load cells were limited to a capacity of 400 kips and served as a backup to the pressure transducer and gauge. Deformation was measured at two locations on each shaft with UniMeasure P510 string pots fixed to reference beams and inclinometer measurements in each shaft. Pressure transducer, string pot, and load cell data was recorded automatically on a laptop computer. Photogrammetry was used as a backup system. Pressure transducer and string pot information was recorded by a laptop and data acquisition system. Inclinometer data was recorded by KDOT personnel with a data logger prior to each test and after each set of load cycles.

Lateral load testing was conducted a part of three separate tests. The first test was conducted July 29, 2009 and consisted of cyclic (load reversal) testing up to 400 kips for a series of primary load increments, where 400 kips was the maximum load that could be achieved in both directions with the equipment configuration used. The equipment was configured such that essentially two separate load frames could load the shafts in opposite directions simultaneously. One set of equipment with three 200 kip hydraulic cylinders was used to jack the shafts apart, and a second set with two 200 kip cylinders was used to pull the shafts together (Figure 3.4). Cycles of loading were applied to the shafts by alternating loading between these sets of equipment. Five or ten cycles were applied at each primary load increment. Additional measurements were taken at intermediate increments.

The second test was conducted on November 10, 2009. For this test the equipment was reconfigured so that all five cylinders could be used together to load the shafts to failure.
as shown in Figure 3.5. Repeated loads were applied at 600 and 800 kip load levels with 10 cycles at each load step. As loading continued above 800 kips, one of the loading beams began to yield, forcing the test to be stopped. The yielding beam was reinforced and the test was restarted on December 21, 2009. Loading proceeded to failure at approximately 1,000 kips for both shafts.

![Figure 12-1 Test 1 setup](image-url)

Figure 12-1 Test 1 setup
12.2.4 Lateral Load Test Results

Three separate test events were conducted on the shafts as described earlier. Figure 12-3 and Figure 12-4 show the deformation for each test event as measured by the top string pots. These figures both show increasing rates of deflection with load to failure, which occurred at approximately 1,000 kips for both shafts. Data for the lower string pots on each shaft were similar.
Figure 12-3 Deflection of the north shaft as measured by the top string pot

Figure 12-4 Deflection of the south shaft with load as measured by the top string pot
12.3 COMPARISON WITH THE RESULTS FROM K-TRAN: KU-09-6

LATERAL LOAD TEST

Mechanical properties of Limestone rock in test site and shaft properties shown in Table 12-2. Figure 12-5 shows comparison between the method proposed in SJN 134137 and the measure movement at the top of the shaft. The comparison was performed for two Geotechnical Strength Index (GSI) values, these are GSI=85 and GSI=100. Figure 12-6 and Figure 12-7 show deflection profile of drilled shaft for GSI=85 and GSI=100. The predicted results using a GSI=100 matches reasonably with measured deflection from the inclinometers.

Table 12-2 Shaft and Rock properties*

<table>
<thead>
<tr>
<th>Shaft Properties</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft Diameter</td>
<td>42 inches</td>
<td></td>
</tr>
<tr>
<td>Concrete Strengths</td>
<td>7500 psi</td>
<td></td>
</tr>
<tr>
<td>Longitudinal Reinforcement</td>
<td>12 - #11 bars</td>
<td></td>
</tr>
<tr>
<td>Distance from pile top (point of loading) to ground surface</td>
<td>12 inches</td>
<td></td>
</tr>
<tr>
<td>Yield stress of steel</td>
<td>60,000 psi</td>
<td></td>
</tr>
<tr>
<td>Steel modulus</td>
<td>29,000,000 psi</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rock Properties</th>
<th>Upper Layer</th>
<th>Lower Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact Rock Strength</td>
<td>1750 psi</td>
<td>5068 psi</td>
</tr>
<tr>
<td>Intact Rock Modulus</td>
<td>370 ksi</td>
<td>1040 ksi</td>
</tr>
</tbody>
</table>

*Information from KDOT Report
Figure 12-5 KDOT- Comparison of Lateral Load vs. Deflection between measured values and prediction from SJN 134137 method.
Figure 12-6 Deflection-Depth Profile for GSI=85
Figure 12-7 Deflection-Depth Profile for GSI=100
CHAPTER XIII: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR IMPLEMENTATION AND FUTURE STUDY

13.1 SUMMARY

Towards the objective of developing a new p-y criterion of anisotropic rock, extensive theoretical and numerical simulation works have been carried out in this research. A detailed literature review was performed to study the existing design and analysis methods of the laterally loaded drilled shafts in rock. A new hyperbolic p-y criterion of transversely isotropic rock and cohesive IGM was developed based on the results of field test data and extensive theoretical work.

Throughout this study, the five elastic constants of a transversely isotropic media (E, E’, G’, ν, and ν’) were varied. The selection of ranges examined for the elastic constants were carefully chosen to include almost all rock types covered in the available literature. The ranges examined in this work are summarized in Table13-1.

Table 13-1: Parameters Ranges for the parametric study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>E(GPa)</th>
<th>E’(GPa)</th>
<th>G’(GPa)</th>
<th>ν</th>
<th>ν’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>5-50</td>
<td>0.5-50</td>
<td>0.25-20</td>
<td>0.05-0.3</td>
<td>0.005-0.3</td>
</tr>
</tbody>
</table>

For a rock with transversely isotropic properties, an empirical formulae for estimating the shear modulus (G’) was developed to simplify the characterization procedures of the elastic properties for this type of rock. For jointed rock, an equivalent transversely isotropic homogeneous model to describe the jointed rock’s stress-strain behavior was
developed to obtain the equivalent five transversely isotropic elastic constants.

3D FE models simulation of the response of the laterally loaded drilled shaft in rock using ANSYS computer program was established to develop the pertinent methods that can be used to estimate the main two parameters required to characterize a hyperbolic p-y curve: the subgrade modulus \( (K_i) \) and the ultimate resistance \( (p_u) \). The first set of FE parametric study was performed for a drilled shaft socketed into a transversely isotropic media to develop a series of design charts for estimating the initial tangent to the p-y curve. The second set of FE simulations was undertaken to investigate the effect of various influencing factors on the maximum side shear resistance, including the interface strength parameters, the moduli of the drilled shaft and rock mass, and the drilled shaft geometry. Based on the results of the series of FE parametric study, an empirical equation was developed to estimate the ultimate side shear resistance of the drilled shaft embedded in rock.

Additionally, theoretical equations for determining the ultimate lateral resistance of the transversely isotropic rock was derived based on the identified failure modes of jointed rock and the transversely isotropic rock strength criterion. The failure modes of rock mass were identified through a series of 3D FE study.

The evaluation of the proposed p-y criterion for rock was done by performing a parametric study on several hypothetical cases of a rock socketed drilled shaft under the lateral load. In these cases, a range of parameters differentiating the isotropic vs. transversely isotropic p-y curves was selected in a systematic numerical study using the

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LPILE computer program with the specific p-y curves.

By employing the results of FE simulations of a drilled shaft socketed into cohesive soil, and in conjunction with the empirical equation recommended by Matlock (1970) for estimating the ultimate lateral resistance of cohesive material, a hyperbolic p-y criterion was developed for the cohesive intermediate geomaterial.

Towards the objective of developing a new methodology to obtain p-y curves of the transversely isotropic rock from the pressuremeter test, extensive studies involving theoretical and numerical simulation works was also carried out in this research. A new method for determining the hyperbolic p-y criterion of the transversely isotropic rock from the pressuremeter test was developed based on the FE numerical simulation of the pressuremeter test and the associated theoretical work.

A 3D finite element model simulating the pressuremeter test in a transversely isotropic rock using ABAQUS computer program was established to develop an empirical correlation equation for estimating the elastic constants of a transversely isotropic rock from the pressuremeter test results. Additionally, a practical procedure was presented for determining the elastic constants from $K_i$ (i.e., tangent to the linear portion of the pressuremeter tests).

A comprehensive database of experimentally determined five independent elastic constants for the transversely isotropic rocks was compiled from literature and presented in this report. In addition, possible statistical cross correlations among these elastic constants were statistically investigated. Empirical equations for estimating the shear
modulus of a transversely isotropic rock $G'$, by using the other elastic constants were developed.

By employing the results of parametric studies performed in this research, a methodology for estimating the main ingredient of a hyperbolic p-y curve using the pressuremeter test results for a transversely isotropic rock was developed. The resistance factor $\eta$ was employed as a key parameter in the estimation of $K_T$ (i.e., initial tangent to the p-y curve). Additionally, Based on FE parametric study, methods for estimating parameters governing the ultimate resistance $p_u$ from the limiting pressure of the pressuremeter test result (i.e., $p_l$) was empirically developed for the transversely isotropic rock media. Empirical estimation for the transversely isotropic rock identification Index $\beta$ in terms of the five elastic constants and the dip angle $\theta$ was developed based on the FE parametric study.

Finally, a step by step procedure for constructing a hyperbolic p-y curve from the pressuremeter test result was developed. For the evaluation and validation purposes, four different case studies were presented. The measured load test data at Dayton, (WAR-48-1), and (JEF-152-1.3) test sites were compared to the predicted shaft response based on the p-y curve derived from the method outlined in this report. Also, a hypothetical case study presented in Chapter VII was used for a comparison purpose.
13.2 CONCLUSIONS

Based on the research work performed in this research, the conclusions regarding the work toward the development of p-y criteria for cohesive IGM and rock mass can be enumerated below.

- The 3D FE simulation provided basic understanding of the mobilization mechanisms of the lateral resistance of rock mass to the drilled shaft, allowing the development of analytical equations for computing the ultimate lateral resistance of transversely isotropic rock \( p_u \).

- Based on the sensitivity study, the influences of rock anisotropy on the predicted response of the rock socketed drilled shaft under the lateral load were clearly observed. Both the orientation of the plane of transversely isotropy and the degree of anisotropy \((E/E')\) can exert great influences on the two main parameters required to characterize the hyperbolic p-y curve: the subgrade modulus \( (K_i) \) and the ultimate lateral resistance \( (p_u) \). Moreover, it was shown that if there was no anisotropy, the proposed p-y curve can be reduced to SJN 134137 (Nusairat et al. 2006) p-y curve. Finally, it was shown that the proposed p-y criterion can provide reasonable predictions of the behaviour of the drilled shaft socketed in rock under the applied lateral loads.

- The favourable evaluation of the proposed p-y criterion for the cohesive IGM based on comparisons between the predicted and measured responses of full-scale lateral load tests on the fully instrumented drilled shafts showed the practical uses of the
proposed p-y criterion. The average prediction error of the maximum moment was around 18%.

- The evaluation of the empirical equation for estimating the ultimate side shear resistance using the available load test data showed that the prediction by the proposed method was conservative but with improved accuracy compared with other existing empirical equations. From the statistical analysis results, the proposed empirical equation can improve our prediction capability for the ultimate side shear resistance of a rock socketed drilled shaft.

- The hyperbolic p-y criterion for the transversely isotropic rock developed in this study can be used in conjunction with a computer analysis program, such as COM624P, LIPL, or FB-Multi Pier, to predict the deflection, moment, and shear responses of a drilled shaft embedded in the rock under the applied lateral loads. Considerations of the effects of joints and discontinuities on the rock mass modulus and strength were included in the proposed p-y criterion.

Regarding the research work performed in this research relating to the development of a pressuremeter test interpretation method for p-y curve of the transversely isotropic rock, the following conclusions can be drawn.

- Varying the elastic constants of the transversely isotropic rock can exert a major influence on the pressuremeter test results in the elastic range ($K_i$). This effect can be more significant when varying $E$, $E'$, and $G'$, but less significant when varying Poisson’s ratio $\nu$. 

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It was found that the dip angle $\theta$ can exert a significant effect on the pressuremeter test results. The study revealed that $K_i$ increases with increasing $\sin \theta$ up to a value of 0.79 (i.e., $\theta = 52^\circ$). Thereafter $K_i$ decreases slightly with increasing $\sin \theta$ for the range between 0.79 and 1 (i.e., $\theta$ between 52° and 90°).

Pressuremeter test results can still be evaluated with minimal inaccuracy for $E$ using the existing equation as long as the angle between a perpendicular to borehole’s axis and the line of dip measured in the plane including both the borehole’s axis and the line of the dip $\hat{\theta} \leq 25^\circ$.

Although the five elastic constants of the transversely isotropic rocks are theoretically independent, this study showed the existence of empirical relations among these constants. The newly developed shear modulus prediction equations presented can predict accurately the experimentally measured shear modulus $G'$.

Varying the elastic constants of a transversely isotropic rock can exert major influences on the lateral resistance factor of a transversely isotropic rock, $\eta$. The larger the elastic contestants (i.e., $E$, $E'$, and $G'$) the stiffer the rock and the larger the lateral resistance factor $\eta$. In addition, varying the elastic constants of the transversely isotropic rock can influence the rock identification number $\beta$. The value of $\beta$ would decrease as the values of $E$, $E'$, and Poisson’s ratios (i.e., $\nu$, and $\nu'$) are increased. The increase in $\beta$ can be noticed with the increase of the shear modulus $G'$ and the dip angle ($\theta$) up to $\theta = 65^\circ$. Thereafter, $\beta$ decreases with increasing $\theta$ between 65° and 90°.

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13.3 RECOMMENDATIONS FOR IMPLEMENTATION

The research work presented in this report has resulted in the development of two implementable items. One is the new p-y curve criterion for the transversely isotropic rock mass and the associated PC based computer program. The second implementable item is the method and the associated PC based computer program for interpreting the pressuremeter results conducted in the transversely isotropic rock to obtain the p-y curve. It is recommended that the ODOT adopt the use of the computer programs within the internal design section to validate the robustness and accuracy of these two computer programs. Once sufficient validation cases have been completed to gain enough confidence, and then ODOT could release these two computer programs to the industry to provide them the design and analysis tools for the drilled shafts in the transversely isotropic rock under the lateral loads.

13.4 RECOMMENDATIONS FOR FUTURE STUDIES

- More lateral load tests on the full-scale drilled shafts socketed in various types of rock, with the accompanying pressuremeter tests at the load test sites, should be performed in order to further validate the developed p-y criteria and the pressuremeter interpretation method.

- It is believed that the group effect of drilled shaft socketed into strong rocks is negligible. However, due to the presence of the inclined bedding and joints, the group effects could be significant in the transversely isotropic rock. Therefore, a future study to investigate the relevant p-multipliers for a drilled shaft group in
the transversely isotropic rock could be desirable. This research objective can be accomplished through a well-planned field test program of drilled shafts groups. Additionally, 3D FE study could also be employed to determine the p-multiplier for a drilled shaft group socketed in the transversely isotropic rock.

- Since the required input for the proposed p-y criterion includes the five elastic constants of the transversely isotropic rock mass, and since the indirect pressuremeter tests are the most frequently used in-situ tests for estimating the rock mass modulus, a future study on the use of pressuremeter test to estimate these five elastic constants would be desirable.
REFERENCES


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